The
ANDERSON
ARITHMETIC

BOOK THREE

Mrs. E. H. Jenkins.

1 - P. 246 - 419.

~~2 - 115 - 4~~

~~3 - 222 - 22~~

~~4 - 305 - 29~~

5 - 282 - 30

~~243 - 17~~

6 - ~~244 - 34~~

~~245 - 26~~

7 - ~~247 - 37~~

8 - 1~~61 - 37~~

9 - 305 - 21

251 - 11

Arithmetic. Inv.

1. P. 163. Ex 3
2. P. 163. Ex 5.
3. P. 169. Ex 7.
4. P. 198. Ex 29
5. 236 - 8
6. 246 - 17
7. 246 - 15
8. 254. 5
7. 255 - 14
8. 305 - 26
9. 306 - 39
10. 306 - 40
11. 306 - 36
12. 306 - 41

Alg Exam.

P 305. 21
293. 5

151

THE

ANDERSON ARITHMETIC

BOOK THREE

BY

ROBERT F. ANDERSON, Sc.D.

PROFESSOR OF MATHEMATICS, STATE NORMAL SCHOOL,
WEST CHESTER, PENNSYLVANIA

SILVER, BURDETT AND COMPANY

BOSTON NEW YORK CHICAGO SAN FRANCISCO

PREFACE

In the preparation of this series of arithmetics the author has utilized not only the results and conclusions of his own experience of many years, but also the results and conclusions of scientific investigations, experiments, and modern school practice in this subject. An examination of these books will reveal their adaptation to modern courses of study and methods of instruction.

Book Three, the third of the series, aims, first, to review and to perfect the work of the preceding years involving processes with integers, fractions, and decimals so that the pupils may attain a high standard of efficiency; second, to make provision for the purposeful application of the processes of arithmetic to problems dealing with the common experiences and interests of children; and, third, to extend the problem material so as to include problems involved in the arithmetical requirements common to many vocations.

Part One consists of four chapters and a general review. Chapter I affords sufficient practice in the processes of addition, subtraction, multiplication, and division with integers, fractions, and decimals to insure the results demanded by current standards. The use of the graph in this chapter will awaken anew the pupil's interest in attaining these high standards. The chapter also provides liberally for the application of this arithmetical knowledge, to the end that the pupil may become skillful in its use.

Chapter II consists of a very elementary presentation of the use of letters as numbers, sufficiently adequate, however, to familiarize the pupil with the fundamental operations in examples and problems involving letters. This chapter will also prepare the pupil for the intelligent use of letters in formulæ as well as acquaint him with the method of solving the simplest kind of equation, and with the application of the equation to the solution of simple problems.

The study of the three problems of percentage and their applications, begun in the preceding book, is carried forward in Chapter III, emphasized, extended, and applied so as to provide for the pupil's mastery of these three problems and their applications to trade discount, profit and loss, commission, and simple interest.

Chapter IV thoroughly reviews and applies the tables of measures and introduces the foundation work in the measurement and construction of the simplest geometrical figures demanded by modern courses.

Part Two consists of five chapters and an extensive general review. Chapter I reviews the work contained in Part One. Experience has shown that not only must the processes involving integers, fractions, and decimals including percentage recur with frequency, but that the applications of them must be recurrent and varied if the desired degree of facility and certainty in their application is to be attained.

Chapter II, Banking and Negotiable Papers, and Chapter III, Investments, Taxes, and Insurance, furnish the pupil with a working knowledge of those modern business forms and usages which will be most helpful in his everyday life. These chapters are characterized by a simplicity of treatment which is, at the same time, in accord with the best business practices of to-day.

Chapter IV extends the work of Part One in measurement and construction of simple geometrical figures and methods of finding their areas. It also familiarizes the pupil with methods of finding the lateral areas and volumes of the most commonly employed solids, with the use and application of formulæ in finding areas and volumes, and with measurements made in and about the home.

Chapter V, Ratio and Proportion, includes all that is essential to a working knowledge of these subjects, freed from needless terms and principles. Sufficient practice and application of these subjects are provided to give the pupil facility in their use.

Part Two closes with an extensive general review, testing out the pupil's knowledge of processes and principles and his ability to work problems, and enabling the teacher to ascertain whether or not the pupil is equipped with a thorough working knowledge of arithmetic.

ROBERT F. ANDERSON.

CONTENTS

PART I

CHAPTER I. FUNDAMENTAL OPERATIONS; FRACTIONS; DECIMALS

CHAPTER II. LETTERS AS NUMBERS

CHAPTER III. PERCENTAGE

CHAPTER IV. MEASURES

PART II

CHAPTER I. REVIEW

CHAPTER II. BANKING AND NEGOTIABLE PAPERS

CHAPTER III. INVESTMENTS, TAXES, AND INSURANCE

CHAPTER IV. MEASURES

CHAPTER V. RATIO AND PROPORTION

BOOK THREE

PART I—CHAPTER I

FUNDAMENTAL OPERATIONS; FRACTIONS; DECIMALS

READING AND WRITING NUMBERS

1. Read the following numbers, reading only one of two that are alike:

978	7842	96,274	851,762	5,167,248
987	6849	96,724	851,672	5,167,002
978	7842	76,248	851,762	9,700,406

2. Automobile license numbers are sometimes written thus: 48–976. Read this number.

3. Sometimes, as in telephoning, a number is read *seven-o-six-four*. Name this number. Write and read four other numbers in the same way.

Sometimes instead of one number, a more convenient number which is nearly equal to it, is used.

Thus, instead of saying that the distance from the earth to the sun is 92,897,400 miles, we usually say that this distance is 93,000,000 miles, which is the distance correct to the nearest 1,000,000 miles, or the distance in "round numbers."

4. Read: The distance from Mars to the sun is 141,546,600 miles. Read this distance correct to the nearest 1000 miles; correct to the nearest 1,000,000 miles.

5. Find out the population of your state by the last census and read this number correct to the nearest 1000.

6. Read this newspaper item: "Uncle Sam's debts at the close of the Civil War were $2,381,530,294; at the close of the Spanish War, $1,046,048,750; August, 1919, $26,596,701,648."

7. How many different figures are used in writing numbers?

8. Why is it that we can express all numbers by ten figures?

9. Beginning with units name the periods to billions.

10. Beginning with units name the orders to millions: thus, *units*, *tens*, *hundreds*, and so on.

11. In 5,786,432, tell what each figure represents.

12. Look at the numbers in example 1 and tell how and when the comma is usually employed in writing numbers.

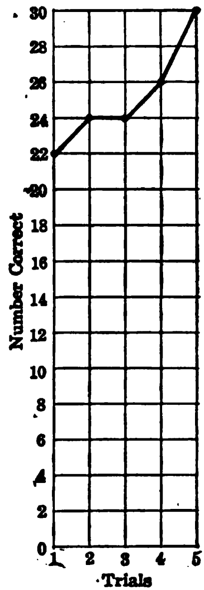
13. Write the numbers in example 1 from dictation, the teacher pronouncing each number but once.

14. Copy the following, writing the numbers in figures: The United States in a recent year produced seven hundred fifty million six hundred forty-eight thousand bushels of wheat, three billion two hundred sixteen million one hundred ninety-two thousand bushels of corn, one billion four hundred forty-four million three hundred sixty-two thousand bushels of oats, and seventy-seven million eight hundred ninety-three thousand bushels of rye.

15. Try to copy these numbers accurately in 2 minutes. Look carefully at each number so as to remember it while you are writing it:

764	326	4876	9807	15,876	91,583
832	758	5941	8075	26,546	80,009
429	901	3820	6320	90,014	89,001
510	190	7503	6230	89,076	76,054
208	745	7530	1576	80,967	70,564

16. In five trials, each lasting 2 minutes, Anna made the following record in copying correctly the numbers in example 15: first trial, 22; second trial, 24; third trial, 24; fourth trial, 26; fifth trial, 30. She kept score of her work as shown in the diagram or *graph.* Copy the numbers five times, make a diagram as Anna did, and compare your score with hers. Each trial should not exceed 2 minutes.



17. Write the numbers in example 15 from dictation in $1\frac{1}{2}$ minutes.

18. Write in figures the numbers used in the following statement: Over one billion acres of land in the United States are not adapted to cultivation, of which at least three hundred sixty million acres may be used for forests, and about six hundred million acres for grazing.

19. 1000 billion make a *trillion.* Read: 1,246,000,000,000; 25,789,684,000,000.

20. 745,683 is sometimes read *seventy-four, fifty-six, eighty-three,* and 25,876 is sometimes read *two, fifty-eight, seventy-six.* Business men usually read large numbers this way. In the same way read the numbers in the last three columns of example 1.

21. Name the letters which are used as Roman numerals and tell what number each represents.

22. Read the following numbers:

IX	IV	XLIV	MCMXXVI	MDCCCLXV
XL	LX	MCIX	MDCCCIX	MCMXXIV
CD	CV	DXCV	MDCCXXII	MDCCCXX

23. Express in Roman numerals:

19	38	119	1776	1863
44	49	409	1492	1927

24. State four uses of Roman numerals.

ADDITION — ACCURACY AND SPEED TESTS

1. Here are 20 of the most difficult of the 45 elementary combinations in addition; try to name all the sums in less than 15 seconds:

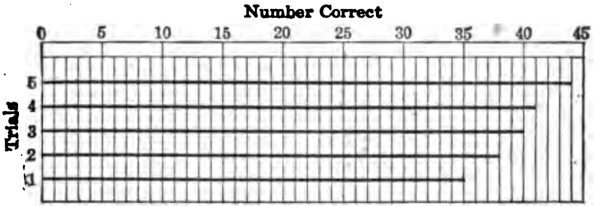
4	7	6	8	4	6	7	9	5	8
6	9	8	8	7	9	8	9	8	9
3	6	3	4	5	5	5	3	3	4
5	7	8	9	6	9	7	9	7	8

2. In adding a number of one figure to a number of two figures, some combinations are more difficult than others; 45 of the most difficult are here given. Try to name all the sums in 1 minute or less.

19	29	19	88	68	47	87	18	99
9	6	5	8	6	9	9	5	9
18	39	59	19	28	46	68	29	98
9	7	5	9	6	8	9	8	6
98	57	77	69	59	39	58	99	89
9	8	7	9	9	8	5	6	8
89	79	37	49	38	28	67	79	88
7	8	7	6	7	8	5	7	7
48	78	28	48	27	58	97	79	69
6	8	7	8	6	9	8	9	6

3. In five trials of 1 minute each Robert made the following record in naming correctly the sums in example 2: first trial, 35;

second trial, 38; third trial, 40; fourth trial, 41; fifth trial, 44. He kept a record of his work as shown in the following graph. Try to surpass Robert's record, then make a graph like his to show your score in the five trials, none of which should exceed 1 minute.



4. Skill in addition depends upon skill in adding single columns. What is your best record in finding these sums?

4	8	4	6	6	4	5	9	8	7
8	7	5	5	1	5	9	8	7	9
3	9	7	8	2	6	7	9	5	8
5	6	8	8	8	6	8	5	6	9
7	8	9	7	7	7	6	7	9	7
9	7	9	7	5	9	9	6	3	8
8	5	5	5	7	7	8	5	8	9
6	8	4	6	6	8	7	0	4	6
3	9	8	9	8	5	4	9	9	9
5	7	3	9	3	8	9	7	5	8
1	3	0	8	8	4	3	8	5	7
9	9	7	7	8	8	7	9	6	8

5. In five trials of 2 minutes each Emily made these records in finding the sums in example 4: first trial, 6 correct; second trial, 7 correct; third trial, 7 correct; fourth trial, 8 correct; fifth trial, 10 correct. Emily made a graph like that on page 3 to show her score. Make the diagram or graph which Emily made; then try to better her record and make a graph of your scores.

Try to get the correct answers to the 12 following examples in 8 minutes. If you cannot, review them frequently and reach this standard as soon as you can. Check results.

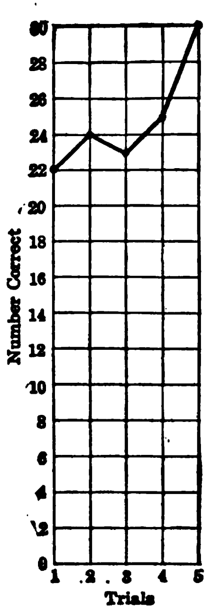
6.	7.	8.	9.	10.	11.
846	796	456	671	658	756
708	458	807	583	497	429
647	306	639	855	706	308
240	421	286	999	846	570
758	987	198	647	123	854
976	365	786	990	989	767
458	829	864	786	767	884
389	476	757	878	584	198
887	507	859	562	426	522

12.	13.	14.	15.	16.	17.
828	284	728	288	488	976
907	962	566	478	367	784
562	578	897	919	749	245
389	176	386	876	506	477
166	999	574	455	998	838
489	878	325	376	742	788
878	757	786	499	658	697
780	638	409	708	986	456
122	959	877	694	687	749

SUBTRACTION — ACCURACY AND SPEED TESTS

1. Albert's record in writing the correct differences for these 30 of the most difficult of the 100 possible elementary subtractions was: first trial, 22; second trial, 24; third trial, 23; fourth trial,

25; fifth trial, 30. Each trial lasted 25 seconds. Albert then checked his answers and made the accompanying graph to show his record. In five trials of 25 seconds each, can you do as well as Albert did? Make a graph of your record.



16 − 9	11 − 7	12 − 3	13 − 6	12 − 5	11 − 3
13 − 9	12 − 8	11 − 5	13 − 4	14 − 5	11 − 6
13 − 7	14 − 8	18 − 9	15 − 7	13 − 8	14 − 6
11 − 4	13 − 5	12 − 4	16 − 7	14 − 9	15 − 8
12 − 7	17 − 9	15 − 9	12 − 9	15 − 6	17 − 8

Find the answers to these subtraction examples if possible in 4 minutes. Check your work. Do not copy the examples.

2.	3.	4.	5.
138599094	10378345	626250541	102406732
73610189	7483071	485707280	65392891

6.	7.	8.	9.
278754376	172909159	149644097	174073189
179746510	93421630	64051436	92549775

10.	11.	12.	13.
527067253	861758930	116813118	1015083799
365861290	488272680	43292635	251304589

MULTIPLICATION

1. These are the most difficult of the 78 multiplication combinations. Can you name the correct products in less than 30 seconds?

11×8	9×9	7×4	8×6	12×9	8×4
11×11	9×5	10×10	7×6	12×6	12×8
12×12	9×3	7×5	11×9	12×3	9×4
12×11	7×7	12×4	8×8	9×6	8×7
11×10	12×5	9×8	12×7	12×10	9×7

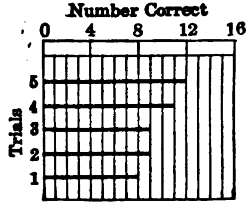
2. Make a graph like that on page 7 of your record of five trials in writing the answers for example 1. Do not exceed 30 seconds for each trial.

3. Write seven different numbers of five figures each, using the figures 5, 6, 7, 8, and 9 in writing each number. Multiply each number by 6; by 7; by 8; by 9. Check each multiplication.

How many of these 16 examples can you work in 6 minutes? Go over your work a second time to see if it is correct.

4.	5.	6.	7.	8.	9.	10.	11.
8524	3468	4829	5739	4756	3478	5467	9376
48	27	560	95	730	208	65	804
12.	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**
3289	9256	2579	3756	6248	6248	2938	7359
460	82	72	65	309	59	370	93

20. The accompanying graph shows Catherine's record of correct answers for examples 4 to 19 in five trials of 6 minutes each. Read her record for each trial. Try to surpass Catherine's record. Make a graph of your record.



Multiply each of the following numbers by 718; by 645; by 239; by 978; by 7056; by 8009; by 6789:

21. 2378	22. 4569	23. 8257	24. 39626
25. 8493	26. 6275	27. 9008	28. 90789

SPECIAL MULTIPLIERS

1. **To multiply a whole number by 10, annex a zero.** Illustrate this principle by five examples.

2. **To multiply a whole number by 100, annex two zeros.** Illustrate this principle by six examples.

3. **To multiply a whole number by 1000, annex three zeros.** Illustrate this principle by seven examples.

4. **To multiply a number, as 38, by a number ending in one zero, as 60, multiply by 6 and annex one zero.** Give four illustrations.

5. **To multiply a number, as 38, by a number ending in two zeros, as 600, multiply by 6 and annex two zeros.** Give four illustrations.

6. Multiply at sight each of the following numbers by 10; by 100; by 1000; by 30; by 50; by 90:

4 7 6 9 11 12 20 50

7. What is a short way of multiplying 26 by 4000?

8. Give four illustrations of the short way of multiplying by a number ending in three zeros; by one ending in four zeros.

Factors. The numbers which when multiplied together will produce a given number are called the *factors* of the number.

Thus, 7 and 3 are the factors of 21.

Square. The product obtained by taking a number twice as a factor is called the *square* of the number.

Thus, the square of 3 is 9, since $3\times3=9$.

3×3 may be written 3^2, read *3 squared.*

Cube. The product obtained by taking a number three times as a factor is called the *cube* of the number.

Thus, the cube of 3 is 27, since $3\times3\times3=27$.
$3\times3\times3$ may be written 3^3, read *3 cubed.*

Power. The result of taking a number any number of times as a factor is called a *power* of the number.

Thus, 9 is called the *second* power of 3; 27 is the *third* power of 3.

SQUARES AND CUBES

1. Name the squares of all whole numbers to 12.

2. Make a list of the squares of all whole numbers to 25, and learn them if you have not already done so.

3. Name the square of 30; 40; 50; 60; 70; 80; 90; 100.

4. Write the cubes of all numbers to 10. How many of them can you name at sight?

5. What is the fourth power of 2? of 3? of 4? of 10?

SHORT METHODS IN MULTIPLICATION

1. Study how the answers in (1), (2), and (3) were found, check the answers by multiplying as indicated, and then state a short way of multiplying by 5; by 50; by 25.

(1) $5\times48=?$ (2) $50\times48=?$ (3) $25\times48=?$

$$\begin{array}{r} 48 \\ \underline{10} \\ 2)\overline{480} \\ 240 \end{array} \qquad \begin{array}{r} 48 \\ \underline{100} \\ 2)\overline{4800} \\ 2400 \end{array} \qquad \begin{array}{r} 48 \\ \underline{100} \\ 4)\overline{4800} \\ 1200 \end{array}$$

Find the products, using a short method:

	a	*b*	*c*	*d*	*e*
2.	5×72	5×84	5×96	5×246	5×324
3.	5×75	5×87	5×98	5×127	5×569

4.	50×34	50×82	50×96	50×175	50×396
5.	50×89	50×91	50×77	50×121	50×786
6.	25×37	25×56	25×54	25×292	25×386
7.	25×55	25×75	25×87	25×491	25×789

8. Try to find out a short way of multiplying by 500; by 125.

Factoring. Finding the numbers which multiplied together will produce a given number is called *factoring*.

Prime Numbers. A number that has no factor other than itself and 1 is called a *prime number*.

Thus, 7 is a prime number, since the only factors of 7 are 7 and 1.

Prime Factors. When all the factors of a number are prime numbers, they are called *prime factors* of the number.

Thus, in $60 = 2\times2\times3\times5$, the factors, being prime numbers, are the prime factors of 60.

Multiple. The product of two whole numbers is called a multiple of either of them.

Thus, 15 is a multiple of 3 and also of 5.

FACTORING

1. Name two factors which will produce each of the following numbers. Name other sets of factors for as many of these numbers as you can.

56	63	77	48	35	88	110	120
42	28	45	54	36	72	132	99

2. Name the two equal factors of:

16	36	64	4	25	9	49	100	81

3. Express each of the numbers in example 1 as the product of its prime factors.

4. Find the sum of all the prime numbers to 100.

5. Name four multiples of 3; four multiples of 6; four multiples of 8; three multiples of 16.

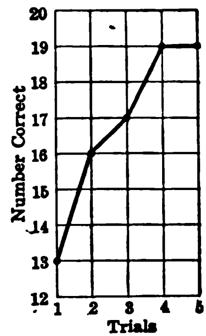
DIVISION

1. Name the quotients and remainders in these examples. For the first say: *quotient 7, remainder 2.*

9)65 6)59 7)60 8)50 9)70 6)45 8)60 9)80 8)70 7)55

7)68 6)50 9)60 6)47 8)79 7)48 8)75 9)76 7)65 9)86

2. In five trials of example 1, of 1 minute each, Joseph made the record shown in this graph. Joseph kept his answers for each trial on separate slips of paper and after the five trials checked the answers and made the graph. Read from the graph Joseph's record. Can you better it?



3. Instead of naming the quotients and remainders in example 1, name the complete quotients. For the first say: $7\frac{2}{9}$.

Work as many of examples 4 to 19 as you can in 8 minutes.

4. 56)26936	5. 73)66649	6. 29)14674	7. 94)19314
8. 47)39950	9. 92)85560	10. 85)40950	11. 19)24809
12. 83)76443	13. 65)45630	14. 57)42488	15. 56)33040
16. 38)28880	17. 74)39294	18. 48)20558	19. 91)37444

20. After checking the answers in examples 4 to 19, try for the best record you can make in five trials of 8 minutes each in working these examples. In each trial use a separate sheet of paper. Make a graph of your record like that on page 8.

Divide; check. Time yourself on each group of examples.

21. 142,272	by	*a.* 416	25. 1,487,563	by	*a.* 1246
22. 211,932		*b.* 609	26. 4,007,658		*b.* 5409
23. 477,006		*c.* 489	27. 6,750,006		*c.* 7005
24. 236,418		*d.* 893	28. 8,300,007		*d.* 8972

Special Divisors. **Cutting off one zero from the right of a whole number divides it by 10; cutting off two zeros divides it by 100; cutting off three zeros divides it by 1000.** Illustrate each case.

Divisibility. A number that will divide a given number without a remainder is an *exact divisor,* or simply a *divisor* of the given number.

Thus, 8 is a divisor of 24 but not of 25.

Even Number. A number that is divisible by 2 is called an *even number;* as 10, 14, and 30.

Odd Number. A number that is not divisible by 2 is called an *odd number;* as 13, 15, and 21.

Some Easy Tests of Divisibility. Illustrate each.

1. A number is divisible by 2 if its units' figure is 0, 2, 4, 6, or 8.

2. A number is divisible by 5 if its units' figure is 0 or 5.

3. A number is divisible by 4 if the number expressed by the two right-hand figures is divisible by 4.

4. A number is divisible by 3 if the number expressed by the sum of the digits is divisible by 3.

5. A number is divisible by 6 if it is divisible by 2 and 3.

6. A number is divisible by 9 if the number expressed by the sum of the digits is divisible by 9.

7. A number is divisible by 12 if it is divisible by 4 and 3.

EXACT DIVISORS

1. Name each number of the following list which is divisible by 2 and state the quotient; by 3; by 4; by 5; by 6; by 9; by 10; by 12; by 100.

21	36	64	360	456	756	192	200
50	150	195	540	621	720	840	204
75	210	110	600	900	180	960	999

2. Which of the following could be set out in 3 equal rows? in 4? in 9? in 6?

210 celery plants	114 cabbage plants
126 fruit trees	240 tomato plants

3. Name all numbers which will divide both numbers in each of the following pairs:

8 and 12	18 and 24	36 and 48	42 and 54	60 and 75
9 and 27	24 and 32	20 and 50	60 and 84	60 and 80

4. Name the even numbers in example 1; the odd numbers.

5. Divide 6200 by 200; then divide 62 by 2. Compare the quotients.

6. Find the quotients by a short way:

$620 \div 20$	$930 \div 30$	$660 \div 60$	$6300 \div 900$	$7500 \div 500$
$720 \div 80$	$960 \div 120$	$5400 \div 600$	$8100 \div 900$	$7200 \div 800$

CANCELLATION

1. How many poor families can be supplied from 36 barrels of flour, each barrel containing 196 pounds, if each family is given 48 pounds?

$$\frac{\overset{3}{\cancel{36}} \times \overset{49}{\cancel{196}}}{\underset{4}{\cancel{48}}} = 147$$

We may indicate the multiplication and division as here shown. Since 36 and 48 are each divisible by 12, the 12 is cancelled, giving 3 and 4. We see by the tests of divisibility that 196 is divisible by 4, so the factor 4 is canceled, giving 49 and 1 (which is not written). Then 3×49, or 147, is the number of families required.

2. Multiply 196 by 36 and divide the product by 48, to prove that 147 is the correct answer for problem 1.

After studying the explanation for problem 1 and checking the result, we see that:

Crossing out (canceling) the same factor from both dividend and divisor does not change the quotient.

Using cancellation, find the value of each of the following:

3. $\frac{12\times124}{24}$ 4. $\frac{56\times72}{63}$ 5. $\frac{48\times121}{88}$ 6. $\frac{250\times132}{165}$ 7. $\frac{248\times720}{192}$

8. $\frac{48\times300}{18\times20}$ 9. $\frac{144\times132}{24\times72}$ 10. $\frac{630\times576}{640\times189}$ 11. $\frac{24\times35\times72}{32\times54}$ 12. $\frac{24\times36\times45}{9\times16\times5}$

How to Solve Problems. Not only should you be able to work examples accurately and rapidly, but you must also strive to become skillful in applying the processes of addition, subtraction, multiplication, and division in solving problems. When you are given a problem to solve, you should read it carefully, and then:

1. Consider what facts are given or implied.
2. Consider what is asked for in the problem.
3. Decide what operations are necessary to solve the problem.
4. Estimate the answer, when practicable.
5. Make the calculations.
6. Compare your answer with the estimate. Check.

Illustration. If Helen had just enough to buy 3 war savings stamps at \$4.12 each and a pair of skates for \$3.95, how much money had she?

1. *Facts given:* The number of war savings stamps bought, the cost of each, and the cost of the skates.
2. *What is required:* The amount of money Helen had.
3. *Operations necessary:* Multiplication and addition.
4. *Estimate:* 3×\$4.12 = \$12, nearly. \$12 + \$3.95 = \$16, nearly.
5. *Calculation:* \$4.12 = the cost of 1 war savings stamp.

$$\begin{array}{rl} \$4.12 & = \text{the cost of 1 war savings stamp.} \\ \underline{3} & \\ 12.36 & = \text{the cost of 3 war savings stamps.} \\ \underline{3.95} & = \text{the cost of the skates.} \\ \$16.31 & = \text{the money Helen had.} \end{array}$$

6. *Answer compared with estimate:* \$16.31 and \$16 are nearly equal, so the estimate was nearly correct. To check, go over the work a second time.

Work

PROBLEMS IN FUNDAMENTAL PROCESSES

(Use cancellation when you can.)

1. Mr. Jones paid $1 a foot for drilling a well to the depth of 40 ft. and $1.25 for each foot over 40. If his well was drilled to a depth of 90 ft., what was the cost?

2. Samuel's father bought a lot for $1800. He built a house on it which cost $6750 and then sold the property for $10,000. How much money did he gain?

3. A falling body descends 256 ft. in 4 seconds and 144 ft. the fifth second. How far does it descend in 5 seconds?

4. Mrs. Hays bought 2 lb. of ham at 35¢ a pound, 2 lb. of butter at 53¢ a pound, 2 lb. of cheese at 28¢ a pound, and 10 lb. of sugar at 9¢ a pound. How much change should she receive from a $10-bill?

5. If a tract of land containing 27 acres is worth $18,000, how much must Mr. Wilson pay for 15 acres of it?

6. State how to check the answer to an example in addition; in subtraction; in multiplication; in division.

7. Recently it was estimated that each person in Philadelphia used 210 gal. of water every 24 hr., but that each person should use not more than 150 gal. in that time, the additional amount being wasted. Estimating the population of Philadelphia at 1,830,000, how much water was wasted every 24 hr.?

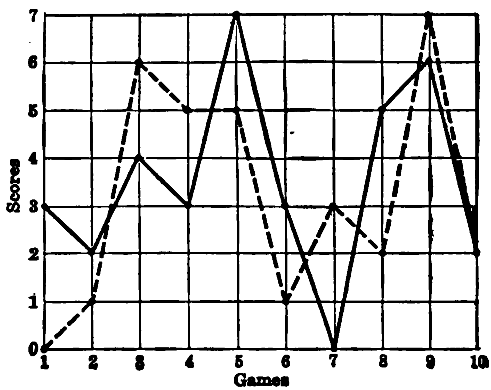
8. How long should it require 6 men each husking 40 shocks of corn a day to husk 2400 shocks of corn?

9. A clock that strikes the hours strikes how many times in a week?

10. Andrew's father worked for a farmer. He received $50 a month for the 12 months of the year, free house rent worth $23 a month, 12 bu. of potatoes worth $1.30 a bushel, and 365 qt. of

milk worth 8¢ a quart. What he received was equivalent to what money wages for the year?

11. The accompanying graph shows the scores made in ten games of baseball played between the Blaine High School and visiting teams. The solid line represents the scores made by the Blaine team, and the dotted line, scores made by the visiting teams. Read the score for each game.



12. To make class colors, two kinds of ribbon were bought, each costing 20¢ a yard. How much did the ribbon cost if 4 in. of each kind were required for each pupil and there were 36 pupils in the class?

FRACTIONS

1. Any one of the equal parts of a unit is called a *fractional unit;* as, *one fourth.* Name three other fractional units.

2. A *fraction* is any number of equal fractional units. Name five fractions.

3. A fraction which is less than 1 is called a *proper fraction.* Name six proper fractions.

4. Draw a line and show *one half* of it; *three fourths* of it; *seven eighths* of it.

5. Express *seven eighths* in figures, and name the *denominator* of the fraction; the *numerator;* the *terms.*

6. Which term of a fraction shows into how many equal parts the unit is divided, or the *size* of the fractional units? Give example.

7. Which term of a fraction shows the *number* of fractional units the fraction contains?

8. Which of the fractional units $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ is the greatest? Which is the least? Draw three lines of equal length and properly divide them to test your answer.

9. Draw two lines of equal length and use them, properly divided, to compare $\frac{2}{3}$ and $\frac{2}{5}$. If two fractions have equal numerators but unequal denominators, which fraction is the greater, that is, has the *greater value,* — the one having the greater denominator or the one having the less?

10. Compare two fractions which have equal denominators but unequal numerators, as you compared the fractions in problem 9.

11. Name the fractions on the first line of the following list in the order of their value, the greatest first; on the second line:

$\frac{3}{7}$ $\frac{3}{4}$ $\frac{3}{10}$ $\frac{3}{2}$ $\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{3}$ $\frac{3}{9}$

$\frac{7}{10}$ $\frac{6}{10}$ $\frac{1}{10}$ $\frac{10}{10}$ $\frac{5}{10}$ $\frac{9}{10}$ $\frac{12}{10}$ $\frac{4}{10}$

MIXED NUMBERS AND IMPROPER FRACTIONS

1. Name four numbers each made up of a whole number and a fraction. What are such numbers called?

2. Name a fraction which is equal to 1; to 2; to 3; to 5.

3. Name a fraction which is greater than 1 but less than 2; greater than 2 but less than 3.

4. When a fraction is equal to 1 or greater than 1, what is it called?

5. Name the mixed numbers in the following list; the improper fractions.

$\frac{3}{2}$ $\frac{7}{8}$ $2\frac{1}{2}$ 4 $3\frac{2}{3}$ $\frac{9}{2}$ $\frac{4}{3}$ $\frac{3}{10}$ $1\frac{1}{2}$

6. Show that $2\frac{1}{2}=\frac{5}{2}$. Give the steps taken in reducing $2\frac{1}{2}$ to $\frac{5}{2}$.

7. Change at sight the following to improper fractions. For the first say $\frac{7}{3}$.

$2\frac{1}{3}$ $3\frac{1}{2}$ $8\frac{2}{3}$ $4\frac{1}{2}$ $6\frac{2}{3}$ $12\frac{1}{2}$ $16\frac{2}{3}$ $33\frac{1}{3}$

$4\frac{4}{5}$ $6\frac{1}{4}$ $3\frac{3}{8}$ $37\frac{1}{2}$ $11\frac{1}{5}$ $1\frac{7}{12}$ $87\frac{1}{2}$ $66\frac{2}{3}$

8. Show that $\frac{7}{3}=2\frac{1}{3}$. Explain how you change $\frac{7}{3}$ to $2\frac{1}{3}$.

9. Change at sight the following to whole or mixed numbers. For the first say $3\frac{1}{3}$. Remember that $\frac{10}{3}$ may be regarded as denoting that 10 is to be divided by 3.

$\frac{10}{3}$ $\frac{5}{2}$ $\frac{20}{3}$ $\frac{25}{4}$ $\frac{12}{4}$ $\frac{20}{2}$ $\frac{25}{3}$ $\frac{25}{6}$

$\frac{16}{4}$ $\frac{20}{9}$ $\frac{15}{8}$ $\frac{25}{8}$ $\frac{75}{2}$ $\frac{50}{3}$ $\frac{100}{9}$ $\frac{100}{3}$

REDUCTION OF FRACTIONS. LEAST COMMON DENOMINATOR

1. Draw two lines of equal length. Show $\frac{3}{4}$ of one of them and $\frac{6}{8}$ of the other. Compare $\frac{3}{4}$ and $\frac{6}{8}$; that is, say that the fractions are equal, or that one of them (naming it) is greater than the other (naming it).

2. Study what you learned by working problem 1 and use it to illustrate that:

Multiplying or dividing both terms of a fraction by the same number does not change its value.

3. When we express $\frac{2}{3}$ as $\frac{4}{6}$, why have we not changed the value of $\frac{2}{3}$?

4. When we change $\frac{2}{3}$ to $\frac{4}{6}$, we say that we have reduced $\frac{2}{3}$ to 6ths. Reduce $\frac{2}{3}$ to 9ths; to 12ths.

5. Reduce at sight each of these fractions to 12ths; to 24ths:

$\frac{1}{2}$ $\frac{7}{4}$ $\frac{1}{6}$ $\frac{1}{4}$ $\frac{5}{6}$ $\frac{3}{4}$ $\frac{3}{2}$ $\frac{5}{4}$

6. Reduce to 48ths:

$\frac{1}{2}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{5}{8}$ $\frac{2}{3}$ $\frac{1}{12}$ $\frac{7}{12}$ $\frac{7}{16}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{5}{16}$ $\frac{7}{24}$

7. Reduce to 60ths:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{2}{3}$ $\frac{1}{12}$ $\frac{7}{20}$ $\frac{9}{10}$ $\frac{3}{10}$ $\frac{2}{15}$ $\frac{4}{5}$ $\frac{11}{20}$

8. Show that $\frac{6}{8}$ can be reduced to *lower terms;* that is, to a fraction equal to $\frac{6}{8}$, but with a less numerator and denominator.

9. Write ten fractions which can be expressed in lower terms; then express each in lower terms.

10. When both terms of a fraction can be divided by no whole number except 1, the fraction is in *lowest terms.* Test the fractions in examples 6 and 7 to see whether or not each is in lowest terms.

11. Write ten fractions which are not in lowest terms and reduce each to lowest terms.

12. The least number which the denominators of two or more fractions will exactly divide, is called the *least common denominator* of the fractions. What is the least common denominator of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$? Reduce these fractions to fractions having the least common denominator.

The least number that two or more numbers will exactly divide is called their *least common multiple.*

Reduce to fractions having the least common denominator:

	a	*b*	*c*	*d*
13.	$\frac{5}{6}, \frac{1}{2}, \frac{2}{5}$	$\frac{7}{12}, \frac{3}{8}, \frac{5}{6}$	$\frac{7}{8}, \frac{5}{16}, \frac{3}{4}$	$\frac{2}{5}, \frac{1}{25}, \frac{3}{50}$
14.	$\frac{2}{9}, \frac{3}{4}, \frac{5}{6}$	$\frac{7}{8}, \frac{5}{12}, \frac{5}{9}$	$\frac{3}{10}, \frac{5}{6}, \frac{7}{12}$	$\frac{5}{12}, \frac{7}{8}, \frac{3}{16}$
15.	$\frac{1}{2}, \frac{1}{4}, \frac{5}{6}, \frac{5}{12}$	$\frac{1}{4}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{16}$	$\frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{7}{10}$

16. Fractions which have a common denominator are called *like* fractions, or *similar* fractions. Name four like fractions.

17. What name is given to fractions which do not have a common denominator? Name four such fractions.

18. Write three fractions which are unlike and show that they may be reduced to fractions which are like fractions. Reduce them to like fractions.

ADDITION OF FRACTIONS

1. Write two like fractions and find their sum.

2. Write two unlike fractions and find their sum.

Add. Remember to reduce all fractions in the results to lowest terms.

	a	b	c	d	e	f	g	h	i	j
3.	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{5}{12}$	$\frac{7}{10}$	$\frac{2}{5}$	$\frac{7}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{12}$	$\frac{5}{6}$	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{3}{16}$
	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{8}$
4.	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{9}$	$\frac{3}{4}$
	$\frac{7}{8}$	$\frac{3}{10}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{4}{5}$
	$\frac{5}{8}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{1}{12}$	$\frac{7}{12}$	$\frac{3}{10}$
	$\frac{3}{8}$	$\frac{7}{10}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{7}{15}$	$\frac{3}{10}$	$\frac{5}{24}$	$\frac{7}{50}$

5.	$7\frac{1}{8}$	$4\frac{3}{8}$	$4\frac{2}{5}$	$8\frac{4}{5}$	$7\frac{9}{16}$	$4\frac{2}{5}$	$4\frac{3}{10}$	$5\frac{3}{4}$
	$5\frac{3}{8}$	$5\frac{7}{8}$	$3\frac{3}{5}$	$3\frac{3}{5}$	$3\frac{1}{4}$	$3\frac{9}{10}$	$2\frac{3}{5}$	$7\frac{5}{12}$
6.	$4\frac{2}{3}$	$7\frac{2}{5}$	$3\frac{3}{5}$	$7\frac{2}{5}$	$4\frac{4}{9}$	$8\frac{5}{6}$	$4\frac{7}{10}$	$8\frac{5}{6}$
	$2\frac{1}{9}$	$8\frac{9}{10}$	$8\frac{3}{8}$	$4\frac{7}{8}$	$5\frac{5}{12}$	$5\frac{5}{8}$	$5\frac{1}{6}$	$9\frac{9}{10}$
7.	$2\frac{1}{2}$	$6\frac{2}{3}$	$8\frac{1}{6}$	$6\frac{1}{6}$	$7\frac{2}{3}$	$2\frac{5}{6}$	$2\frac{3}{10}$	$3\frac{3}{16}$
	$3\frac{2}{3}$	$2\frac{1}{2}$	$9\frac{5}{8}$	$3\frac{1}{9}$	$8\frac{3}{4}$	$3\frac{3}{8}$	$4\frac{1}{5}$	$4\frac{5}{6}$
	$5\frac{1}{6}$	$8\frac{3}{4}$	$6\frac{1}{12}$	$6\frac{2}{3}$	$5\frac{5}{12}$	$3\frac{5}{12}$	$3\frac{1}{2}$	$3\frac{3}{8}$
8.	$2\frac{3}{4}$	$3\frac{1}{8}$	$1\frac{3}{20}$	$9\frac{3}{8}$	$2\frac{2}{9}$	$3\frac{3}{16}$	$2\frac{2}{7}$	$2\frac{1}{4}$
	$1\frac{7}{8}$	$4\frac{1}{2}$	$4\frac{7}{10}$	$4\frac{1}{6}$	$4\frac{1}{6}$	$5\frac{7}{12}$	$3\frac{1}{6}$	$3\frac{2}{5}$
	$4\frac{1}{2}$	$5\frac{1}{6}$	$2\frac{1}{2}$	$3\frac{7}{12}$	$2\frac{2}{3}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$4\frac{7}{10}$
	$2\frac{1}{4}$	$2\frac{2}{3}$	$5\frac{1}{5}$	$4\frac{2}{3}$	$5\frac{7}{12}$	$5\frac{5}{6}$	$3\frac{1}{3}$	$5\frac{9}{50}$

9.

$2\frac{1}{4}$	$2\frac{3}{8}$	$7\frac{5}{12}$	$3\frac{2}{3}$	$4\frac{1}{16}$	$7\frac{2}{3}$	$4\frac{4}{9}$	$3\frac{3}{10}$
$3\frac{3}{4}$	$4\frac{1}{2}$	$3\frac{1}{4}$	$5\frac{5}{6}$	$3\frac{3}{8}$	$4\frac{4}{9}$	$3\frac{5}{12}$	$2\frac{2}{5}$
$2\frac{1}{2}$	$6\frac{3}{4}$	$2\frac{1}{8}$	$3\frac{1}{10}$	$2\frac{1}{6}$	$3\frac{5}{6}$	$7\frac{3}{4}$	$7\frac{1}{2}$
$1\frac{1}{2}$	$2\frac{1}{4}$	$5\frac{1}{3}$	$2\frac{1}{2}$	$3\frac{2}{3}$	$2\frac{1}{12}$	$5\frac{5}{6}$	$3\frac{7}{50}$
$3\frac{1}{4}$	$5\frac{5}{8}$	$2\frac{1}{2}$	$1\frac{1}{5}$	$1\frac{1}{4}$	$7\frac{1}{4}$	$2\frac{2}{3}$	$2\frac{2}{25}$

10. Time yourself on each of the following tests in addition; make a record of the number of correct answers you have in each test.

Test 1

$\frac{1}{4}$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{4}{5}$	$\frac{1}{6}$	$\frac{5}{9}$	$\frac{5}{6}$	$\frac{2}{3}$
$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{9}{10}$	$\frac{7}{10}$	$\frac{4}{15}$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{4}{15}$	$\frac{5}{6}$

Test 2

$\frac{3}{4}$	$\frac{1}{7}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{1}{12}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{6}$
$\frac{7}{12}$	$\frac{5}{14}$	$\frac{7}{12}$	$\frac{9}{10}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{3}{16}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{14}{15}$

Test 3

$\frac{1}{8}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{7}{12}$	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{11}{16}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{9}{10}$	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{7}{15}$	$\frac{8}{15}$	$\frac{1}{6}$	$\frac{13}{16}$

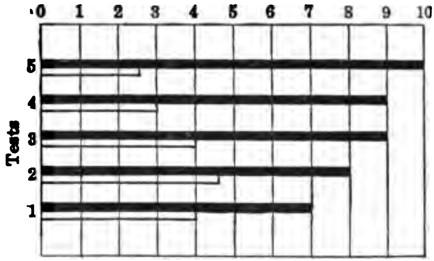
Test 4

$\frac{1}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{9}$	$\frac{5}{6}$	$\frac{1}{6}$
$\frac{2}{15}$	$\frac{7}{8}$	$\frac{2}{9}$	$\frac{3}{8}$	$\frac{3}{10}$	$\frac{11}{15}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{9}{10}$	$\frac{11}{15}$

Test 5

$\frac{3}{4}$	$\frac{7}{8}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{7}{10}$	$\frac{3}{10}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{13}{16}$	$\frac{4}{5}$
$\frac{7}{8}$	$\frac{1}{12}$	$\frac{8}{15}$	$\frac{4}{15}$	$\frac{7}{12}$	$\frac{13}{15}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{3}{4}$	$\frac{7}{10}$

11. In this graph the black bars represent the number of answers David had correct in the tests of example 10, and the light bars represent the number of minutes he required for the tests. Read his score for each test and the number of minutes he required for the test.



12. Make a graph like that in example 11 to show your score in the above tests and your record for time.

SUBTRACTION OF FRACTIONS

1. Write two like fractions and find their difference.

2. Write two unlike fractions and find their difference.

3. Time yourself on each of the following tests in subtraction, make a record of the number of correct answers you have in each test, then make a graph like that above to show your record.

Test 1

$\frac{9}{10}$	$\frac{7}{8}$	$\frac{13}{15}$	$\frac{9}{16}$	$\frac{3}{8}$	$18\frac{1}{3}$	$12\frac{3}{4}$	$7\frac{1}{8}$	9	$7\frac{1}{2}$
$\frac{2}{5}$	$\frac{4}{5}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{1}{10}$	5	$\frac{5}{8}$	$3\frac{3}{4}$	$7\frac{3}{4}$	$\frac{9}{10}$

Test 2

$\frac{11}{12}$	$\frac{4}{5}$	$\frac{11}{12}$	$\frac{9}{10}$	$\frac{5}{6}$	21	$51\frac{7}{8}$	$8\frac{2}{3}$	$66\frac{2}{3}$	$9\frac{1}{6}$
$\frac{5}{6}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{3}{10}$	$9\frac{1}{9}$	12	$\frac{7}{8}$	$\frac{1}{2}$	$2\frac{2}{3}$

Test 3

$\frac{3}{4}$	$\frac{5}{8}$	$\frac{4}{5}$	$\frac{5}{8}$	$\frac{7}{15}$	56	$18\frac{3}{4}$	$10\frac{1}{4}$	$14\frac{2}{7}$	$12\frac{2}{3}$
$\frac{3}{16}$	$\frac{7}{12}$	$\frac{7}{15}$	$\frac{2}{5}$	$\frac{3}{10}$	$24\frac{3}{5}$	$\frac{1}{12}$	$\frac{5}{6}$	7	$9\frac{3}{4}$

Test 4

$\frac{9}{10}$	$\frac{4}{5}$	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{7}{10}$	$20\frac{3}{8}$	71	$18\frac{3}{4}$	$58\frac{7}{12}$	$15\frac{3}{8}$
$\frac{2}{3}$	$\frac{11}{15}$	$\frac{1}{4}$	$\frac{8}{15}$	$\frac{3}{16}$	$\frac{5}{12}$	$21\frac{5}{8}$	7	$5\frac{1}{3}$	$7\frac{5}{6}$

Test 5

$\frac{7}{15}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{9}{10}$	56	$58\frac{5}{12}$	$8\frac{11}{24}$	$8\frac{1}{12}$	$33\frac{1}{3}$
$\frac{5}{12}$	$\frac{4}{5}$	$\frac{7}{12}$	$\frac{5}{16}$	$\frac{3}{4}$	$37\frac{5}{6}$	$9\frac{3}{4}$	$2\frac{1}{8}$	$5\frac{3}{8}$	$8\frac{5}{6}$

MULTIPLICATION OF FRACTIONS

Study how each answer has been found in examples 1 to 9 and, wherever possible, state a short rule for finding it.

1. $2\times\frac{2}{7}=\frac{4}{7}$

2. $2\times\frac{3}{8}=\frac{3}{4}$

3. $\overset{3}{\cancel{9}}\times\frac{5}{\underset{4}{\cancel{12}}}=\frac{15}{4}=3\frac{3}{4}$

4. $\frac{7}{\underset{4}{\cancel{12}}}\times\overset{3}{\cancel{9}}=\frac{21}{4}=5\frac{1}{4}$

5. $\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{2}{\cancel{4}}}=\frac{1}{2}$

6. $\frac{\cancel{8}}{\underset{3}{\cancel{9}}}\times\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}}=\frac{5}{6}$

7.
$$\begin{array}{r} 56\frac{2}{3} \\ \underline{5} \\ 3\frac{1}{3} \\ \underline{280} \\ 283\frac{1}{3} \end{array}$$

8.
$$\begin{array}{r} 22 \\ \underline{6\frac{2}{3}} \\ 14\frac{2}{3} \\ \underline{132} \\ 146\frac{2}{3} \end{array}$$

9.
$$\begin{array}{r} 63\frac{1}{2} \\ \underline{15\frac{1}{4}} \\ \frac{1}{8} \\ 15\frac{3}{4} \\ 7\frac{1}{2} \\ 315 \\ \underline{63} \\ 968\frac{3}{8} \end{array}$$

When the multiplicand or the multiplier or both are small mixed numbers, as $2\frac{1}{2}$ and $3\frac{1}{3}$, the product may readily be found if the mixed numbers are changed to improper fractions. Give four illustrations. Compare this method with the method used in example 9. Which is the shorter method when the numbers are small?

Time yourself on each of the following tests in multiplication, make a record of the number of correct answers you have in each test, then make a graph like that on page 23 to show your record.

Test 1	*Test 2*	*Test 3*	*Test 4*	*Test 5*
$2\times\frac{1}{3}$	$\frac{3}{4}\times8$	$\frac{9}{10}\times\frac{8}{9}$	$\frac{7}{8}\times1\frac{3}{5}$	$2\frac{1}{2}\times\frac{4}{5}$
$2\times\frac{3}{4}$	$3\frac{1}{3}\times6$	$\frac{3}{4}\times3\frac{1}{3}$	$5\frac{1}{4}\times\frac{2}{3}$	$1\frac{1}{2}\times1\frac{1}{3}$
$\frac{2}{3}\times7$	$\frac{5}{6}\times\frac{6}{7}$	$3\frac{2}{3}\times\frac{3}{8}$	$\frac{3}{4}\times\frac{8}{9}$	$\frac{3}{5}\times1\frac{2}{3}$
$3\frac{1}{3}\times2$	$\frac{3}{4}\times2\frac{2}{3}$	$8\times7\frac{1}{2}$	$2\frac{1}{2}\times4$	$\frac{2}{5}\times\frac{4}{5}$
$7\times5\frac{1}{2}$	$3\times\frac{5}{8}$	$2\frac{2}{3}\times10$	$\frac{1}{6}\times3$	$6\times\frac{2}{3}$
$\frac{3}{4}\times\frac{8}{9}$	$8\times\frac{3}{8}$	$3\times\frac{2}{9}$	$5\times\frac{3}{8}$	$\frac{3}{5}\times8$
$\frac{2}{3}\times4\frac{1}{5}$	$9\times6\frac{2}{3}$	$\frac{3}{5}\times10$	$8\times33\frac{1}{3}$	$5\frac{1}{3}\times9$
$3\frac{1}{5}\times\frac{5}{4}$	$7\frac{1}{2}\times\frac{4}{5}$	$5\frac{5}{6}\times2\frac{2}{3}$	$6\frac{1}{4}\times12\frac{1}{2}$	$6\times\frac{5}{18}$
$2\frac{1}{2}\times2\frac{1}{2}$	$3\frac{1}{3}\times2\frac{2}{5}$	$2\times\frac{1}{9}$	$12\times6\frac{2}{3}$	$10\times3\frac{1}{3}$
$5\times165\frac{3}{4}$	$8\frac{1}{3}\times50$	$7\frac{1}{2}\times125$	$5\times\frac{3}{10}$	$37\frac{1}{2}\times87\frac{1}{2}$

DIVISION OF FRACTIONS

1. If we *invert* 5 ($\frac{5}{1}$) we get $\frac{1}{5}$. $\frac{1}{5}$ is called the *inverse* of 5. The inverse of $\frac{2}{3}$ is $\frac{3}{2}$. What is the inverse of 2? of $\frac{1}{4}$? of $\frac{4}{5}$? of $1\frac{1}{2}$?

Study how each answer has been found in examples 2 to 8, and, wherever possible, state a short rule for finding it.

2. $\frac{9}{16}\div3=\frac{3}{16}$

3. $\frac{3}{4}\div2=\frac{3}{8}$

4. $6\div\frac{3}{4}=\overset{2}{\cancel{6}}\times\frac{4}{\cancel{3}}=8$

5. $\frac{3}{4}\div\frac{5}{6}=\frac{3}{\underset{2}{\cancel{4}}}\times\frac{\overset{3}{\cancel{6}}}{5}=\frac{9}{10}$

6. $15)\ 66\frac{2}{3}$

$\frac{45}{3})\frac{200}{3}(4\frac{4}{9}$

7. $16\frac{2}{3})120$

$\frac{50}{3})\frac{360}{3}(7\frac{1}{5}$

8. $16\frac{2}{3})\ 66\frac{2}{3}$

$\frac{50}{3})\frac{200}{3}(4$

When the dividend or divisor or both are small mixed numbers, as $3\frac{1}{3}$ and $2\frac{1}{2}$, the quotient may readily be found if the mixed numbers are changed to improper fractions. Give four illustrations.

Time yourself on each of the following tests in division, make a record of the number of correct answers you have in each test, then make a graph like that on page 23 to show your record.

Test 1	*Test 2*	*Test 3*	*Test 4*	*Test 5*
$\frac{1}{2} \div 4$	$\frac{4}{5} \div 2$	$7\frac{1}{2} \div 3$	$25 \div 6\frac{1}{4}$	$15 \div \frac{3}{4}$
$\frac{6}{7} \div 3$	$4 \div \frac{3}{4}$	$10 \div \frac{2}{3}$	$\frac{15}{16} \div \frac{25}{32}$	$12\frac{1}{2} \div 5$
$3\frac{1}{3} \div 3$	$\frac{5}{6} \div \frac{15}{16}$	$7\frac{1}{2} \div \frac{3}{10}$	$12\frac{1}{2} \div \frac{3}{4}$	$\frac{15}{16} \div 3$
$8 \div \frac{2}{3}$	$6 \div 2\frac{1}{2}$	$\frac{2}{3} \div 3$	$10 \div \frac{4}{5}$	$6\frac{2}{3} \div \frac{4}{9}$
$\frac{3}{4} \div \frac{5}{8}$	$2\frac{1}{2} \div \frac{2}{5}$	$\frac{8}{15} \div \frac{32}{25}$	$\frac{7}{8} \div 7$	$15 \div 3\frac{3}{4}$
$3\frac{1}{3} \div \frac{1}{4}$	$\frac{3}{4} \div 1\frac{1}{5}$	$8 \div 1\frac{1}{3}$	$\frac{1}{2} \div 2$	$\frac{5}{4} \div 2$
$4 \div 2\frac{2}{3}$	$\frac{3}{4} \div 4$	$\frac{7}{8} \div 1\frac{5}{16}$	$10\frac{2}{3} \div 8$	$\frac{9}{16} \div \frac{3}{8}$
$\frac{2}{3} \div 1\frac{1}{3}$	$6\frac{2}{3} \div 2$	$6\frac{2}{3} \div 1\frac{2}{3}$	$\frac{9}{16} \div 3\frac{3}{8}$	$\frac{5}{8} \div 2\frac{2}{3}$
$2\frac{2}{3} \div 1\frac{1}{3}$	$3\frac{3}{4} \div 1\frac{1}{4}$	$\frac{15}{16} \div 5$	$2\frac{1}{2} \div 7\frac{1}{2}$	$87\frac{1}{2} \div 62\frac{1}{2}$
$62\frac{1}{2} \div 4$	$75 \div 8\frac{1}{3}$	$62\frac{1}{2} \div 12\frac{1}{2}$	$87\frac{1}{2} \div 4$	$8\frac{1}{3} \div 1\frac{2}{3}$

COMPARISON

1. Each number on the first line is how many times the number under it?

6	5	3	5	7	9	10	10	$7\frac{1}{2}$	$\frac{1}{2}$
2	2	$1\frac{1}{2}$	$2\frac{1}{2}$	2	2	$2\frac{1}{2}$	$3\frac{1}{3}$	$2\frac{1}{2}$	$\frac{1}{4}$

2. Each number in the second line in problem 1 is what part of the number above it?

Supply at sight the missing fractions, as shown in the first three examples, expressing all fractions in lowest terms:

3.	$3 = \frac{1}{3}$ of 9	$8 = \frac{2}{3}$ of 12	$12 = \frac{4}{3}$ of 9	$6 = \cdot\cdot$ of 12
4.	$1 = \cdot\cdot$ of 5	$3 = \cdot\cdot$ of 5	$5 = \cdot\cdot$ of 4	$3 = \cdot\cdot$ of 7
5.	$9 = \cdot\cdot$ of 6	$10 = \cdot\cdot$ of 15	$20 = \cdot\cdot$ of 15	$10 = \cdot\cdot$ of 8
6.	$1\frac{1}{2} = \cdot\cdot$ of 6	$3\frac{1}{3} = \cdot\cdot$ of 20	$3\frac{1}{2} = \cdot\cdot$ of 7	$4\frac{1}{2} = \cdot\cdot$ of 9
7.	$\frac{1}{2} = \cdot\cdot$ of 2	$\frac{1}{4} = \cdot\cdot$ of $\frac{3}{4}$	$\frac{3}{4} = \cdot\cdot$ of $1\frac{1}{2}$	$\frac{3}{8} = \cdot\cdot$ of $\frac{3}{4}$
8.	$1 = \cdot\cdot$ of $1\frac{1}{2}$	$2 = \cdot\cdot$ of $2\frac{1}{3}$	$.3 = \cdot\cdot$ of $.5$	$3 = \cdot\cdot$ of $3\frac{1}{3}$

Supply the missing numbers as shown in the first three examples:

9.	$3=\frac{1}{2}$ of 6	$8=\frac{4}{5}$ of 10	$8=\frac{4}{3}$ of 6	$5=\frac{1}{3}$ of . .
10.	$6=\frac{2}{3}$ of . .	$2\frac{1}{2}=\frac{1}{2}$ of . .	$3\frac{1}{3}=\frac{1}{3}$ of . .	$2\frac{1}{2}=\frac{1}{3}$ of . .
11.	$\frac{1}{4}=\frac{1}{3}$ of . .	$\frac{1}{8}=\frac{1}{2}$ of . .	$2\frac{1}{2}=\frac{1}{4}$ of . .	$5=\frac{5}{3}$ of . .
12.	$6=\frac{3}{2}$ of . .	$\frac{1}{2}=\frac{1}{2}$ of . .	$\frac{1}{6}=\frac{1}{5}$ of . .	$2\frac{1}{2}=\frac{1}{6}$ of . .

PROBLEMS IN COMPARISON

1. Draw on the blackboard a line which you think is 2 ft. long. Test, and make it 2 ft. long if your estimate was wrong. Whose estimate is nearest to the correct length? Who drew a line within an inch of the correct length?

2. Tell quickly what part of the length of a 2-foot line there is in each of the following:

12 in.	6 in.	8 in.	4 in.	3 in.	2 in.	15 in.	18 in.
21 in.	16 in.	9 in.	14 in.	20 in.	$\frac{1}{2}$ ft.	$1\frac{1}{2}$ ft.	10 in.

3. What part of a pound is 8 oz.? 6 oz.? 4 oz.? 10 oz.? 12 oz.? 15 oz.?

4. When cheese is sold at 24¢ a pound, what should be charged for a slice of cheese weighing 8 oz.? 6 oz.? 4 oz.? 10 oz.? 12 oz.? 15 oz.?

5. When eggs are 60¢ a dozen, what is the cost of 6 eggs? 4? 8? 9? 10?

6. Three neighbors, Mrs. Hays, Mrs. Martin, and Mrs. Joyce, bought a case containing 6 cans of bacon for which they paid \$24.90. Mrs. Hays took 1 can, Mrs. Martin, 2 cans, and Mrs. Joyce, 3 cans. How much should each pay?

7. These three neighbors (see problem 6) bought a case of tomatoes containing 24 cans, for which they paid \$2.64. Mrs. Hays paid 66¢, Mrs. Martin, 88¢, and Mrs. Joyce, \$1.10. How many cans should each receive?

8. Marion has started a savings account. Her father told her that for each quarter dollar she saves he will add a half dollar to her fund. When she has saved \$1.50, how much has he added to her fund?

Complex Fractions. Such expressions as $\frac{2}{\frac{4}{5}}$, $\frac{\frac{3}{4}}{4}$, $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{37\frac{1}{2}}{100}$, $\frac{3\frac{1}{2}}{7}$, $\frac{2\frac{1}{2}}{\frac{3}{4}}$, $\frac{\frac{7}{8}}{1\frac{3}{4}}$, and $\frac{2\frac{1}{2}}{7\frac{1}{2}}$ are called *complex fractions.* The first, $\frac{2}{\frac{4}{5}}$, means that 2 is to be divided by $\frac{4}{5}$. It is reduced to its simplest form by performing the indicated operation, thus:

$$\frac{2}{\frac{4}{5}} = 2 \div \frac{4}{5} = 2 \times \frac{5}{4} = 2\frac{1}{2}.$$

What does the second expression, $\frac{\frac{3}{4}}{4}$, mean? What does each of the other expressions given above mean?

REDUCTION OF COMPLEX FRACTIONS

Reduce to simplest form, but, before doing so, read each carefully to see what operation is involved.

1. $\frac{\frac{3}{4}}{4}$	2. $\frac{\frac{2}{3}}{\frac{4}{5}}$	3. $\frac{37\frac{1}{2}}{100}$	4. $\frac{3\frac{1}{2}}{7}$
5. $\frac{2\frac{1}{2}}{\frac{3}{4}}$	6. $\frac{\frac{7}{8}}{1\frac{3}{4}}$	7. $\frac{\frac{7}{8}}{\frac{3}{8}}$	8. $\frac{2\frac{1}{2}}{7\frac{1}{2}}$
9. $\frac{12\frac{1}{2}}{100}$	10. $\frac{62\frac{1}{2}}{100}$	11. $\frac{87\frac{1}{2}}{100}$	12. $\frac{33\frac{1}{3}}{100}$
13. $\frac{66\frac{2}{3}}{100}$	14. $\frac{6\frac{2}{3}}{100}$	15. $\frac{8\frac{1}{3}}{100}$	16. $\frac{6\frac{1}{4}}{100}$
17. $\frac{14\frac{2}{7}}{100}$	18. $\frac{3\frac{1}{3}}{100}$	19. $\frac{\frac{3}{4}}{2\frac{1}{2}}$	20. $\frac{\frac{3}{5}}{\frac{4}{5}}$

PROBLEMS

1. It is estimated that $\frac{3}{4}$ lb. of market dressed turkey will produce one order of turkey. A market dressed turkey weighing 15 lb. should produce how many orders?

2. About $1\frac{1}{8}$ lb. of market dressed goose is necessary to produce one order. How many orders should a market dressed goose weighing 9 lb. make?

3. A half pound of tenderloin steak is considered an order. How many orders can be made from a tenderloin weighing 10 lb.?

4. When Warren was 13 years old his height was 57 in. When he was 14 years old his height was $59\frac{1}{4}$ in. What was his average growth per month for the year?

5. In a basketball game a player tried 18 throws at the basket and missed 8 times. What part of his throws proved successful?

6. Market Street in Henry's home town is $\frac{3}{10}$ of a mile long. $\frac{2}{3}$ of it has been repaved. What part of a mile has been repaved? How many feet have been repaved? (1 mi. = 5280 ft.)

7. Andrew has 25¢, which is 5 times as much money as George has. How much money have both?

8. James sold 4 copies of a magazine at 20¢ each and with $\frac{1}{2}$ of the money bought daily papers at 3 for 4¢. How many daily papers did he buy?

9. Mr. Meyers paid $\frac{1}{2}$ of the expenses of an automobile trip and Mr. Clark paid $\frac{1}{3}$ of the expenses. Mr. Meyers paid \$4 more than Mr. Clark paid. What were the expenses of the trip, and how much did each of the two men pay?

10. Clara spent $\frac{1}{16}$ of her money for Christmas seals and three times as much for decorations for her Christmas tree. One fourth of what she had left was 12¢. How much money had she at first?

11. When 15¢ is charged for a serving of $\frac{1}{6}$ of a mince pie, what is charged for $\frac{1}{2}$ of the pie? for $\frac{1}{3}$ of it? for $\frac{5}{6}$ of it? for $\frac{2}{3}$ of it? How many servings will the pie make and how much is received for it?

12. How many feet of molding are required for the four sides of a room $21\frac{3}{4}$ ft. long and $15\frac{1}{2}$ ft. wide?

13. A 6-in. spike was driven through a $2\frac{1}{2}$-in. plank into a log. How far did it extend into the log? What fractional part of the spike is in the log?

14. Allowing a soldier $1\frac{1}{4}$ lb. of fresh meat daily, how much meat must be provided for him in a year (365 da.)? How much must be provided for 100 soldiers in one year?

15. There are 16 oz. in 1 lb. What part of a pound is 5 oz.? When 5 oz. of salted peanuts are sold for 25¢, how much is that a pound?

16. How many bushels of wheat are required to seed a field of $13\frac{1}{2}$ A. if $1\frac{1}{2}$ bu. are sowed to each acre?

17. If a $3\frac{1}{2}$-lb. dressed chicken cost $1.47, how much was that a pound?

18. The following are the ingredients necessary to make a filling for a small lemon pie for a family of 4 persons. How much of each of these ingredients must be used to make 6 such pies? to make a dozen?

$1\frac{1}{2}$ tablespoonfuls of cornstarch	1 teaspoonful of butter
$\frac{1}{2}$ cup of sugar	$\frac{1}{2}$ lemon
$\frac{1}{2}$ cup of boiling water	$\frac{1}{2}$ teaspoonful of pulverized sugar

1 egg

19. The distance between the first plant in a row and the last one is 24 ft. Each end plant is $1\frac{1}{2}$ ft. from the end of the row. How long is the row? The plants are 3 ft. apart in the row. How many plants are there in the row?

20. Two fifths of a mixture of black and green tea is green tea. There are $2\frac{1}{4}$ lb. of black tea in the mixture. What is the weight of the mixture?

21. Mrs. Light paid 48¢ for a pound of sugar and a pound of coffee. She paid $\frac{1}{5}$ as much for the sugar as for the coffee. How much did she pay for each?

22. What was the decrease in the cost of 100 lb. of sugar, when the price dropped from 18¢ to $8\frac{1}{2}$¢ a pound?

23. To make plain lemonade use $2\frac{1}{2}$ lemons and $\frac{1}{2}$ cup of sugar with sufficient water to make a quart. Estimating the cost of lemons at 2¢ each and sugar at 8¢ a pound, what is the cost of a quart of lemonade, a cup of sugar being $\frac{1}{2}$ lb.? How many quarts are required to serve 30 persons with 2 glasses of lemonade apiece, there being 4 glasses to a quart?

24. Twelve tablespoonfuls of liquid make a cup. What part of a cup is 1 tablespoonful? What part of a pint, there being 2 cups to a pint? A quart of liquid is equal to how many tablespoonfuls?

25. Show that $\frac{2}{3}+\frac{4}{5}$ does not equal $\frac{2+4}{3+5}$. What does $\frac{2}{3}+\frac{4}{5}$ equal?

26. Show that $\frac{5}{6}-\frac{2}{3}$ does not equal $\frac{5-2}{6-3}$. What does $\frac{5}{6}-\frac{2}{3}$ equal?

27. Obtain current prices at a local butcher shop and find the cost of $2\frac{1}{4}$ lb. of sirloin steak, $1\frac{1}{2}$ lb. of sausage, and $\frac{3}{4}$ lb. of dried beef.

28. Anna and Emily bought a piece of ribbon containing $3\frac{1}{2}$ yd. After Anna had used $\frac{3}{4}$ yd. of the piece, they divided the remainder equally between them. How much of the $3\frac{1}{2}$ yd. did each take?

29. How many feet of baseboard are required to go around a room $18\frac{3}{4}$ ft. long and $16\frac{1}{2}$ ft. wide, deducting for 2 doors each $3\frac{5}{12}$ ft. wide and a closet door $2\frac{5}{8}$ ft. wide?

30. Simplify by cancellation: $\dfrac{7\frac{1}{2}\times4\frac{1}{2}\times1\frac{1}{4}}{3\frac{3}{4}\times2\frac{1}{4}\times5}$.

31. When a grocer reduced sugar from 10¢ to $7\frac{1}{2}$¢ a pound, what reduction did he make on a pound? The reduction was what part of 10¢? what part of $7\frac{1}{2}$¢?

32. If the value of a fraction is equal to 1, test to see that adding the same number to both terms does not change its value. If the value of a fraction is not equal to 1, does adding the same number to both terms change its value? When the fraction is not equal to 1, does subtracting the same number from both terms change its value?

33. Illustrate each of the following rules: Multiplying or dividing both terms of a fraction by the same number does not change its value.

34. Simplify by cancellation: $\dfrac{37\frac{1}{2}\times12\frac{1}{2}\times62\frac{1}{2}}{18\frac{3}{4}\times87\frac{1}{2}\times6\frac{1}{4}}$.

35. Taking the weight of milk to be $2\frac{1}{2}$ lb. to the quart, what price is received per 100 lb. for milk that is retailed at 6¢ a pint?

36. Mr. Taylor is a tenant farmer. He receives $\frac{1}{3}$ of all the crops he raises and the owner receives $\frac{2}{3}$. If Mr. Taylor's share of the crops is 206 bu. of oats, 262 bu. of wheat, 578 bu. of corn, and 175 bu. of potatoes, what is the owner's share?

37. If a girl in the seventh grade spends $5\frac{1}{2}$ hr. in school, 10 hr. in sleep, $1\frac{3}{4}$ hr. at meals, $1\frac{1}{4}$ hr. studying, $\frac{3}{4}$ hr. helping with housework, and $1\frac{1}{3}$ hr. going to and from school, how much time has she each day for recreation?

38. Add the following numbers without writing them in a column: $6\frac{1}{2}$, $5\frac{1}{4}$, $6\frac{3}{4}$, $7\frac{1}{2}$, $9\frac{3}{4}$.

39. Show that $\frac{3}{8}\div\frac{5}{8}=3\div5$, or $\frac{3}{5}$. State a short way of finding the quotient of two fractions which have a common denominator. Test with four other examples.

DECIMALS

1. $\frac{2}{10}$ may be written .2; $\frac{2}{100}$ may be written .02; $\frac{2}{1000}$ may be written .002. Write without a denominator each of the following: $\frac{3}{10}$ $\frac{5}{10}$ $\frac{7}{100}$ $\frac{9}{100}$ $\frac{7}{1000}$ $\frac{1}{1000}$ $\frac{8}{1000}$

2. A fraction whose denominator is some power of 10 but is not written is called a *decimal fraction,* or a *decimal.* Write five decimals.

3. Beginning at tenths, name in order the places shown to the right of the decimal point in the table below; then study the table until you can name the places at any time without looking at it.

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
1	4	5	9	8	7	6	.	2	5	7	8	3	4

4. In the number in the table, the figure 2 is in tenths' place and stands for 2 *tenths.* The figure 5 is in what place and for what does it stand? Answer the same question for each of the other figures to the right of the decimal point.

5. .75 is read *75 hundredths;* read .45
.275 is read *275 thousandths;* read .416
.7456 is read *7456 ten-thousandths;* read .8775
.08325 is read *8325 hundred-thousandths;* read .04278
.000045 is read *45 millionths;* read .000025

To read a decimal, read the number as if it were an integer; then add the name of the last order.

6. Read these decimals: .92 .096 .2056 .8009 .70465
.75 .405 .6705 .9756 .894568

7. The number 1.609 is read *1 and 609 thousandths.* Notice that *and* is read at the decimal point. Read:

2.45   7.025   2.006   7.0756   3.1416

NOTE. 2.00 may be read *2* or *200 hundredths.*

8. The decimal $.33\frac{1}{3}$ is read *$33\frac{1}{3}$ hundredths.* The number $2.66\frac{2}{3}$ is read *2 and $66\frac{2}{3}$ hundredths.* Read:

$.08\frac{1}{3}$ $.12\frac{1}{2}$ $.37\frac{1}{2}$ $2.62\frac{1}{2}$ $7.33\frac{1}{3}$ $6.08\frac{1}{3}$

9. Study these three statements, observing carefully the denominator of the fraction and the name of the order in which the right-hand figure of the corresponding decimal stands:

$\frac{75}{100} = .75$ $\frac{275}{1000} = .275$ $\frac{265}{10000} = .0265$

10. Write each of the following without a denominator:

$\frac{7}{10}$ $\frac{27}{100}$ $\frac{56}{100}$ $\frac{475}{1000}$ $\frac{856}{10000}$ $\frac{9875}{10000}$ $\frac{45}{10000}$

$2\frac{5}{10}$ $3\frac{75}{100}$ $8\frac{96}{100}$ $7\frac{35}{1000}$ $9\frac{256}{10000}$ $8\frac{406}{10000}$ $7\frac{14}{10000}$

11. Write from dictation the numbers in example 6; in example 7; in example 8.

12. Write in decimal form: forty-six hundredths; four hundred eight thousandths; four hundred *and* eight thousandths; sixty-two ten-thousandths; $66\frac{2}{3}$ thousandths; $87\frac{1}{2}$ ten-thousandths.

13. Write without a denominator each of the following:

$\frac{6\frac{2}{3}}{10}$ $\frac{6\frac{2}{3}}{100}$ $\frac{14\frac{2}{7}}{100}$ $\frac{66\frac{2}{3}}{100}$ $\frac{26\frac{2}{3}}{1000}$ $\frac{18\frac{3}{4}}{1000}$ $\frac{8\frac{1}{3}}{10000}$

14. Express each of the following correct to the nearest hundredth; correct to the nearest tenth; to the nearest integer:

7.483 ft. $2.756 $3.7832   2.484 oz. $8.876   3.1416

15. Tell what each 1 represents in .1111. Then count this way: 1 tenth is 10 times 1 hundredth, 1 hundredth is · · · times 1 thousandth, 1 thousandth is · · · times 1 ten-thousandth; also,

1 ten-thousandth is $\frac{1}{10}$ of 1 thousandth, 1 thousandth is · · · of 1 hundredth, 1 hundredth is · · · of 1 tenth.

16. Write .8 and .80 as common fractions and compare their values; also .8 and .800; .8 and .8000.

17. Annexing one or more zeros to a decimal has what effect upon its value? Removing one or more zeros from the right of a decimal has what effect upon its value? Illustrate.

18. Write .08 and .8 as common fractions and compare their values; also .008 and .8; .0008 and .8. What is the effect upon the value of a number when the decimal point is moved one place to the right? two places to the right? three places to the right? one place to the left? two places to the left? three places to the left? Illustrate each case.

19. In the following pairs of numbers name those in which the two numbers have the same value:

8 and 8.0	3.5 and 3.50	6 and 60	5.0 and 5.00
.4 and .40	.4 and .04	1.3 and 1.300	.7 and 7.0
0.2 and .2	.2 and .200	2 and 2.0	0.07 and .07

20. Multiply at sight each of the following by 10; by 100; by 1000:

.07	.075	2.75	1.756	2.7	8.04	.0006	.5

21. Divide at sight each of the following by 10; by 100; by 1000:

75	3.5	75.5	4800	650	8	.7	.06

22. Express each of the following as a common fraction in lowest terms:

.2	.25	.05	.125	.625	.325	.425
.5	.75	.04	.375	.875	.3125	.4375
.7	.15	.36	.075	.175	.1875	.0625

23. When the denominator of a common fraction is an exact divisor of 10, 100, or 1000, the fraction can readily be changed to a decimal.

Thus, $\frac{3}{5}=\frac{6}{10}=.6$; also, $\frac{1}{4}=\frac{25}{100}=.25$; and $\frac{3}{8}=\frac{375}{1000}=.375$. Notice that 5 is an exact divisor of 10, that 4 is an exact divisor of 100, and that 8 is an exact divisor of 1000.

Express each of the following as a decimal:

$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{3}{25}$	$\frac{1}{50}$	$\frac{7}{25}$	$\frac{11}{20}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{7}{40}$
$\frac{1}{5}$	$\frac{4}{5}$	$\frac{2}{25}$	$\frac{7}{20}$	$\frac{3}{50}$	$\frac{9}{20}$	$\frac{17}{20}$	$\frac{5}{8}$	$\frac{1}{40}$	$\frac{9}{40}$

24. We may reduce $.33\frac{1}{3}$ to a common fraction in lowest terms, thus:

$$.33\tfrac{1}{3}=\frac{33\frac{1}{3}}{100}=\frac{3\times 33\frac{1}{3}}{3\times 100}=\frac{100}{300}=\frac{1}{3}$$

Reduce each of the following decimals to a common fraction in lowest terms; memorize these fractional and decimal equivalents:

$.12\frac{1}{2}$	$.37\frac{1}{2}$	$.06\frac{2}{3}$	$.08\frac{1}{3}$	$.83\frac{1}{3}$	$.16\frac{2}{3}$
$.62\frac{1}{2}$	$.87\frac{1}{2}$	$.66\frac{2}{3}$	$.14\frac{2}{7}$	$.06\frac{1}{4}$	$.18\frac{3}{4}$

25. Since $\frac{7}{8}$ may be regarded as $7\div 8$, we may reduce $\frac{7}{8}$ to a decimal, thus:

$$8\overline{)7.000}\atop{.875}$$

In like manner reduce each of the following to a decimal:

$\frac{5}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{3}{8}$	$\frac{43}{80}$	$\frac{7}{16}$	$\frac{25}{32}$	$\frac{29}{32}$

26. A common fraction may be expressed as a decimal of any order desired.

Thus, $\frac{7}{8}=8\overline{)7.00}$ Here $\frac{7}{8}$ is reduced to *hundredths.*
$.87\frac{1}{2}$

Reduce each of the following fractions to hundredths:

$\frac{1}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{9}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

Some fractions cannot be reduced to decimals without still retaining a fraction in the decimal; as,

$$\tfrac{5}{12}=.4\tfrac{1}{6}=.41\tfrac{2}{3}=.416\tfrac{2}{3}=.4166\tfrac{2}{3}.$$

For such fractions we frequently use convenient decimals nearly equal to the fractions, carrying the decimals out to hundredths, thousandths, ten-thousandths, etc., according to the degree of accuracy desired in the result.

27. Express each of the following fractions as a decimal, correct to the nearest hundredth; correct to the nearest thousandth; correct to the nearest ten-thousandth:

$\frac{3}{7}$ $\quad\frac{7}{12}$ $\quad\frac{5}{6}$ $\quad\frac{1}{3}$ $\quad\frac{2}{9}$ $\quad\frac{2}{3}$ $\quad\frac{5}{9}$ $\quad\frac{11}{12}$ $\quad\frac{2}{15}$

28. Express each of the following in complete decimal form, as shown in examples *a* to *f*:

a. $.2\frac{1}{2}=.25$	*b.* $.3\frac{3}{4}=.375$	*c.* $.07\frac{3}{4}=.0775$
d. $.2\frac{1}{4}=.225$	*e.* $.1\frac{3}{8}=.1375$	*f.* $.022\frac{3}{8}=.022375$
g. $.3\frac{1}{8}=$	*h.* $.6\frac{5}{8}=$	*i.* $.015\frac{1}{8}=$
j. $.9\frac{1}{8}=$	*k.* $.12\frac{1}{8}=$	*l.* $.003\frac{1}{4}=$
m. $.1\frac{3}{10}=$	*n.* $.10\frac{3}{4}=$	*o.* $.020\frac{5}{8}=$

ADDING AND SUBTRACTING DECIMALS

Add; check each result:

1.	2.	3.	4.
7.23	2.003	0.45	0.27
46.1	0.8	4.205	4.0625
0.682	0.46	0.6	0.005
0.52	18.006	14.78	7.5
1.46	0.25	9.25	7.85
0.28	8.14	0.035	0.4
7.14	0.006	0.2	2.9
9.28	0.7084	2.75	0.75

5.	6.	7.	8.
14.27	9.78	7.05	7.256
9.07	14.234	96.056	0.37
6.29	0.685	1.8	12.0785
10.34	1.0749	0.0953	9.006
7.83	5.6	0.29	0.7427
6.50	19.457	4.3	6.87
20.25	6.78	0.754	9.7
5.74	8.9015	42.92	28.004
19.39	16.54	8.0407	7.7708
0.08	0.092	3.7	5.7
17.59	12.79	5.68	0.76
0.05	6.02	46.805	3.2

Express in complete decimal form and add; check:

9.	10.	11.	12.
$0.21\frac{1}{4}$	$0.2\frac{3}{8}$	$0.022\frac{3}{8}$	$0.84\frac{5}{8}$
$0.075\frac{1}{2}$	$0.07\frac{3}{4}$	$0.01\frac{3}{16}$	$0.007\frac{3}{4}$

Subtract and check:

13.	14.	15.	16.	17.
2.8	9.08	7.004	2.7	0.82
1.075	7.605	2.7065	1.009	0.0753

18.	19.	20.	21.	22.
7.05	9.1	8.2	0.909	0.801
1.1786	7.1009	6.188	0.899	0.6998

23. In examples 9 to 12 subtract instead of adding.

MULTIPLICATION OF DECIMALS

1. Show that the answers in (1), (2), and (3) are correct, by substituting for each decimal its equivalent common fraction, performing the multiplication, and then changing the answer to a decimal:

(1) $5 \times .03 = .15$ (2) $.005 \times 3 = .015$ (3) $.05 \times .3 = .015$

In (1), (2), and (3) observe the position of the decimal point in the multiplicand, in the multiplier, and in the product.

There are as many decimal places in the product of two numbers as there are decimal places in the multiplicand and multiplier together.

2. How many decimal places in the product of each of the following? Name each product. To check, multiply a second time.

.2×.3	.02×.3	.2×.003	.002×.03	.002×.003
2×.3	.2×.03	.02×.03	.2×3	.02×.003

Are the answers correct for examples 3 to 6? Give reasons.

3.	4.	5.	6.
.025	.025	.025	.025
.25	25	250	2.5
.00625	.625	$6.25\cancel{0}$	.0625

In examples 7 to 9, how many decimal places will there be in each product? Find each product; check.

	a	*b*	*c*	*d*	*e*	*f*
7.	.72	8.7	.36	9.6	.035	.24
	.04	2.5	40	.28	.035	2.4
8.	.75	.075	920	2.46	.045	.032
	.86	.04	.46	.75	.024	460
9.	$72.88	$83.25	$9.86	$950	$70.75	$99.68
	.06	.03	.05	.07	.035	.075

DIVIDING A DECIMAL BY AN INTEGER

1. Divide .08 by 2.

$.08 \div 2 = \frac{8}{100} \div 2 = \frac{4}{100} = .04$ *In practice write:* $2\overline{).08}$ .04

Check: 2×.04 = .08

2. In example 1, how many decimal places are there in the dividend? How many in the quotient? Make a rule for fixing

the decimal point in the quotient obtained when a decimal is divided by an integer.

Divide and check:

	a	*b*	*c*	*d*	*e*	*f*
3.	3).45	4).12	5).005	6).0036	7).0049	8).00064
4.	7)4.2	8)4.48	6)3.36	5)2.425	4)31.44	9)9.081
5.	4).7	SUGGESTION. Annex zeros to the dividend.				5).6
6.	4).5	4).56	6)3.3	8)4.2	8).97	8)2.1

7. 6).29 $\dfrac{6\overline{).29000}}{.04833^{+}}$ = .0483, correct to the nearest ten-thousandth.

8. Find the quotients correct to the nearest ten-thousandth; then read each quotient correct to the nearest thousandth:

9).35 7).62 9).82 12)7.7 15)3.7 52).25

DIVIDING BY A DECIMAL

1. Divide .8 by 2. What is the quotient? Multiply both dividend and divisor by 10 and find the quotient; by 100. Compare the three quotients.

2. Use another illustration as in example 1 to show that:

Multiplying or dividing both dividend and divisor by the same number does not change the quotient.

3. Complete the following:

$.6 \div .3 = \cdots \div 3$	$.009 \div .03 = \cdots \div 3$	$.75 \div .5 = \cdots \div 5$
$.08 \div .4 = \cdots \div 4$	$.012 \div .004 = \cdots \div 4$	$7.5 \div .05 = \cdots \div 5$
$.014 \div .7 = \cdots \div 7$	$1.2 \div .04 = \cdots \div 4$	$.075 \div .005 = \cdots \div 5$

Divide:

4. 3.024 by .48

5. 302.4 by .048

Before dividing, move the decimal point to the right in both divisor and dividend as many decimal places as will make the divisor a whole number; then divide and check. The work may be arranged thus:

$.48\overline{)3.024} = 48\overline{)302.4}$ $\qquad$ $.048\overline{)302.4} = 48\overline{)302400.}$

6. In the second form of example 4 shown above, how many decimal places are there in the divisor? in the dividend? in the quotient? Answer the same question for example 5.

The number of decimal places in the divisor and quotient must together equal the number of decimal places in the dividend (with one zero or more annexed if necessary).

Using this rule, tell where the decimal point should be placed in each number above the line in the following in order to express the quotients. Check.

	a	*b*	*c*	*d*
7.	$\begin{array}{r}135\\ .25\overline{)3.375}\end{array}$	$\begin{array}{r}135\\ .25\overline{).3375}\end{array}$	$\begin{array}{r}135\\ 2.5\overline{)337.5}\end{array}$	$\begin{array}{r}135\\ 2.5\overline{).03375}\end{array}$
8.	$\begin{array}{r}135\\ .025\overline{)3.375}\end{array}$	$\begin{array}{r}135\\ 25\overline{).03375}\end{array}$	$\begin{array}{r}135\\ .25\overline{)337.50}\end{array}$	$\begin{array}{r}135\\ .25\overline{)337.5}\end{array}$

Compare examples 8, *c*, and 8, *d*. What must be done before you divide if the dividend does not contain as many decimal places as there are in the divisor?

Divide as indicated; remember how you found the answers in examples 4 and 5. Check each result.

	a	*b*	*c*	*d*
9.	$.32\overline{)2.4896}$	$7.6\overline{).35112}$	$.084\overline{)49.728}$	$.075\overline{)648.75}$
10.	$.28\overline{).19152}$	$.0054\overline{)414.72}$	$.089\overline{).71556}$	$.076\overline{)570}$
11.	$6.1\overline{)37.759}$	$.068\overline{)558.96}$	$4.7\overline{).28529}$	$.0059\overline{)4.1831}$

Find the quotients correct to the nearest thousandth; to the nearest ten-thousandth.

12.	$.15\overline{)37}$	$21\overline{)4.8}$	$.036\overline{).83}$	$4.9\overline{)18}$
13.	$35\overline{)8}$	$37\overline{).08}$	$5.8\overline{)1.7}$	$.86\overline{).04}$

ACCURACY AND SPEED TESTS

(Answer as many as possible at sight.)

Test 1 — Addition (3 minutes)

1.	2.	3.
1.04	7.09	6.08
0.37	3.02	2.48
6.08	5.76	7.56
0.30	0.78	2.09
0.96	0.95	1.97
1.58	0.12	0.08
2.62	0.28	1.48
1.09	5.76	9.02
5.07	3.22	8.32
1.09	8.49	4.56
7.06	7.68	2.89
3.49	3.28	7.28
2.26	9.15	3.46
1.96	6.07	2.88
7.20	1.09	7.06

Test 2 — Subtraction (2 minutes)

1.	.7	.06	.81	.36
	.3	.004	.1	.07
2.	.32	.8	.6	.9
	.17	.301	.209	.299
3.	.7	.72	.9	.8
	.69	.08	.619	.199
4.	.85	.701	.501	.49
	.8	.282	.402	.199
5.	.3	.75	.42	.38
	.2486	.1056	.1008	.2098

Test 3 — Multiplication (3 minutes)

1.	.2	.06	.3	.005
	.6	.05	.06	.03
2.	7.2	9.01	.4	.075
	8	.08	20	30
3.	.36	.042	.87	.054
	.05	200	8	.09
4.	$.12\frac{1}{2}$	$8.7\frac{1}{2}$	$.037\frac{1}{2}$	$.62\frac{1}{2}$
	.6	.04	6	10
5.	3.3	.06	.81	9.6
	9	$.66\frac{2}{3}$	$.06\frac{2}{3}$	$.33\frac{1}{3}$

Test 4 — Division (5 minutes)

1.	$3\overline{).06}$	$.3\overline{).006}$	$.03\overline{).6}$	$.03\overline{)6}$
2.	$10\overline{).6}$	$.5\overline{).2}$	$.6\overline{).03}$	$1.5\overline{).3}$
3.	$.02\overline{).002}$	$.02\overline{).2}$	$.6\overline{).12}$	$.4\overline{)1.8}$
4.	$.05\overline{)1}$	$.06\overline{).3}$	$20\overline{).6}$	$.8\overline{).6}$
5.	$.2\frac{1}{2}\overline{).05}$	$.2\frac{1}{2}\overline{)1.2\frac{1}{2}}$	$.3\frac{1}{3}\overline{).06\frac{2}{3}}$	$.12\frac{1}{2}\overline{)3.7\frac{1}{2}}$

Review Test

What is the best record you can make on this test?

$.3 \div 4$	$1.25 \times .5$	$.33\frac{1}{3} \div .01$	$10 \times .1$
$.03 \times .9$	$.03 \div .4$	$6 \div .8$	$8 \times .125$
$7 - 5.07$	$.1 \div .01$	$66\frac{2}{3} \times .06$	$1 \div .125$
$.2 + .002$	$6\frac{1}{4} \div .5$	$6\frac{1}{8} \times 1.6$	$3 \div .375$
$.02 - .007$	$6\frac{3}{4} + .5$	$3 - 1.12\frac{1}{8}$	$100 \div .5$

BILLS

1. If possible, you should secure blank bill forms of various kinds. You should study these forms and make others like them. Fill them in appropriately, with your name as purchaser and using current prices. You should also study bills sent out by mechanics, gas companies, and telephone and electric light companies. How is a bill *footed?* Who *receipts* a bill, and when?

2. Rule bill forms and use them to make out bills for the following, supplying at least six items for each:

Equipment for tennis	Supplies for a picnic lunch
Equipment for baseball	Material for a serge dress

Use the date on which you do this; use your name as that of the buyer and the name of some one who sells these articles as that of the seller. Find the cost of the separate items in each bill; foot and receipt it.

3. Bills for merchandise purchased or for services performed during one month are usually rendered on the first of the following month. The bill given below is an example of a bill of this kind. Read it and tell at whose store the articles mentioned in it were bought, and who was the purchaser.

MONTHLY SETTLEMENTS REQUIRED

STRAWBRIDGE & CLOTHIER

PHILADELPHIA

29 Folio 330/124 Purchases for the month of SEPTEMBER, 1921

Sold to MR. JOSEPH S. DOE
4823 SOUTH BROAD ST.
PHILADELPHIA, PA. $61.70

PAID BY CHECK NO.______ DATE______ STRAWBRIDGE & CLOTHIER

DATE	DESCRIPTION		CHARGES	CREDITS	BALANCE
1921	AMOUNT OF BILL PREVIOUSLY RENDERED		0		0
SEPT.					
13	1 COAT		35.00		
	1 PR. SHOES		10.75		
	1 BABY COACH		45.00		
	1 PR. HOSE		2.50		93.25
23	2 PR. PILLOW CASES	1.20	2.40		95.65
25	1 BOX PENCILS		.45		
	1 YD. RIBBON		.60		96.70
30 CR	1 COAT			35.00	61.70

THE LAST AMOUNT IN RIGHT HAND COLUMN IS BALANCE NOW DUE.

Usually, when a bill like that on page 44 is paid by check, only the upper part, or *stub*, is detached and sent with the check, the remainder of the bill being kept by the customer for reference. In many cases business houses no longer return receipted bills to the customer, the latter's canceled check being sufficient receipt.

4. Name the items purchased Sept. 13, the cost of each, and the cost of all.

5. Name the item purchased Sept. 23 and its cost; the items purchased Sept. 25 and their total cost. What is the total of the bill on Sept. 25?

6. Goods returned are credited as shown in this bill. What article was returned?

7. What is the amount due Strawbridge and Clothier on Sept. 30?

Using appropriate names for dealer and purchaser, render bills like that shown on page 44 for goods purchased and those returned last August as follows:

8.

Aug.		
4.	2 lb. butter	$ .48
	$2\frac{1}{2}$ lb. bacon	.38
	1 lb. coffee	.40
11.	$\frac{1}{2}$ lb. tea	.60
	5 cans tomatoes	.15
16.	3 cans corn	.14
	1 broom	.85
25.	3 doz. eggs	.52
	1 pkg. raisins	.22
	5 lb. sugar	.09
	1 bag flour	1.75
30.	1 broom (returned)	.85

9.

Aug.		
2.	1 pr. sandals	$ 1.75
	1 fern dish	1.50
	1 hat	4.75
	1 velocipede	12.75
9.	1 book, "Friendship"	.90
	1 velocipede (returned)	12.75
	$2\frac{1}{2}$ yd. dimity	.42
	$1\frac{1}{4}$ yd. georgette	1.80
22.	1 hat (returned)	4.75
	$2\frac{1}{4}$ yd. flannel	.80
24.	1 book, "Cheer"	.90
	$\frac{3}{4}$ yd. lawn	.28

ACCOUNTS

1. Here is a record of each sum received or paid out by a boy during one month. It is his *cash account* for the month. The difference between the total receipts and total expenses is called the *balance*. How much were his receipts for the month? his expenses? How much was the balance? How was *balance* on the right hand side of the account found? How much cash had he on hand at the close of the month?

1922		Receipts			1922		Expenses		
Apr.	1	Cash on hand	3	60	Apr.	1	Roller skates	1	50
	4	Planting potatoes		50		5	Fishing trip		75
	11	Clearing flower beds		75		10	Cap		75
	15	Washing windows		50		14	Relief fund		25
	22	Delivering goods		75		20	Church fund		50
	30	Delivering papers	3	50			*Balance*	5	85
			9	60				9	60
May	1	Cash on hand	5	85					

Make out and balance these accounts:

2. Receipts: Dec. 1, cash on hand, \$4.33; Dec. 1, allowance, \$0.50; Dec. 8, delivering circulars, \$0.25; Dec. 17, cleaning cellar, \$0.25; Dec. 23, shoveling snow, \$0.50.

Expenses: Dec. 3, 1 pr. gloves, \$1.50; Dec. 10, show, \$0.25; Dec. 20, 1 pr. hockey skates, \$2.00; Dec. 22, gifts, \$1.25.

3. Receipts: Dec. 1, cash on hand, \$2.10; Dec. 15, Christmas savings, \$12.50; Dec. 20, doll's dress, \$0.75; Dec. 22, gift from brother, \$1.00; Dec. 24, helping mother, \$1.00; Dec. 24, fudge, \$0.50.

Expenses: Dec. 2, cap and scarf, \$2.00; Dec. 16, Red Cross seals, \$0.10; Dec. 20, decorations, \$0.75; Dec. 24, sweater, \$7.50; Dec. 24, gifts, \$2.25.

4. Mr. Adams reports the following items of expense in producing his crop of wheat from a ten-acre field, and his receipts therefrom. Supply the missing numbers:

Expenses		Receipts	
Land rental	$30.00	250 bu. wheat @ $1.60	. .
Preparation of soil	64.10	20 T. straw @ $12.00	. . .
Fertilizer	60.00	Total receipts	. . .
Seed and drilling	47.00	Expenses to be deducted	. . .
Harvesting	30.00	Net profit from 10 acres	. . .
Threshing	37.00	Net profit per acre	. . .
Baling straw	40.00		
Marketing	80.00		

MONEY ORDERS

United States Post Office Money Orders. A sum of money not exceeding $100 may be paid by means of a postal money order purchased at any post office. This order directs the postmaster at a specified post office to pay a certain sum of money to the person named in the order, upon his presentation of the order.

Fees. For money orders payable in the United States, the fees are as follows:

From $ 0.01 to $ 2.50	3¢	From $30.01 to $ 40.00	15¢
From $ 2.51 to $ 5.00	5¢	From $40.01 to $ 50.00	18¢
From $ 5.01 to $10.00	8¢	From $50.01 to $ 60.00	20¢
From $10.01 to $20.00	10¢	From $60.01 to $ 75.00	25¢
From $20.01 to $30.00	12¢	From $75.01 to $100.00	30¢

Express Money Orders. Money may be paid by means of express money orders, which may be obtained at the offices of express companies. The rates for express money orders are as follows:

From $ 0.01 to $ 2.50	5¢	From $ 50.01 to $ 75.00	18¢
From $ 2.51 to $ 5.00	7¢	From $ 75.01 to $100.00	20¢
From $ 5.01 to $10.00	10¢	Over $100.00 at a combination of above rates	
From $10.01 to $25.00	12¢		
From $25.01 to $50.00	15¢		

The cost of a postal money order or an express money order is the sum of the amount of the order and the fee charged for it.

USING MONEY ORDERS

1. Albert's father owes the following sums of money, each of which he pays by post office money order: \$1.75, \$3.25, \$6.30, \$15.85, \$26.82, \$35. What does each money order cost?

2. What would each have cost him if he had paid by express money order?

3. Find the cost of a post office money order sent in payment of the following purchases: 6 Niagara grapevines at 35¢ each; 2 Norway maples at \$1.50 each; 4 peach trees at 60¢ each.

4. Eleanor sent to a dry goods firm in New York City for the following: $2\frac{1}{2}$ yd. of organdie at 42¢; $2\frac{3}{4}$ yd. of outing flannel at 28¢; $5\frac{1}{2}$ yd. of plaid dress gingham at 40¢. She inclosed an express money order in payment; what was the amount of the money order?

5. Edgar ordered from a firm in Chicago a suit of clothes costing \$19.25. He paid by express money order. What would a post office money order have cost? Which was the cheaper way?

6. If you wish to send a post office money order to a mail order house to pay for a bicycle costing \$32.50, explain what you must do to obtain this order and find what it will cost. If you wish to send an express money order instead of a post office money order, what will be the difference in cost?

7. If you wish to send an express money order in payment of a tennis net costing \$10.50 and a tennis racket costing \$4.50, explain what you must do to obtain this order and tell what it will cost. If you wish to send a post office money order instead of an express money order, what will be the difference in cost?

REVIEW PROBLEMS

(Check by working each problem a second time.)

1. What is the profit on a quart of milk bought for 15¢ and sold at 10¢ a glass, there being 2 glasses to a pint?

2. When apples are bought at $1.60 a bushel and sold at the rate of 2 apples for 5¢, what is the profit per bushel if the apples average 250 to the bushel?

3. Which is the better position for a stenographer, one that pays $15 a week with two slack periods of 6 weeks each during which time she is laid off without pay, or a steady position that pays $12.50 a week, basing the estimate on service for 1 year (52 weeks)? How much better is one offer than the other?

4. Norman's father bought an automobile for $2450. What was it worth at the end of 3 years if the annual depreciation was .18 of the cost?

5. .08 of the cost of furniture is considered a fair amount to allow for depreciation each year. At this estimate what is the depreciation at the end of two years on furniture which cost new as follows: bookcase, $125; rug, $75; set of parlor furniture, $400; curtains, $65; blankets, $120; tableware, $200?

6. The postage on letters and sealed matter is 2¢ an ounce or fraction thereof. What is the postage on a letter which weighs 2 oz.? $1\frac{1}{4}$ oz.? $1\frac{3}{4}$ oz.? $2\frac{1}{4}$ oz.? $\frac{3}{4}$ oz.?

7. A savings bank in Fred's town offers to pay, at the end of 50 weeks, $63.75 to each member of the Five Cent Club who pays dues of 5¢ the first week of the year, 10¢ the second week, 15¢ the third week, and so on for 50 weeks. Fred joins this club and pays his dues regularly. Compare the amount which he pays in dues with the sum which the bank pays him at the end of the 50 weeks.

8. The same savings bank organized a Ten Cent Club (see problem 7), which pays 10¢ the first week of the year, 20¢ the second week, and so on. How much does a member of this club pay in dues in 50 weeks? How much should he receive at the end of 50 weeks?

9. What is the purpose of sending a letter *special delivery?* What postage, in addition to the regular postage, is required? What is the cost of sending a special delivery letter weighing $1\frac{1}{4}$ oz.?

10. It is estimated that skim milk for feeding purposes is worth half as much per 100 lb. as corn is worth per bushel. What is 200 lb. of skim milk worth when corn is worth $0.90 a bushel? 50 lb. of skim milk? 60 lb.? 80 lb.?

11. Pigs just after weaning should be fed about 5 lb. of skim milk with each pound of corn. What is the skim milk worth which should be fed such pigs with each bushel of corn, when corn is worth $0.80 a bushel and 100 lb. of skim milk is worth half as much as a bushel of corn?

12. Dorothy saved 36¢ one week, 48¢ the second week, 51¢ the third week, and 45¢ the fourth week. What were her average savings for the four weeks?

13. The rainfall at New Orleans for the first six months of a recent year was: 4.6 in., 4.5 in., 5.3 in., 4.9 in., 3.9 in., and 6.2 in. What was the average rainfall at New Orleans for these six months? Answer correct to the nearest .01 in.

14. To find the standing of a baseball team, divide the number of games it won by the number it played, computing the result to the nearest thousandth. What is the standing of a baseball team which won 88 games in a season and lost 52? which won 75 and lost 51?

15. The batting average of a player is found by dividing the number of hits he made by the number of times he was at bat. Find the batting average of a player who was 422 times at bat and made 132 hits. Reckon the average to the nearest .001.

16. The postage on newspapers and periodicals is 1¢ for each 4 ounces or fraction thereof. What is the postage on a package of newspapers weighing 12 oz.? 6 oz.? 10 oz.? 9 oz?

17. Find out the meaning and purpose of registering a piece of mail. What is the cost?

18. If a carpenter who is earning $2000 a year puts $120 of this in a savings bank, what decimal part of his earnings does he put in the savings bank?

19. Mildred's father has an income of $3000 a year. He sets aside .25 of this for food, .20 of it for rent, .15 of it for clothing, .15 of it for operating expenses, .15 for savings, and the remainder of it for amusement, church, charity, etc. How much money does he set aside for each?

20. A farmer in northern Michigan grew 2700 bu. of potatoes on 12 acres of land for which he paid $12.50 an acre. He sold the potatoes at $0.80 a bushel. How much money had he left from the sale of the potatoes after paying for the land?

21. The postage on miscellaneous printed matter, weighing 4 pounds or less, is 1¢ for each 2 ounces or fraction thereof. What is the postage on a package of address tags weighing 8 oz.? weighing 5 oz.? 1 lb.? 20 oz.?

22. Be prepared to tell the class what matter may be sent parcel post, and what the rates are for parcels sent not more than 50 miles from the office of mailing. (See Book Two, pages 191, 192.)

23. What should be paid a plumber for $6\frac{1}{2}$ hours' work if he charges $7.20 per day of 8 hr.?

24. The number of cows in Wisconsin in a recent year was 1,819,000, and the production of milk per cow averaged 4926 lb. Estimating the value of the milk at $2.82 per hundredweight, what was the value of the milk produced in Wisconsin that year?

25. Reports from 150 New Jersey poultry farms show that in a recent year the average receipts per hen were $3.82 and the average current expenses, not including the operator's labor, were $2.34 per hen. How much more were the receipts than the expenses on one of these farms which maintained a flock of 760 hens?

26. Authorities on stock feeding estimate that 1 lb. of corn will produce .1 of a pound of beef or .2 of a pound of pork. At this estimate, how many pounds of corn are required to produce 100 lb. of beef? 100 lb. of pork?

27. In a recent period of five years, bran rose from $27.91 to $50.23 per ton. What was the increase per 100 lb. during this period, correct to the nearest cent?

28. How much more is paid a year (52 wk.) by renting a room at $3.50 a week than by the month at $12.50 a month?

29. The following is from the *Portland* (Oregon) *Journal:* "Two prune cuttings from his home in France were planted in 1857 by a French immigrant on his brother's ranch near San Jose. This was the beginning of the California prune industry, which to-day yields an annual crop of 225,000,000 pounds." How many tons (2000 lb.) are there in this annual crop?

30. It is estimated that in a recent year 29,000 tons of commercial fertilizer were used in Lancaster County, Pa., and that the average amount used on each acre was 290 lb. At this estimate, on how many acres was it used, and what was the cost if the fertilizer averaged $34.75 per ton?

CHAPTER II

LETTERS AS NUMBERS

SIMPLE OPERATIONS WITH LETTERS

1. In arithmetic how many different figures are used to write numbers? Name all the figures used.

2. What are the four simple operations in arithmetic?

3. What is the answer to the addition operation called? to the subtraction operation? to the multiplication operation? to the division operation?

4. Without working out, how do you *indicate* the sum of 6 and 2? the difference? the product? the quotient?

5. In addition and subtraction operations the numbers added or subtracted are called *terms*. What is the sum of the two terms $7+5$? What is the sum of the three terms $8+4+2$? What is the difference of the two terms $9-5$? of $12-7$?

6. Without working out, indicate the sum of 3 and 2; 7 and 5; 8 and 4; 9 and 3. Indicate the difference in each case also.

7. Letters are sometimes used to represent numbers. For example, w may represent the number of ounces in the weight of a candy box. Indicate the number of ounces in the weight of the box and 8 oz. of candy.

8. If n represents a number, what represents the number that is 1 greater than n? 2 greater than n? 10 greater than n?

9. If l represents one number and w another, what represents their sum?

10. What number is represented by $l+w$, when l represents 4 and w represents 2? when $l=5$ and $w=3$? when $l=10$ and $w=5$?

11. If m represents one number, n another, and p another, what represents their sum? What is their sum if $m=3$, $n=2$, and $p=1$?

12. If n represents a number, what represents the number that is 1 less than n? 2 less than n? 5 less than n? a less than n?

13. What number is represented by $n-1$, when $n=2$? when $n=3$? when $n=10$?

14. When $x=6$, $y=4$, and $z=1$, what number is represented by:

$x+y$	$x+z$	$y+z$	$x+y+z$	$x-y+z$
$x-z$	$y-z$	$x-y$	$x+y-z$	$x-y-z$

15. If one boy weighs p pounds and another q pounds, what represents their combined weight? If the one weighing p pounds is the heavier, what represents the difference of their weights? If the one weighing q pounds is the heavier, what represents the difference of their weights?

16. What is meant by the factors of a number? Illustrate.

17. If a stands for 5, what does $2\times a$ represent? $2\times a$ is usually written $2a$ and read *two a.*

18. What number is represented by $2n$ when $n=3$? when $n=4$? when $n=5$? when $n=10$? when $n=12$?

19. If x cents represents the cost of a quart of milk, what represents the cost of 4 qt. of milk? 8 qt.? 16 qt.? 3 gal.?

20. If a train runs r miles an hour, how far will it run in 4 hr. at the same speed? in 8 hr.? in 12 hr.? in 24 hr.? in 2 days?

21. What number does $l\times w$ represent, when $l=10$ and $w=4$? when $l=20$ and $w=10$?

22. What number does $l \times w \times n$ represent, when $l=6$, $w=4$, and $n=2$?

23. What number does $p \times r \times t$ (which may be written prt) represent, when $p=200$, $r=.06$, and $t=2$?

24. What number does $\frac{1}{2}\, a \times b$ represent, when $a=12$ and $b=6$? when $a=6$ and $b=9$?

25. What number does $B \times H$ represent, when $B=6$ and $H=10$? when $B=8$ and $H=9$?

26. When $n=6$, what does $\frac{1}{2}\, n$ represent? when $n=10$? when $n=7$?

$\frac{1}{2}\, n$ is frequently written $\frac{n}{2}$ and read *n divided by 2* or *n over 2.*

27. Name the number which each of the following represents, when $n=24$; when $n=48$:

$$\frac{n}{2} \qquad \frac{n}{3} \qquad \frac{n}{4} \qquad \frac{n}{6} \qquad \frac{n}{8} \qquad \frac{n}{12} \qquad \frac{2\,n}{6} \qquad \frac{2\,n}{8}$$

28. At c cents a yard, represent the cost of 4 yd. of cloth; $\frac{1}{2}$ yd. of cloth; $\frac{1}{4}$ yd.; $\frac{1}{3}$ yd.; $\frac{1}{8}$ yd.

29. If c cents represents the cost of a peck of potatoes, what represents the cost of a half peck? What is the cost of a peck when $c=40$? of a half peck?

30. When $n=8$, what does $\frac{3}{4}\, n$ $\left(\text{frequently written } \frac{3\,n}{4}\right)$ represent? What does $\frac{3\,n}{4}$ represent when $n=12$? when $n=20$?

31. What number does $\frac{a \times b}{2}$ represent, when $a=8$ and $b=10$? when $a=2.6$ and $b=4$? when $a=6$ and $b=3.5$?

32. When the speed of an airplane is r miles an hour, what distance does it travel in $\frac{3}{4}$ of an hour? How far does it travel in $\frac{3}{4}$ of an hour, when $r=120$?

33. What represents the perimeter of a rectangle l feet long and w feet wide? What is the perimeter, when $l=20$ and $w=10$? when $l=100$ and $w=40$?

34. If the sides of a triangle are a feet, b feet, and c feet, what is the perimeter of the triangle? What is the perimeter if $a=15$, $b=14$, and $c=13$? What is one half the perimeter? Using a, b, and c, represent one half the perimeter.

35. What number is represented by n^2, when $n=1$? when $n=2$? when $n=3$? when $n=10$? when $n=\frac{1}{2}$? when $n=.25$?

36. What number is represented by n^3, when $n=1$? when $n=2$? when $n=3$? when $n=10$? when $n=\frac{1}{2}$?

THE EQUATION

1. A certain number increased by 4 is equal to 12. What is the number?

What sign in arithmetic stands for "increased by"? "is equal to"? If you take a letter of the alphabet, say n, to stand for the number you wish to find, you may with these symbols write the statement in the above question in this short way:

$$\underbrace{\text{A certain number}}_{n}\quad\underbrace{\text{increased by}}_{+}\quad \underset{4}{4}\quad\underbrace{\text{is equal to}}_{=}\quad \underset{12}{12.}$$

Since you now have the two parts *equal* to each other, the statement when written in this short way is called an *equation*. For what number does n stand in this equation?

If you put the value of n in the place of n in the above equation, you have:

$$8+4=12$$
$$12=12.$$

This shows that the value of n as found is correct. This is *checking* the correctness of the result.

2. The number of peach trees bought by a farmer plus 10 apple trees equals 28 trees. How many peach trees did the farmer purchase?

Taking n to represent the unknown number, the whole statement may be written in a short way thus:

$$n \text{ trees} + 10 \text{ trees} = 28 \text{ trees.}$$

What is the unknown number n? Check.

3. A boy, after hoeing a certain number of rows of corn, finds that he has 20 rows yet to hoe. If he had 50 rows in all to hoe, how many has he done? Work this by forming an equation and then finding the number. Check. Solve this problem by arithmetic.

4. A boy has saved \$25. How much more does he need to buy a pony for \$75? Supposing him to yet need \$$x$, form an equation from the fact that the money he needs and the money he has saved must together make up the money he must pay for the pony. From the equation find the value of x. Check. Solve this problem by arithmetic.

5. A certain number diminished by 6 becomes 4. What is the number?

What sign may stand for "diminished by"? "becomes"? Taking n to represent the number which you wish to find, write the statement in a short way.

What have you formed? For what number does n stand?

6. Helen took 20¢ from her bank and then found that it still contained 25¢. How much money had she in her bank at first?

Take n to represent the unknown number of cents. Then this number of cents, less 20¢ withdrawn, leaves 25¢. Express this in symbols. What number is n? Check.

Solve the following equations; that is, find in each case the number for which the letter stands; check:

7. $n+6=10$	8. $n-7=3$
9. $9+x=15$	10. $9-y=4$
11. $x+2\frac{1}{2}=7$	12. $x-1\frac{1}{2}=2\frac{1}{2}$
13. $12=7+x$	14. $11=x-5$
15. $13=t+5$	16. $9=14-x$
17. p lb. $+5$ lb. $=7$ lb.	18. 7 oz. $-x$ oz. $=4$ oz.

19. 5 times a certain number is 20; what is the number?

Here if x is put for the number, how is 5 times the number written? What sign may stand for "is" in this statement? Express the statement in the question as an equation. Solve and check.

20. 7 times a certain number is 28; what is the number?

Taking x to represent the unknown number, form an equation and find the number. Check.

Solve the following equations; check:

21. $4n=12$	22. $5x=20$	23. $4x=12$	24. $3y=30$
25. $7p=28$	26. $8r=96$	27. $4l=24$	28. $5k=35$

29. Two times a certain number with 3 added makes 11. What is the number? That is, if $2x+3=11$, what is $2x$? What is x? Check.

30. If 3 times a certain number with 4 added makes 19, what is the number?

Think of the number as n; then form an equation and find $3n$; find n. Check.

31. Four times a certain number when diminished by 3 leaves 17. What is the number? Check.

If z stands for the number, what stands for 4 times the number diminished by 3? Form the equation and find $4z$; find z. Check.

32. If 6 times a certain weight diminished by 6 lb. leaves 18 lb., what is the weight? Check.

Solve the following equations; check:

33. $3x+4=13$ (What is $3x$? What is x?)

34. $4y+1=25$

35. $2n-1=7$

36. $5z-3=17$

37. $6y-5=1$

38. $5+2x=13$

39. $4p-2=6$

40. What is the number whose half is 4?

If x stands for the number, express in two ways half the number. Are these expressions equal to each other? What is the resulting equation?

41. What is the number whose third part is 2?

Let n stand for the number. Form an equation and find n. Check.

Solve the following equations; check:

42. $\frac{n}{4}=2$

43. $\frac{y}{4}=1$

44. $\frac{z}{6}=4$

45. $\frac{r}{8}=5$

46. $\frac{x}{5}=2$

47. $\frac{n}{10}=6$

48. If half a number increased by 3 makes 8, what is the number? If x stands for the number, what expression equals 8? What is x?

49. What is the number whose half when increased by 4 makes 10? Think of the number as y, form the equation, and find y. Check.

50. What is the number whose fourth part diminished by 5 leaves 1? Check.

51. One tenth of the length of a line increased by 4 in. is 24 in. What is the length of the line? Check.

Solve the following equations; check:

52. $\frac{1}{2}n+2=12$ (What is $\frac{1}{2}n$? What is n?)

	a	*b*	*c*
53.	$\frac{1}{3}n+4=20$	$\frac{1}{4}x+1=3$	$\frac{1}{4}m+5=10$
54.	$\frac{1}{6}n+3=4$	$\frac{1}{10}y+2=4$	$\frac{1}{6}k+4=12$
55.	$\frac{1}{2}p+1\frac{1}{2}=3$	$\frac{1}{3}x+\frac{1}{2}=2\frac{1}{2}$	$\frac{1}{5}m+9=11$

56. If $\frac{3}{4}x=6$, what is $\frac{1}{4}x$? What is x?

57. If $\frac{2}{3}x=8$, what is $\frac{1}{3}x$? What is x?

58. If $\frac{3}{10}n=6$, what is $\frac{1}{10}n$? What is n?

59. Illustrate that $\frac{3}{4}x=\frac{1}{4}$ of $3x$. (Let x = some number.)

60. Illustrate that $\frac{3}{8}n=\frac{1}{8}$ of $3n$.

Members of an Equation. You have learned that such statements as:

$$x=4 \qquad 2x=6 \qquad x+4=7 \qquad 2x+3=x+7$$

are called *equations*. The equation $x=4$ means that the number represented by the letter x is equal to 4. In the equation $x=4$, x is called the *first member* and 4 is called the *second member*. Name the first and second members in the other equations.

Principles Used in Solving Equations. Finding the number for which a letter stands (the *value* of a letter) in an equation is called *solving* the equation.

Any changes may be made in an equation provided these changes do not destroy the equality of its members.

(1) Does $3x+2=20$ when $x=6$? If we add 5 to each member will the results be equal? That is, will $3x+7=25$?

If the same number is added to each member of an equation, the results are equal.

(2) Does $2x+4=12$ when $x=4$? If we subtract 2 from each member will the results be equal? That is, will $2x+2=10$?

If the same number is subtracted from each member of an equation, the results are equal.

(3) Does $4x+3=5x$ when $x=3$? If we multiply each member by 2 will the results be equal? That is, will $8x+6=10x$?

If each member of an equation is multiplied by the same number, the results are equal.

(4) Does $6x+12=8x$ when $x=6$? If we divide each member by 2 will the results be equal? That is, will $3x+6=4x$?

If each member of an equation is divided by the same number (not zero), **the results are equal.**

If x stands for 4, does

$$(1)\quad 6x-2=3x+10?$$
$$(2)\quad 6x-10=3x+2?$$

In equations (1) and (2) observe that the 2 and also the 10 are in different members, and that the signs before each of these numbers are different in the two equations; hence,

A term may be transposed from one member of an equation to the other, if the sign before it is changed.

SOLVING EQUATIONS

1. If $x+6=8$, what is the value of x?

$$x+6=8$$
Transposing 6, $\quad x=8-6$
$$x=?$$
Check

2. If $x-2=6$, what is the value of x? What number must be added to 6 to make the sum equal to x?

$$x-2=6$$
Transposing 2, $\quad x=6+2$
$$x=?$$
Check

3. If $5w = 40$, what is the value of w? Check.

4. If $\frac{1}{2}n = 6$, what is the value of n? Check.

5. If $\frac{3}{4}x = 6$, what is the value of x? Check.

6. If $4x - 3 = 2x + 9$, what is the value of x?

$$4x - 3 = 2x + 9$$

(1) Transpose 3 to the second member and $2x$ to the first member thus:

$$4x - 2x = 9 + 3$$

(2) Unite the terms of each member thus:

$$2x = 12$$

$x = ?$ Check in the first equation.

Solve the following equations and check:

7. $3x = 21$
8. $4x = 11$
9. $3n = 8$
10. $\frac{1}{2}r = 4$
11. $\frac{1}{2}m = 6$
12. $\frac{1}{3}x = 7$
13. $\frac{3}{4}n = 6$
14. $\frac{2}{3}n = 8$
15. $\frac{4}{5}n = 16$
16. $\frac{1}{2}x = \frac{3}{4}$
17. $\frac{1}{5}x = 1\frac{1}{2}$
18. $\frac{1}{8}x = 12\frac{1}{2}$
19. $4y - 2 = y + 7$
20. $5n - 1 = 3n + 7$
21. $7r + 2 = 5r + 6$
22. $7m + 8 = 4m + 11$
23. $10x + 3 = 4x + 6$
24. $9n + 1 = 6n + 3\frac{1}{4}$
25. $x + 1 = \frac{1}{2}x + 2$
26. $\frac{3}{4}x + 1 = \frac{1}{2}x + 5$

SOLVING PROBLEMS

1. If your weight increased by 10 lb. is equal to 92 lb., what is your weight?

Let w = the number of pounds in your weight

Then, $w + 10 = 92$

$w = 92 - 10$

$w = ?$

Check: $? + 10 = 92$

2. Catherine and Ruth together sold 350 Christmas Red Cross seals. Catherine sold 50 more than Ruth. How many did each sell?

If x represents the number Ruth sold, what represents the number Catherine sold? Indicate the sum of the two numbers, form an equation, and solve it.

3. If n represents a number, what represents 2 times the number? What represents the number and 1 more? If 2 times the number plus 1 is equal to 17, what is the number?

4. If n represents a number, what represents a number 4 times as large? What represents the sum of the two numbers? If the sum is 30, what are the numbers?

5. If x represents a number, what represents $\frac{1}{2}$ of the number? Indicate the sum of the number and $\frac{1}{2}$ the number. If the sum is 30, what is the number?

6. If n is the number of cents which Frank has, and if John has 4 times as many, what number represents John's money? Indicate the sum of the two numbers. If the sum is 50, how many cents has each?

7. Mrs. Ray bought y yards of lining and 2 times as many yards of dress goods. What stands for the number of yards of dress goods she bought? What indicates the number of yards of both which she bought? She bought 30 yd. of both; how many yards of each did she buy?

8. In the equation $K = a \times b$, find the value of K when $a = 10$ and $b = 6$.

9. In the equation $K = \frac{a \times b}{2}$, find the value of K when $a = 6$ and $b = 4$.

10. In the equation $K = \frac{B + b}{2} \times a$, find the value of K when $B = 10$, $b = 6$, and $a = 5$.

11. In the equation $K=a\times b$, find the value of K when $a=.6$ and $b=.42$. What is the value of a when $K=24$ and $b=8$? What is the value of b when $K=.48$ and $a=.6$?

12. In the equation $K=\frac{a\times b}{2}$, what is the value of K when $a=1.5$ and $b=2.4$? What is the value of a when $K=54$ and $b=9$? What is the value of b when $K=40$ and $a=16$?

13. In the equation $s=\frac{a+b+c}{2}$, what is the value of s when $a=13$, $b=14$, and $c=15$?

14. In the equation $I=Prt$, what is the value of I when $P=500$, $r=.05$, and $t=2$?

15. In the equation $I=Prt$, what is the value of P when $I=80$, $r=.04$, and $t=2$?

16. In the equation $I=Prt$, what is the value of r when $I=210$, $P=1200$, and $t=3\frac{1}{2}$?

17. In the equation $h^2=a^2+b^2$, what is the value of h^2 when $a=4$ and $b=3$? What is the value of a^2 when $h=10$ and $b=6$? What is the value of b^2 when $h=13$ and $a=5$?

18. In the equation $A=P+Prt$, what is the value of A when $P=500$, $r=.06$, and $t=4$?

19. In the equation $K=\frac{B+b}{2}\times a$, what is the value of B when $K=84$, $b=8$, and $a=6$?

20. In the equation $K=3.1416\times r^2$, what is the value of K when $r=10$?

21. The distance s passed over by a falling body in t seconds is given in feet by the equation $s=16t^2$. Find s when $t=4$; when $t=5$.

$a = P + PRT$

$a = P + 5$

CHAPTER III

PERCENTAGE

Meaning of Per Cent. Instead of saying that Helen's father saves 16 *hundredths* of his earnings, we may say that he saves 16 *per cent* of his earnings. *Per cent* means *hundredths*. The symbol for *per cent* is %.

Thus,	1 per cent means .01	1% of = .01 time
	10 per cent means .10	10% of = .10 time
	$12\frac{1}{2}$ per cent means $.12\frac{1}{2}$	$12\frac{1}{2}$% of = $.12\frac{1}{2}$ time
	100 per cent means 1.00	100% of = 1 time
	125 per cent means 1.25	125% of = 1.25 times

The number or amount of which a given per cent is taken is sometimes called the *base*. The number of hundredths of the base taken is called the *rate per cent*, or the *rate*, and the result obtained by taking a given per cent of the base is called the *percentage*. These terms are much less frequently used than formerly.

EQUIVALENTS — FRACTIONAL, DECIMAL, AND PER CENT

1. Read the following as per cents:

$\frac{3}{100}$	$\frac{6}{100}$	$\frac{7}{100}$	$\frac{15}{100}$	$\frac{25}{100}$	$\frac{60}{100}$	$\frac{75}{100}$	$\frac{100}{100}$
$\frac{2\frac{1}{2}}{100}$	$\frac{6\frac{2}{3}}{100}$	$\frac{8\frac{1}{3}}{100}$	$\frac{6\frac{1}{4}}{100}$	$\frac{12\frac{1}{2}}{100}$	$\frac{37\frac{1}{2}}{100}$	$\frac{87\frac{1}{2}}{100}$	$\frac{33\frac{1}{3}}{100}$
.04	.09	.18	.10	.50	.75	.92	1.00
$.06\frac{1}{4}$	$.12\frac{1}{2}$	$.37\frac{1}{2}$	$.66\frac{2}{3}$	1.75	2.50	2.00	1.37

2. Write each of the following as a decimal and as a common fraction; thus, for the first write $5\% = .05 = \frac{5}{100}$.

5%	3%	12%	16%	20%	25%	75%	100%
$7\frac{1}{2}\%$	$16\frac{2}{3}\%$	$8\frac{1}{3}\%$	$66\frac{2}{3}\%$	150%	200%	275%	250%

3. Express each of the following as a decimal and as a common fraction in lowest terms; thus, for the first write $20\% = .20 = \frac{1}{5}$, and for the second, $62\frac{1}{2}\% = .62\frac{1}{2} = \frac{5}{8}$.

20%	$62\frac{1}{2}\%$	25%	$12\frac{1}{2}\%$	30%	$6\frac{2}{3}\%$	50%	$87\frac{1}{2}\%$
75%	$8\frac{1}{3}\%$	$6\frac{1}{4}\%$	12%	15%	4%	5%	$33\frac{1}{3}\%$
80%	$66\frac{2}{3}\%$	32%	$11\frac{1}{9}\%$	24%	$16\frac{2}{3}\%$	$22\frac{2}{9}\%$	$7\frac{1}{2}\%$

4. Express each of the following as a per cent:

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{2}{5}$	$\frac{3}{5}$
$\frac{3}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{50}$	$\frac{1}{9}$

5. Memorize this table:

$20\% = \frac{1}{5}$	$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$8\frac{1}{3}\% = \frac{1}{12}$
$40\% = \frac{2}{5}$	$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$60\% = \frac{3}{5}$	$75\% = \frac{3}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$80\% = \frac{4}{5}$	$100\% = 1$	$87\frac{1}{2}\% = \frac{7}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$

6. Express as a whole or mixed number:

100%	600%	300%	200%	150%	250%
500%	750%	125%	106%	104%	1000%

FINDING A GIVEN PER CENT

1. Find 25% of 36.

SUGGESTION. 25% of 36 = $\frac{1}{4}$ of 36 = ?

2. Find $62\frac{1}{2}\%$ of 24.

SUGGESTION. $62\frac{1}{2}\%$ of 24 = $\frac{5}{8}$ of 24 = ?

Find the per cents as required in examples 3 to 10:

	a	*b*	*c*
3.	50% of 6 yd.	20% of 25 ft.	25% of 40 in.
4.	30% of 50 A.	80% of 35 mi.	25% of 11 oz.
5.	5% of $40	4% of $100	20% of 40¢
6.	35% of $60	8% of $75	3% of $100

$\frac{18}{2} \times \frac{1}{100} = \frac{18}{200}$?

	a	*b*	*c*
7.	12½% of 24 lb.	37½% of 16 boys	62½% of 32%
8.	37½% of $100	12½% of $4	62½% of 16 ft.
9.	33⅓% of 90 gal.	66⅔% of 48 bu.	11⅑% of 18%
10.	18¾% of 16 qt.	6⅔% of 30¢	14²⁄₇% of 21%

11. Name 50% of each of the following numbers; then 25% of each; 20% of each; 75% of each; 100% of each:

12 10 24 40 6 5 15 ½

12. Name 33⅓% of each of the following; 66⅔% of each; 16⅔% of each:

6¢ $12 9¢ 24¢ $3 15 bu. $1.80 $2.40

13. Name 12½% of each of the following; 37½% of each; 62½% of each; 87½% of each:

8¢ 16 oz. $40 $0.80 4 bu. $0.16 $1.60 ½

14. Name 16⅔% of each of the following; 8⅓% of each:

12 24 48¢ $4.80 12 in. 144 sq. in.

15. Supply at sight the missing numbers:

25% of a bushel = ··· pk.	37½% of a peck = ··· qt.
12½% of a bushel = ··· pk.	50% of a square foot = ··· sq. in.
62½% of a gallon = ··· qt.	87½% of a pound = ··· oz.
33⅓% of a foot = ··· in.	16⅔% of a foot = ··· in.
200% of 1¢ = ···¢	20% of a dime = ···¢
66⅔% of a yard = ··· ft.	2½% of $100 = $···

16. Name 100% of each of the following:

6¢ 2 ft. $1.50 3 in. .2 ¾ 1 gal. 1.5

17. Name 200% of each number or amount in example 16; 300% of each; 400% of each; 600% of each; 1000% of each.

18. Name 125% of: $4; 8¢; 12 bu.; 16 in.; 24 hr.; $1.20.

19. Name 150% of: 6¢; 2 pk.; $2; $3; 8 ft.; $0.24.

20. Name 250% of each of the amounts in example 19; 350% of each; 1000% of each.

21. Count this way: 1% of 100 is 1, 2% of 100 is 2, and so on to 10% of 100 is 10. Change 100 to 200 and count as before.

22. If you know 1% of a number, how can you find 3% of it? 5% of it? 6% of it?

23. Name the following per cents of $100:

6%	3%	3½%	4½%	12½%	4.8%	6¼%

FINDING A GIVEN PER CENT

1. Express each of the following as a decimal; thus, for the first write 3.3% = .033, and for the second write 12½% = .125.

3.3%	12½%	3.6%	37½%	2½%	4½%
4.8%	6.75%	4¼%	4¾%	3.5%	5.25%

2. Find 4% of $465.

SUGGESTION. Multiply $465 by .04.

3. Find 4¾% of $250.

SUGGESTION. Multiply $250 by .0475.

Find the per cents as required in examples 4 to 8:

	a	*b*	*c*
4.	6% of 400 bu.	3.3% of 4.50 ft.	7½% of 37.5 days
5.	5% of 775 pt.	4¼% of 875¢	3¼% of 2000 pupils
6.	3% of 495 in.	4¾% of 3.6 lb.	4.8% of 650 ft.
7.	4% of $396.50	3.6% of 475 yd.	5½% of 200 girls
8.	2% of $125.25	4.2% of $125.50	12½% of 1000 soldiers

PROBLEMS

1. The light area of a schoolroom should be at least 20% of the floor area. What is the least number of windows 4′×8′ necessary to meet this requirement in a schoolroom if its floor space is 40′×24′?

2. A merchant's sales for a certain day were $1746, 45% of which was profit. What was the profit, and what was the cost of the goods sold?

3. How much was the weekly pay of 20 laborers reduced if they were working 44 hours a week at 75¢ an hour and the reduction in wages was 10%?

4. In an experiment with 724 head of pure Holstein cows, it was found that the average production for each animal for a period of 330 days was 14,985.9 lb. of milk, which tested 3.42% butter fat. What was the weight of the butter fat from this average production of milk?

NOTE. Butter fat is the principal constituent of butter. It is the pure oil contained in milk, cream, or butter.

5. During a recent period of seven years, the cost of food advanced 109%. Food that cost $100 at the beginning of this period cost how much at the close of it?

6. During the same period (see problem 5) hard coal advanced 151%. Coal that cost $7 per ton at the beginning of this period cost how much at the close of it?

7. A boy who received a grade of 98% on a test of 50 spelling words spelled how many of the words correctly?

8. Calves in dressing lose about 40% of their weight. What should be the dressed weight of a calf whose live weight is 110 lb.?

9. A half veal weighing 62 lb. was cut up into standard hotel cuts as here shown. Find the weight of each cut:

Cut	Per Cent	Cut	Per Cent
Leg	38.71	Neck	9.68
Shoulder	16.12	Loin	8.06
Rack	14.51	Kidney	1.61
Breast	11.31		

10. It is estimated that rugs depreciate 8% of their cost for each year of wear. Upon this basis what is the value of a rug which cost $120 four years ago?

11. A sample of stove coal was found to be 83.65% carbon (the substance which burns). How many pounds of carbon were there in a ton of 2000 lb. of this coal?

12. A $6\frac{1}{4}$-pound strip of bacon was bought for 28¢ per pound and sold sliced without rind at 50¢ a pound. What was the profit, the weight of the rind being .16 of the weight of the strip?

13. A family of five with an income of $3000 per year apportioned this income thus: 25% for food, 20% for rent, 15% for operating expenses, 20% for clothing, and 20% for savings and higher life. How much did this allow for each item?

Chemists have found that of the elements necessary for plant food, one or more than one of the following three is deficient in many soils: *nitrogen*, *phosphoric acid*, and *potash*. Various commercial fertilizers are sold which contain all three of these elements. In stating the composition of a fertilizer of this kind, use is made of such expressions (*formulas*) as 2–8–3, 4–10–6, etc. The first of these formulas means that the fertilizer contains 2% of nitrogen, 8% of phosphoric acid, and 3% of potash.

14. If nitrogen is valued at 18¢ a pound and phosphoric acid and potash each at 4.5¢ a pound, what is the value of each of these elements in a ton of a 2–8–3 fertilizer?

15. At the prices given in problem 14, what is the value of each element in a ton of fertilizer which contains .82% nitrogen, 8% phosphoric acid, and 2% potash?

Three important materials are constituents of our foods; they are (1) *protein*, which builds tissue; (2) *fats*, which yield energy or are stored as fats; (3) *carbohydrates*, which yield energy or are stored as fats.

16. A sample of 100 lb. of milk was found to be 87% water, 3.3% protein, 4% fat, 5% carbohydrates, .7% mineral matter. How many pounds of each of these substances were there in this quantity of milk?

17. A sample of white bread was found to be 35.4% water, 9.5% protein, 1.2% fat, 52.8% carbohydrates, and 1.1% mineral matter. How many pounds of each of these substances did 10 lb. of such bread contain?

18. If the author of a United States history receives 6% of the wholesale price of a book, how much will he receive from the sales of 40,000 books, if the wholesale price is 75% of the retail price of $1.60?

FINDING A NUMBER WHEN A PER CENT OF IT IS GIVEN

1. Instead of saying *50% of a number = 6*, we may say *½ of a number = 6*. If ½ of a number is 6, what is the number?

2. What is the number if 50% of it is:

2	5	8	.5	.25	1.2	2½

3. Change 50% to 25% in example 2 and answer the question; change it to 20%; to 10%.

4. What is the number if 12½% of it is:

2	3	½	¼	.25	1.5	2.5	¾

5. What is the amount if 40% of it is:

6¢	8¢	12¢	10¢	$24	$30	$2.40

6. What is the amount if 60% of it is:

$9 $12 $1.50 75¢ 45¢ $4.50 $400

7. Change 60% to $37\frac{1}{2}$% in example 6 and answer the question.

8. Supply the missing numbers:

2 gal. = 50% of ··· gal.	6 oz. = $37\frac{1}{2}$% of ··· lb.
1 ft. = $33\frac{1}{3}$% of ··· yd.	10¢ = 5% of $···
2 qt. = 25% of ··· pk.	5¢ = 4% of $···
2 qt. = $12\frac{1}{2}$% of ··· pk.	2¢ = 1% of $···

9. 8¢ is 200% of what sum? 400% of what sum? 800% of what sum?

10. 15¢ is 150% of what sum? 250% of what sum?

11. If 48¢ was 200% of what a dealer paid for a pound of bacon, how much did he pay for it?

12. If shoe strings were sold at 15¢ a pair, which was 300% of their cost, what did they cost?

13. If milk was retailed at 15¢ a glass, which was 400% of the cost, what did it cost per quart, there being 4 glasses to the quart?

14. If Edna was given a grade of 90% in a spelling test, what fractional part of the words did she spell correctly? She spelled 45 words correctly; how many words were given in the test?

FINDING A NUMBER WHEN A PER CENT OF IT IS GIVEN

We frequently have to solve more difficult problems than those of the last exercise. For example:

1. If Martha's father saves each year $260, which is 8% of his income, what is his income?

Find the answer thus:

$.08\overline{)260.00}$

EXPLANATION

We know that if we have the product of two factors and one of them, we can find the other.

Here .08 × his income = $260.

Therefore, his income = $260 ÷ .08.

In examples 2 to 8, find the number of which:

	a	*b*	*c*
2.	300 is 15%	22.5 is 15%	14.4 is 12%
3.	4.48 is 4%	1.75 is 5%	5.1 is 17%
4.	8.46 is 9%	5.76 is 6%	160 is 80%
5.	250 is $12\frac{1}{2}\%$ (Change $12\frac{1}{2}\%$ to .125)		25 is $62\frac{1}{2}\%$
6.	450 is $37\frac{1}{2}\%$	145 is $62\frac{1}{2}\%$	217 is $87\frac{1}{2}\%$
7.	660 is 3.3% (Change 3.3% to .033)		156 is 60%
8.	960 is 4.8%	220 is 5.5%	18.75 is 5.75%

9. Mr. Williams paid $27.50 for the support of the schools, which is 1% of the valuation of his property. What is the valuation of his property?

10. If 64% of the members of a class voted for the class officers and 128 votes were cast, how many members were there in the class?

11. How many pounds of milk testing 4.1% butter fat will yield 82 lb. of butter fat?

SUGGESTION. Write .041 for 4.1%.

12. 75% of the eggs which James placed in an incubator hatched. How many eggs did he place in it if 150 of them hatched?

13. At a mine 40% of the ore mined was found to be copper. How many tons of ore produce a ton of copper?

14. 29% of the men drafted for service from a certain city were found unfit for service. How many were drafted from the city if 580 were rejected?

15. In a test made at a city hotel it was found that a piece of beef after roasting weighed 14 lb. and that it had lost 30% of its weight in roasting. What did it weigh when prepared for roasting?

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

1. Frank raised 8 turkeys and sold 6 of them. What per cent of them did he sell?

SUGGESTION. $6 = \frac{3}{4}$ of 8, or ···% of 8.

Supply the missing numbers as in example 2.

	a	*b*
2.	5 is $\frac{1}{2}$ of 10	5 is 50% of 10
3.	4 is ··· of 12	4 is ···% of 12
4.	3 is ··· of 15	3 is ···% of 15
5.	9 is ··· of 12	9 is ···% of 12
6.	9 is ··· of 15	9 is ···% of 15
7.	2 is ··· of 6	2 is ···% of 6
8.	4 is ··· of 6	4 is ···% of 6
9.	2 is ··· of 16	2 is ···% of 16
10.	9 is ··· of 24	9 is ···% of 24
11.	15 is ··· of 24	15 is ···% of 24

What per cent of:

	a	*b*	*c*
12.	20 is 10?	40 is 10?	20 is 4?
13.	15 is 5?	15 is 10?	8 is 1?
14.	24 is 3?	16 is 6?	16 is 10?
15.	40 is 16?	40 is 24?	40 is 32?
16.	12 is 2?	30 is 2?	18 is 2?

17. What per cent of a gallon is a quart? a pint?

18. A number is how many per cent of itself?

19. What per cent of a number is 2 times the number? 3 times the number? $1\frac{1}{2}$ times the number? $1\frac{1}{4}$ times the number? $1\frac{3}{4}$ times the number?

20. What per cent of a number is 5 times the number? 1.5 times the number? 1.25 times the number? 2.75 times the number?

21. Read the following, supplying the missing numbers as in the first two:

\$1 is 1% of \$100	\$1.75 is 1.75% of \$100
\$20 is ···% of \$100	\$3.42 is ···% of \$100
\$75 is ···% of \$100	\$27.56 is ···% o \$100

22. In a certain town 750 men were drafted for war service. Out of this number 218 were rejected. What per cent of the drafted men were rejected? Give the result to the nearest per cent.

23. In a recent month the employees in a post office handled 58,000,000 pieces of mail matter. Of these, 119,000 were addressed defectively. What per cent were defectively addressed?

24. John's height was $5\frac{1}{2}$ ft. One year later it was $5\frac{3}{4}$ ft. What was the per cent of increase in his height?

25. A corner lot increased in value in 2 years from \$500 to \$750. What was the per cent of increase in the 2 years? What was the annual per cent of increase?

26. Owing to lack of care, a house and lot worth \$3000 in 3 years depreciated in value to \$2500. What was the per cent of depreciation?

27. A hog weighed 300 lb. on foot. When killed and dressed it weighed 243 lb. What per cent of its weight did it lose in being dressed? Its dressed weight is what per cent of its live weight?

28. In a seventh-grade test in arithmetic the average for the grade was 85%. In a test one month later, the average was increased by 5%. This increase indicated what per cent improvement upon the first test?

$\frac{57}{300}$ = $\frac{243}{300}$ = .81

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

It is quite important for you to become skillful in finding the rate per cent. Many problems like the following occur which are more difficult than those of the last exercise.

1. Samuel sold a calf for $12.60 more than he paid for it. It cost him $36 including feed. His profit was what per cent of the cost of the calf?

EXPLANATION

$$\begin{array}{r} 35 \\ .36\overline{)12.60} \end{array}$$

.01×$36, or $0.36 = the profit, if the rate were 1%.
12.60 ÷ .36 = 35. Therefore, 35% is the required rate.

Compare the way problem 1 is worked above with the following, and then work your problems by the method you can use best:

EXPLANATION

$$\begin{array}{r} .35 \\ 36\overline{)12.60} \end{array}$$

Some number × $36 = $12.60; therefore, $12.60 ÷ $36, or .35 = the number. Therefore, 35% is the required rate.

In examples 2 to 4 find what per cent of:

	a	*b*	*c*	*d*
2.	450 is 27	850 is 34	720 is 36	850 is 17
3.	276 is 69	425 is 204	568 is 85.2	455 ft. is 36.4 ft.
4.	475 is 114	932 is 74.56	$282 is $14.10	$98.50 is $5.91

5. An automobile priced $2500 was sold at a reduction of $355. What was the per cent of reduction?

6. A rug which cost $165 was estimated to have depreciated in value $11.55 after one year's service. What was the per cent of depreciation?

7. A short ton is 2000 lb. and a long ton, 2240 lb. A long ton is what per cent of a short ton? A short ton is what per cent of a long ton?

8. Of the 250 apples in a bushel taken from a tree which was not sprayed, 100 apples were found to be defective. What per cent of the apples were defective? If there were 25 bushels on

the tree and apples are worth $1½ per bushel, what was lost by failing to spray the tree?

9. When the wages of a carpenter are advanced from 75¢ an hour to 90¢ an hour, what is the per cent of advance?

10. Lumber and building material which could be bought for $100 in 1913 cost $271 in 1920. What was the per cent of increase?

11. In the seventh grade of the schools of a certain city, 225 minutes are assigned per week to the subject of arithmetic. The school is in session 1500 minutes per week. What per cent of the total time is assigned to arithmetic?

12. Richard is 22 years old and is earning $1000 a year. He left school when he was 18 years old. Walter is also 22 years old and is earning $575 a year. He left school when he was 14 years old. His earnings are what part of Richard's earnings? what per cent of Richard's earnings?

13. In ten years the population of St. Cloud, Minn., increased from 10,600 to 15,873. What was the per cent of increase in the ten years, correct to the nearest .01%?

14. The population of Arkansas was 1,574,449 in 1910 and 1,750,995 in 1920. What was the per cent of increase for the ten years correct to the nearest .01%? What was the average yearly per cent of increase?

15. In a recent season the pennant winners played 154 games of baseball and won 98 of them. What per cent of the games played did the team win? Answer correct to the nearest .1%?

16. When Sarah was 12 years old she weighed 79 lb. When she was 13 years old she weighed 90 lb. What per cent did she increase in weight during the year? Answer correct to the nearest .1%.

The Three Problems of Percentage. Finding a given per cent *of* a number is simply finding the product when the two factors of the product are given. Thus, in finding 6% of 50, we have:

$$6\% \text{ of } 50 = .06 \times 50, \text{ which is equal to } 3.$$

It is evident that since $.06 \times 50 = 3$, we may say

$$3 \div .06 = 50; \text{ also, } 3 \div 50 = .06.$$

That is, we may find a number of which a number is a given per cent (see page 71), or we may find what per cent one number is of another (see page 74). The principles involved above are but applications of:

(*1*) *Factor × factor = product.* (*2*) *Product ÷ factor = factor.*

It is easily seen from what is stated above that in each case there are three numbers involved, two of which are given, the third to be found.

THE THREE PROBLEMS OF PERCENTAGE

Before taking up the work required on pages 79 to 82, try to understand the relationship of the three problems of percentage. As an aid to this, supply the missing numbers in the following:

1.	50% of 8 is ···	4 is 50% of ···	4 is ···% of 8
2.	25% of 12 is ···	3 is 25% of ···	3 is ···% of 12
3.	20% of 15 is ···	3 is 20% of ···	3 is ···% of 15
4.	$12\frac{1}{2}$% of 16 is ···	2 is $12\frac{1}{2}$% of ··	2 is ···% of 16
5.	40% of 30 is ···	12 is 40% of ···	12 is ···% of 30
6.	$37\frac{1}{2}$% of 48 is ···	18 is $37\frac{1}{2}$% of ···	18 is ···% of 48
7.	$6\frac{1}{4}$% of 32 is ···	2 is $6\frac{1}{4}$% of ···	2 is ···% of 32
8.	$8\frac{1}{3}$% of 24 is ···	2 is $8\frac{1}{3}$% of ···	2 is ···% of 24
9.	6% of 200 is ···	12 is 6% of ···	12 is ···% of 200
10.	3% of 148 is ···	4.44 is 3% of ···	4.44 is ···% of 148

.04½ = .045 .0475

REVIEW OF PERCENTAGE

1. Express each of the following per cents as a decimal:

$4\frac{1}{2}\%$ $4\frac{3}{4}\%$ 12% 1.8% 6.45% $1\frac{3}{4}\%$ 4.2% 7.42%

2. Express each of these decimals as a per cent:

.03 .25 $.03\frac{3}{4}$ .065 034 .0345 .0641

3. Express in lowest terms the fraction which is equivalent to $12\frac{1}{2}\%$; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$; $16\frac{2}{3}\%$; $8\frac{1}{3}\%$; $6\frac{1}{4}\%$; $6\frac{2}{3}\%$.

4. Name the answers at sight:

40% of 20	$12\frac{1}{2}\%$ of 24	$33\frac{1}{3}\%$ of 21	$8\frac{1}{3}\%$ of 36
60% of 25	$37\frac{1}{2}\%$ of 16	$66\frac{2}{3}\%$ of 15	$6\frac{1}{4}\%$ of 16
75% of 40	$62\frac{1}{2}\%$ of 32	$16\frac{2}{3}\%$ of 30	$6\frac{2}{3}\%$ of 30

5. Read, supplying the missing amounts:

\$2 = 50% of ⋯	20¢ = $33\frac{1}{3}\%$ of ⋯	8¢ = 4% of ⋯
\$6 = $12\frac{1}{2}\%$ of ⋯	60¢ = $66\frac{2}{3}\%$ of ⋯	2¢ = 1% of ⋯
\$6 = $37\frac{1}{2}\%$ of ⋯	8¢ = $16\frac{2}{3}\%$ of ⋯	3¢ = 5% of ⋯

6. Read, supplying the missing numbers:

2¢ = ⋯% of 8¢	\$6 = ⋯% of \$18	\$2 = ⋯% of \$50
20¢ = ⋯% of 40¢	\$6 = ⋯% of \$9	\$6 = ⋯% of \$100
5¢ = ⋯% of \$1	\$18 = ⋯% of \$20	\$3 = ⋯% of \$1.50

Read, supplying the missing numbers or amounts:

7. $12\frac{1}{2}\%$ of 72 = ⋯	9 = $12\frac{1}{2}\%$ of ⋯	9 = ⋯% of 72
8. $33\frac{1}{3}\%$ of 36 = ⋯	12 = $33\frac{1}{3}\%$ of ⋯	12 = ⋯% of 36
9. 200% of 10 = ⋯	20 = 200% of ⋯	20 = ⋯% of 10
10. 4% of \$300 = ⋯	\$12 = 4% of ⋯	\$12 = ⋯% of \$300

REVIEW PROBLEMS IN PERCENTAGE

1. Mr. White bought a house for \$6000. He made one payment of 10% of this purchase price and later paid an additional 10% of this price. How much did he then owe on the house?

2. A bushel of clover seed (60 lb.) was found to contain only 42 lb. of good seed. What per cent of good seed did the bushel contain?

3. To test seed corn, Chester planted 50 grains. He found that 45 of them germinated. What per cent of them germinated?

4. A certain town was asked to subscribe $1000 to the Red Cross. It subscribed $1250. This subscription was what per cent of the quota (*i.e.* what it was asked to raise)? what per cent above the quota?

5. Duchess Skylark Ormsby, a Holstein-Friesian champion cow, produced in one year 27,761.07 lb. of milk which tested 4.32% butter fat. How many pounds of butter fat did she produce in that year?

6. Bess, a champion Jersey cow, produced in one year 18,783 lb. of milk, which yielded 962.8 lb. of butter fat. What per cent of butter fat did this milk yield, correct to the nearest .01%?

7. If the present price of eggs, 85¢ a dozen, is 425% of the price they sold for ten years ago, what did they sell for ten years ago?

8. School was in session 20 da. in December. Ralph's per cent of attendance marked on his report was 90. How many days did he attend?

9. Name a measure that is 800% of a quart; 50% of a quart.

10. George bought a second-hand bicycle for $15. The bicycle cost $40 when new. George paid what per cent of $40 for it? How much less than $40 did he pay for it? what per cent less?

11. Mr. Harvey purchased a home for $7500. He made a cash payment of 10% on the purchase price and later a payment of $1750 more. What per cent of the purchase price did he still owe?

$\frac{4}{5} = \frac{5}{4} = 1.25$

12. A merchant wishing to compare current prices with prices of the year previous advertised as follows; find the per cent of reduction on each commodity.

	PRICE LAST YEAR	PRICE TO-DAY
Rump steak, pound	60¢	32¢
Sugar corn, can	15¢	10¢
California prunes, pound	28¢	21¢
Lima beans, pound	16¢	10¢
Tapioca, pound	$12\frac{1}{2}$¢	10¢

13. A boy who attended school 19 days of the 20 school days of November, attended what per cent of full time during that month?

14. A contractor estimated the cost of the material for a certain house at $3600 and the cost of building the house at 10% more than this amount. At what price must the contractor take the job to make 10% of the cost of the material and labor?

15. At an experiment station it has been determined that out of each 100 bu. of corn, by weight, placed in a crib Nov. 30, the quantity remaining on May 31 after shrinkage is 93.1 bu. by weight. What is the per cent of shrinkage in the six months?

16. In an investigation made to ascertain who got the money paid by the consumer for Wisconsin butter, it was shown that for each pound sold at an average price of 34.55¢, the farmer received 23.33¢. What per cent of the price paid did the farmer receive, correct to the nearest .01%?

17. 200 lb. of potatoes were found to contain 32.4 lb. of starch. From this experiment, what per cent of potatoes proved to be starch?

18. What per cent fat is corn if 1000 lb. of corn were found to contain 54 lb. of fat?

19. After a laborer's wages were increased 20%, they were 72¢ an hour. What were they before the increase?

20. Diminish $1000 by 10% of itself, the result by 33⅓% of itself, and that result by 12½% of itself. What is the final result? How many per cent of $1000 is it?

21. The attendance at the Grove School last year was 612; the attendance this year is 708. Find the per cent of increase correct to the nearest .01%.

22. Wood ashes contain about 5% potash, 1.5% phosphoric acid, and 32.5% lime. How many pounds of each of these substances are there in 1000 lb. of wood ashes?

23. How much does a merchant lose on a bill of $62.20 if he is able to collect only 75% of it?

24. The standard gold coin of the United States is 9 parts pure gold and 1 part alloy; what per cent is alloy? What per cent is pure gold?

25. Draw a line 8 in. long to represent 24 hr. Divide the line so as to indicate how James spends a day if he spends 37½% of it in sleep, 25% in school, 6¼% at meals, 6¼% in home work and the rest in recreation.

Trade Discount. Edward read the following advertisement: "*Great February Sale of Furniture.* Every piece of furniture in our stock is marked this morning on the regular price tags with a deduction of 33⅓%, with the exception of certain lots at a deduction of 50%." This was a special sale, and the deduction, called a *trade discount*, was given to attract customers.

List Price. Standard retail prices are sometimes fixed for certain commodities by manufacturers, publishers, or wholesale dealers. These prices, called *list prices*, are set forth in printed lists or catalogues. Goods on sale at retail stores are usually marked at the price at which the retailer expects to sell them, *i.e.* at the list price or, as it is sometimes called, the *regular price.*

Reductions are sometimes made by retailers from list prices, as on special sales for the purpose of attracting trade or disposing of an oversupply of goods which otherwise would be carried over to another season.

Net Price. Any reduction made from list price, time price, or marked price is called trade discount or commercial discount. The sum remaining after the deduction of trade discount is the *net price* or the *net amount* of the bill.

DISCOUNTS

1. Further on in the advertisement which Edward read occurred the following:

	Regular Price	Now
Rockers	$30	$20
Dining room set	$300	$200
Solid mahogany chairs	$30	$15
Brass bedsteads	$35	$17.50

Which of these articles were offered at a discount of 50%? Which were offered at a discount of $33\frac{1}{3}$%? What discount was offered on each article?

2. At a discount of 50%, find the discount on each of the following and the net price of each:

A desk marked $80	A lamp marked $32
A rug marked $110	A desk calendar marked $1

3. At a discount of $33\frac{1}{3}$%, find the discount on the following and the net price of each:

A winter coat marked $45	A dress marked $15
Woolen blankets marked $9	A table marked $30

4. Find the net cost of a bill of hardware amounting to $187.40 on which a discount of 15% was allowed.

5. Find the net cost of a bill of dry goods amounting to $125.50 on which a discount of 16% was allowed.

6. Mrs. Trent had her grocer deliver her orders. Her bill for one month was \$42. A "cash-and-carry store" sells goods of the same grade at a reduction of 5% from the prices which Mrs. Trent paid. How much money would she have saved by buying her groceries for that month at the latter store? How much would she save in a year, if her grocery bill averaged \$42 a month?

7. What must your school district pay for 100 copies of an arithmetic sold at a discount of $16\frac{2}{3}$% from the catalogue price of \$0.72?

8. A department store offered 600 worsted suits at \$20.75 each. The regular price was \$43.50. What was the total discount offered, and what was the rate of discount?

9. A merchant offered a \$450 three-piece dining room set for \$250. What rate of discount did he offer?

10. What rate of discount was allowed on a see-saw bought for \$8, the catalogue price of which was \$10?

11. What rate of discount was allowed on a playground slide bought for \$62.50, the catalogue price of which was \$75?

Find the rate of discount offered on:

12. A cedar box marked down from \$25 to \$15.

13. A dinner set marked down from \$50 to \$32.

14. Boys' shoes marked down from \$10 to \$6.50.

15. Ribbon marked down from 60¢ a yard to 40¢ a yard.

16. Handkerchiefs marked down from \$3 a dozen to \$2.50 a dozen.

BILLS, OR INVOICES, FROM WHOLESALER TO RETAILER

Articles are generally advertised at retail prices and a sufficient discount made by the wholesaler to the retailer to allow the latter to make a reasonable profit by selling the article at the list price.

Thus, an automobile listed to retail at $2000, was sold to the retail dealer at a discount of 20%. It is easily seen that the retail dealer made a reasonable profit by selling the automobile at list price. What was his profit?

Bills sent from wholesalers to retailers show the discounts and the terms of sale.

1. The first item of the following invoice is *extended*. Extend the other items, check the amount of the invoice as given, and find the net amount, the invoice having been paid within ten days from date.

SCHWENCK & CALDWELL
Wholesale Grocers

No. 35 North Third Street
Philadelphia, 2-28-22

Sold to Mr. C. J. Carroll
Lancaster, Pa.

TERMS: 1% IN 10 DAYS.

QUANTITY ORDERED			
1	Pail chocolate drops, 32 lb. 0.25	8.00	
1	Pail broken chips, 35 lb. 0.22		
1	Can peanut brittle, 35 lb. 0.20		
1	Can jelly beans, 35 lb. 0.21		30.05

Make out bills for the following and find the net amount of each if paid within the time required to obtain the discount:

2. 478 lb. marrow beans at $0.04; 250 lb. California prunes at $0.15; 58 gal. sirup at $0.60; 2 kegs of mustard at $2.65. Terms, 2% in 10 days.

3. 12 pr. baseball shoes at $5.00; 12 pr. lawn tennis shoes at $3.60; 12 pr. golf shoes at $6.00; 6 pr. moccasins at $4.50. Terms, 2% in 10 days.

DISCOUNT SERIES

Frequently two or more discounts are made on the same bill. When two or more discounts are thus applied, they are called a *discount series* or *successive discounts*.

The net amount of a bill for $160 subject to discounts of 25%, $12\frac{1}{2}$%, and 5% may be found thus:

$160 − 25% of $160 = $120; $120 − $12\frac{1}{2}$% of $120 = $105;
$105 − 5% of $105 = $99.75, the net amount of the bill.

In a discount series the first discount is based on the amount of the bill, the second is based on the amount of the bill after the first discount has been deducted, and the third discount is based on the amount of the bill after the first two discounts have been deducted.

1. Find the net amount of a bill of groceries amounting to $250, sold subject to discounts of 20% and 5%.

2. Find the net amount of a bill of dry goods amounting to $275, sold subject to discounts of 20% and 5%.

3. Find the net amount of a bill of wall paper amounting to $160, sold subject to discounts of 20%, $12\frac{1}{2}$%, and 5%.

4. Find the single rate of discount equivalent to 50% and 20%; 30% and 20%; 50% and 10%; 20% and 20%; 10% and 10%; 25% and 10%.

5. Show that a discount of 20% and 10% on a bill of goods amounting to $200 is the same discount as one of 10% and 20% on the same bill; and hence show that it makes no difference in which order discounts are taken.

PROFIT AND LOSS

1. Henry bought 20 pennants at $0.35 each and paid expressage on them amounting to $0.50. He sold the pennants at $0.50 each. What was Henry's per cent of profit?

The *rate* or *per cent* of profit which Henry made may be determined thus:

(1) Find the selling price of the pennants.

(2) Find the cost of the pennants, including expenses (or overhead charges).

(3) Find the difference between these two amounts (net profit).

(4) Find what per cent the net profit is of the cost, including overhead charges.

Besides the actual cost of the goods bought and sold, there may be such items of expense (overhead charges) as freight charges, storage, operating expenses, and so on. If the selling price of goods is less than the cost (including overhead charges), the goods are sold at a loss.

The per cent of profit or loss is sometimes reckoned on the cost (including overhead charges) and sometimes on the selling price.

In this book, unless it is otherwise stated, the per cent of profit or loss is reckoned on the *cost*, which as used here includes overhead charges.

2. State at sight the profit or loss and the selling price:

Cost	Rate of Profit	Cost	Rate of Loss
32¢	$12\frac{1}{2}\%$	\$15	$33\frac{1}{3}\%$
\$16	25%	48¢	$37\frac{1}{2}\%$
\$48	$33\frac{1}{3}\%$	\$32	$12\frac{1}{2}\%$
\$40	$37\frac{1}{2}\%$	50¢	40%

3. State at sight the rate of profit or loss:

Cost	Profit	Cost	Loss	Cost	Selling Price
5¢	1¢	10¢	1¢	20¢	40¢
15¢	5¢	20¢	1¢	\$100	\$112
1¢	3¢	\$100	\$5	8¢	7¢
\$1	\$1.25	\$300	\$12	\$500	\$480

4. If the profit was \$1.50 and the rate of profit was 25%, evidently $\frac{1}{4}$ of the cost was \$1.50. Find the cost; the selling price.

5. If the loss was \$0.75 and the rate of loss was 20%, the loss was what part of the cost? The selling price is what part of the cost? What was the cost? What was the selling price?

6. Find the selling price:

Profit	Rate of Profit	Loss	Rate of Loss
\$3	$33\frac{1}{3}$%	$2\frac{1}{2}$¢	25%
18¢	$37\frac{1}{2}$%	\$30	$37\frac{1}{2}$%
\$400	40%	1¢	20%
12¢	200%	45¢	$12\frac{1}{2}$%

PROBLEMS

1. Find the selling price of pen points per dozen when they are bought at \$0.72 a gross and sold at a profit of 100%.

2. Find the selling price of ink per bottle when it is bought at \$0.72 per dozen bottles and sold at a profit of $66\frac{2}{3}$%.

3. A grocer bought hams at 18¢ a pound. At what price must he sell them to gain $33\frac{1}{3}$% of the cost? His profit will then be what per cent of the selling price?

4. Belts that cost \$10 a dozen were retailed at a profit of 50% of the cost. What was the selling price of each belt? The profit was what per cent of the selling price?

5. What per cent of the cost is gained on pencils bought at \$3 per gross and retailed at 5¢ each?

6. Mr. Hooper bought 20 doz. fresh eggs at 60¢ a dozen. He sold 5 doz. of them at 80¢ a dozen and 5 doz. at 75¢. The price of eggs suddenly dropped and he sold the remaining 10 doz. at 65¢ a dozen. What per cent of the cost of the eggs did he make?

7. Mr. Russell buys books at a discount of 20% from the list price and sells them at list price. What per cent of cost does he make? What per cent of list price?

14.
12
12.

8. What per cent profit is made on eggs bought at 80¢ a dozen and served at a lunch counter at a charge of 35¢ for 2 boiled eggs, supposing overhead charges to be 20¢ a dozen?

9. What per cent of the cost is the profit on goods which are sold at double their cost? What per cent of the selling price is the profit?

10. A dealer bought apples at \$1.50 a bushel. Estimating the overhead charges at \$0.50 a bushel, what must he charge per half peck for the apples to make a profit of 60%?

11. Mr. Hays made a profit of 60% by selling oranges at 80¢ a dozen. How much did they cost him?

The selling price was what per cent of the cost? Represent this per cent by a fraction. 12

12. What per cent of the cost does a newsboy make on papers that he buys at the rate of 3 for 4¢ and sells at 2¢ each? What per cent of the selling price does he make? What per cent of the selling price does the news dealer receive? What per cent of the selling price does the newsboy receive?

13. Mr. Elliot sold 1000 lb. of sugar at $7\frac{1}{2}$¢ a pound; the sugar cost him 10¢ a pound. What was his loss? What was his loss per cent?

14. When a grocer sells coffee which cost him 20¢ a pound at the rate of 3 lb. for \$1, what per cent of profit is he making?

15. In one year Mr. Foster sold groceries to the amount of \$20,000. He estimated that the cost of doing business was 20% of the sales. The goods cost him 60% of the selling price. How much profit did he make? His profit was what per cent of the sales? what per cent of the money he paid for the goods? what per cent of the money paid for the goods plus the cost of doing business?

16. Mr. Martin's automobile was worn out in 5 years. What was the average yearly per cent of depreciation?

17. Mr. Jordan estimates that the cost of running his department store is 30% of the sales. At this estimate, what was his profit on an overcoat which he bought for $30 and sold for $60? His profit was what per cent of the cost? what per cent of the selling price? what per cent of the cost plus the cost of making the sale?

18. Estimating the cost of carrying charge accounts and of delivery at $12\frac{1}{2}$% of the selling price, at what price could the dealer afford to sell groceries on the "cash-and-carry" plan for which he charges $1.20 to deliver? for which he charges $4.80? $10? $6.20?

19. When candy that cost $0.80 a pound is marked to sell at $1.50 a pound and is sold at a discount of 20% from the asking price, what per cent of the cost is gained? what per cent of the selling price?

20. A sweater that cost $2.40 was marked to gain $66\frac{2}{3}$% of the cost. What was the marked price? It was sold at a discount of 25% from the marked price. What per cent of the cost was gained? what per cent of the selling price?

21. A merchant's annual sales were $40,000. The cost of the goods sold was $25,000. His operating expenses were $10,000. What per cent of net profit did he make upon his sales?

22. The yearly sales of a manufacturer having an investment of $50,000 in a plant were $200,000. The value of the plant depreciated 10% in one year. What per cent of the sales is represented in plant depreciation?

23. Find the gain on goods listed at $480, bought at a discount of $12\frac{1}{2}$% and 10%, and sold at a profit of 20% of cost.

COMMISSION

1. Edith's brother helps to earn his way through college by taking orders for maps during his vacation. His orders for one summer amounted to $850. The publishers paid him for his services by allowing him to retain 40% of this amount. The money he earned is called *commission.*

How much was his commission on these orders?

2. One week William's father bought 1000 bu. of wheat for a grain dealer at $1.50 a bushel, receiving a commission of 2% of the cost of the wheat. The money he received is called his commission. What was his commission?

A person who transacts business for another is called an *agent.* The sum charged by an agent for transacting business, that is, the commission, is usually a certain per cent of the amount involved in the transaction. What was the amount involved in the transaction in problem 1? in problem 2?

When a sale is made, commission is reckoned on the amount of sales, and when a purchase is made, on the amount of purchase.

A quantity of goods sent by one person to another for sale is called a *consignment.* The *net proceeds* of a consignment is the sum left after all expenses of the sale are paid. What were the net proceeds in problem 1?

3. State at sight the commission and the net proceeds in each:

Sales	Rate of Commission	Sales	Rate of Commission
$400	$12\frac{1}{2}$%	$150	$33\frac{1}{3}$%
$300	1%	$250	2%

4. State at sight the commission on each of the following:

Purchase Price	Commission	Purchase Price	Commission
$1	4%	$100	$1\frac{1}{2}$%
$1000	$2\frac{1}{2}$%	$320	$12\frac{1}{2}$%

5. A real estate agent sold a farm for $16,500, receiving a commission of 5%. What was the amount of his commission?

6. Edward sold books on commission, being paid $50 a month and $12\frac{1}{2}\%$ on the amount of sales. His sales one month were $640 and another month, $740. What were his earnings for each month?

7. What must a dealer pay for 200 bu. of potatoes which he bought through a commission agent for $1.20 a bushel, if the rate of commission charged was 5%? What was the cost of each bushel of potatoes including commission?

8. Fred received $10.20 for selling vegetables. He was paid a commission of $16\frac{2}{3}\%$. How many dollars' worth of vegetables did he sell?

9. Mr. Weaver drew plans for a building costing $25,000 and supervised its erection. He charged a commission of 5% of the cost. What was his commission?

10. Mr. West charged $120 for selling a house for which the purchaser paid $6000. What rate of commission did Mr. West charge?

11. An auctioneer whose commission is 2% of the selling price of goods should charge how much for selling $1650 worth of furniture?

12. A lawyer collected 75% of a debt of $800 and charged his client a commission of 5% of the sum collected. How much should he pay over to his client? What per cent of the debt was paid over to the client?

13. An agent rented a house for Mr. Appleton for $60 a month, charging a commission of 5%. How much was Mr. Appleton's income from this house for the year, after deducting the agent's commission, together with taxes, repairs, and insurance amounting to $212.40? His income is what per cent of his investment, if the house cost him $6000?

14. William sold aluminum ware during a summer vacation. He received $50 per month and 20% on all sales in excess of $300. What did he earn during July if his sales amounted to $480 for that month? What did he earn during August, his sales being $610?

15. Edith solicits subscriptions for three magazines. One costs $4 a year, one $2.50 a year, and the third $3.50 a year. She receives a commission of 25% on all subscriptions she secures. What was her commission for a month in which she secured 42 subscriptions for the first magazine, 30 for the second, and 38 for the third?

16. Jane's older sister is a professional shopper. For one day her orders amounted to $170. She charged 5% commission on each purchase. Her expenses for the day were $1.75. How much did she have clear for that day's work?

17. What is an agent's commission, at $\frac{1}{8}$%, on a sale of property amounting to $10,000?

What is 1% of $10,000? What is a short way of finding 1% of a number?

18. Mr. Groves sells clothing on commission. He receives $80 a month and in addition commission as follows:

No commission on sales up to $1000.
1% on sales in excess of $1000.
1½% on sales in excess of $2000.
2% on sales in excess of $3000.

How much did he make in each of these months?

	Sales		Sales		Sales
Aug.	$960	*Oct.*	$2200	*Dec.*	$3200
Sept.	$1450	*Nov.*	$2600	*Jan.*	$2250

19. An automobile agency made two different propositions to Henry to act as their salesman: (1) an offer of $125 per month and his expenses; (2) his expenses and a commission of 5% on all sales up to $1000 and 6% on all sales above that sum, per month. Henry accepted the second offer. His sales the first month were $2575. Did he gain or lose by his choice?

Interest. Mr. Wells, having saved some money, decided to buy a home. He found that the one he wished to purchase would cost $3000 more than the amount he had saved. He decided, however, to buy the home and borrow $3000. The money he borrowed being the property of another, he had to pay for its use. Money paid for the use of money is called *interest.*

The sum of money on which interest is paid is called the *principal.* Thus, the $3000 which Mr Wells borrowed is the principal. The sum of the principal and the interest is called the *amount.*

Reckoning Interest. Interest is usually reckoned as a certain per cent of the principal for one year. This per cent is called the *rate of interest.* Thus, if Mr. Wells borrows $3000 at 6% interest, he pays 6% of $3000, or $180, as interest for one year. What interest would he pay for two years? for one half year?

FINDING INTEREST

1. What is the interest on $100 for 1 yr. at 6%? at 5%?
2. What is the interest on $200 for 1 yr. 6 mo. at 4%? at 5%?
3. What is the interest on $200 for 3 yr. at 6%? at 4%?
4. What is the interest on $500 for 2 yr. at $4\frac{1}{2}$%? at $4\frac{3}{4}$%?
5. What is the interest on $500 for 6 mo. at 6%? for 9 mo.?

What is the interest on:

6. $300 for 4 yr. at 6%?
7. $800 for 2 yr. at $4\frac{1}{2}$%?
8. $700 for 3 yr. at 6%?
9. $600 for 3 yr. at $3\frac{3}{4}$%?
10. $700 for 6 mo. at 6%?
11. $400 for 9 mo. at 5%?
12. $450 for $1\frac{1}{2}$ yr. at 4%?
13. $175 for $1\frac{3}{4}$ yr. at 4%?
14. $212.40 for $3\frac{1}{2}$ yr. at 6%?
15. $250 for $2\frac{3}{4}$ yr. at 4.8%?

Discard any part of a cent less than $0.005 in the answers. Count any other part of a cent as an additional cent.

16. $175.80 for $1\frac{3}{4}$ yr. at $4\frac{1}{2}$%?
17. $375.20 for 9 mo. at 6%?

18. What is the interest on a \$1000 bond of the first Liberty Loan for 1 yr., the rate being $3\frac{1}{2}\%$? on a \$5000 bond of this loan?

19. What is the interest on a \$50 bond of the second Liberty Loan for 1 yr., the rate being 4%? on a \$10,000 bond of this loan?

Interest for a Period of Less than a Year. Money borrowed for periods of less than a year is usually borrowed from banks. Mr. Wells borrowed \$3000 from a bank June 15 and paid it back Sept. 13. The banker charged him for interest for 15 days in June, 31 days in July, 31 days in August, and 13 days in September, 90 days in all. Notice that June 15 is not counted, but that Sept. 13 is counted.

Common Interest. In computing interest for a period of time less than a year, the general custom is to count 360 da. to a year, or 12 mo. of 30 da. each, thus counting 1 day's interest as $\frac{1}{360}$ of a year's interest. The interest thus computed is called *common interest.*

FINDING INTEREST FOR PERIODS OF LESS THAN ONE YEAR

1. Find the interest on \$475 from Sept. 3 to Dec. 17 at 6%.

\$475	The number of days from Sept. 3 to Dec. 17 is	Sept. 27
.06	here shown.	Oct. 31
\$28.50	The time is 105 da., or $\frac{105}{360}$ yr. or $\frac{7}{24}$ yr.	Nov. 30
7	$.06 \times \$475$, or $\$28.50 =$ the interest for **1 yr.**	Dec. 17
24)\$199.50	$\frac{7}{24} \times \$28.50 =$ the interest for 105 da.	105
\$8.31		

Find the interest on:

2. \$120 for 30 da. at $5\frac{1}{2}\%$.
3. \$250 for 60 da. at 5%.
4. \$175 for 90 da. at 4.5%.
5. \$675 for 141 da. at 6%.
6. \$320 from Mar. 1 to May 30 at 6%; at 5%; at 4%.
7. \$45 from June 10 to Nov. 7 at 6%; at 4.5%; at 3.6%.

8. $972 from Apr. 20 to Sept. 2 at 6%; at 7%; at 8%.

9. $180 from Feb. 1 (of a leap year) to June 7 at 6%.

10. $96.50 from Feb. 1 (of a common year) to Apr. 1 at 6%.

11. If $120 is borrowed June 15 for 60 da. at 6% interest, when is the loan due and what is the interest?

12. If $250 is borrowed Sept. 15 for 90 da. at 5%, when is the loan due and what is the interest?

Interest for a Period of More Than a Year. In computing interest for a period of time greater than a year, custom varies in counting the time between dates. For example, the time from May 20, 1923, to Sept. 17, 1925, may be counted thus:

2 yr. from May 20, 1923, to May 20, 1925, and 120 da. from May 20, 1925, to Sept. 17, 1925, making in all 2 yr. 120 da.

Again the time may be counted thus:

yr.	mo.	da.
1925	9	17
1923	5	20
2	3	27

September is the 9th month and May the 5th month of the year. Since 20 da. cannot be subtracted from 17 da., 1 mo. of the 9 mo. is changed to 30 da., which added to 17 da. gives 47 da. Then, subtracting as here shown, the time by this method of counting is found to be 2 yr. 3 mo. 27 da., or 2 yr. 117 da., counting 1 mo. as 30 da.

In the first method of counting, we see that the time is $2\frac{120}{360}$ yr., while by the second it is $2\frac{117}{360}$ yr. Ask your banker how he counts this time. In working examples in interest, follow the method used by the bankers of your state.

Interest is usually paid at intervals of not more than a year, so that in practice it seldom becomes necessary to reckon interest on a sum of money for a time greater than a year.

FINDING THE INTEREST FOR PERIODS OF MORE THAN ONE YEAR

In examples 1 to 8 of the following, count the time by the method used in your state. The answers are given for both methods.

Find the interest on:

1. \$1000 from Sept. 1, 1923, to Dec. 15, 1924, at 6%.
2. \$275 from Oct. 15, 1922, to Feb. 1, 1924, at 6%.
3. \$586 from Dec. 15, 1923, to Jan. 15, 1925, at 5%.
4. \$678 from May 20, 1922, to June 16, 1925, at 4%.
5. \$65 from Oct. 25, 1924, to Sept. 15, 1926, at 5.4%.
6. \$480.75 from Nov. 20, 1924, to Dec. 31, 1925, at 5.5%.
7. \$356.40 from Apr. 1, 1923, to June 1, 1925, at 3.5%.
8. \$360 from Jan. 6, 1923, to Sept. 3, 1925, at 6%.
9. In the equation,

$$I = Prt$$

let $P=200$, $r=.06$ and $t=2$; what is the value of I? What is the interest on \$200 for 2 yr. at 6%? Compare the number of dollars in this interest with the value of I. What use can you make of this equation? What does I represent? P? r? t?

10. In the equation $I=Prt$, substitute 24 for I, 200 for P, and .06 for r, and find t.

11. In the equation $I=Prt$, substitute 24 for I, 200 for P, and 2 for t, and find r.

12. In the equation $I=Prt$, substitute 24 for I, .06 for r, and 2 for t, and find P.

Special Methods of Reckoning Common Interest. When the rate of interest is 6% and 1 yr. is counted as 360 da. or 1 mo. as 30 da., which is done in common interest, certain relations are easily observed which give rise to methods of reckoning interest different from that explained on page 95. Two of the most generally used of these *special methods* are the *six per cent method* and the *60-day method.*

The Six Per Cent Method.

(1) \$0.06 = the interest on \$1 for 1 yr. (12 mo.) at 6%.
(2) \$0.00½ = the interest on \$1 for 1 mo. (30 da.) at 6%. Explain.
(3) \$0.000⅙ = the interest on \$1 for 1 da. at 6%. Explain.

Observe that in (1), (2), and (3) the principal is \$1, and the rate is 6%; that in (2) ½ of the number of months is the number of cents in the interest; that in (3) ⅙ of the number of days is the number of mills in the interest. Therefore:

To find the interest on \$1 at 6%, reckon \$0.06 for each year in the time; \$0.00½ for each month in the time; \$0.000⅙ for each day in the time.

Illustrative Example. Find the interest on \$360 from Jan. 6, 1923, to Sept. 3, 1925, at 6%.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1923, to Sept. 3, 1925, is 2 yr. 240 da.

\$0.12	= the interest on \$1 for 2 yr. at 6%.
0.04	= the interest on \$1 for 240 da. (or 8 mo.) at 6%.
\$0.16	= the interest on \$1 for 2 yr. 240 da. at 6%.
360	
\$57.60	= the required interest.

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.	
1925	9	3	
1923	1	6	
2	7	27	= the required time.

\$0.12	= the interest on \$1 for 2 yr. at 6%. Explain.
\$0.035	= the interest on \$1 for 7 mo. at 6%. Explain.
\$0.0045	= the interest on \$1 for 27 da. at 6%. Explain.
\$0.1595	= the interest on \$1 for the required time at 6%.
360	
95700	
4785	
\$57.42	= the required interest.

To find the interest at any per cent other than 6%, divide the interest at 6% by 6, and multiply by the required per cent.

FINDING INTEREST BY THE SIX PER CENT METHOD

Find by the six per cent method the interest on the following at 6%; at 4%; at 5%.

1. $7500 for 81 da.; for 96 da.; for 2 yr. 8 mo. 18 da.
2. $9450 for 5 mo.; for 7 mo.; for 1 yr. 9 mo. 10 da.
3. $250 from March 18, 1922, to June 12, 1923.
4. $775 from Sept 1, 1922, to June 15, 1925.

The 60-Day Method.

(1) .06 of the principal = the interest for 1 yr. (360 da.) at 6%.
(2) .01 of the principal = the interest for 60 da. at 6%.
(3) .001 of the principal = the interest for 6 da. at 6%.
From (2) and (3) we have the following:

To find the interest on any principal at 6%, reckon .01 of the principal as the interest for each 60 da. in the time; and .001 of the principal as the interest for each 6 da. in the time.

Illustrative Example. Find the interest on $360 from Jan. 6, 1923, to Sept. 3, 1925, at 6%.

(1) *Solution with the time reckoned in years and days.*

The time from Jan. 6, 1923, to Sept. 3, 1925, is 2 yr. 240 da.

2 yr. 240 da. = 960 da. = 16 × 60 da.

The interest for 60 da. at 6% = $3.60.

$$\begin{array}{r} \$3.60 \\ \underline{16} \\ 2160 \\ \underline{360} \\ \$57.60 \end{array} = \text{the interest for 2 yr. 240 da.}$$

(2) *Solution with the time reckoned in years, months, and days.*

yr.	mo.	da.
1925	9	3
1923	1	6
2	7	27 = the required time.

$3.60 = the interest for 60 da. at 6%.
15 (2 yr. 6 mo. = 15×60 da.)
1800
360
$54.00 = the interest for 2 yr. 6 mo. (15×60 da.).
1.80 = the interest for 1 mo. (½ of 60 da.).
1.44 = the interest for 24 da. (4×6 da.).
0.18 = the interest for 3 da. (½ of 6 da.).
$57.42 = the interest for 2 yr. 7 mo. 27 da. at 6%.

Notice that by each of the methods explained the interest is $57.60 when the time is counted in years and days, and $57.42 when it is counted in years, months, and days.

FINDING INTEREST BY THE 60-DAY METHOD

1. State at sight the interest on $100 for 60 da. at 6%; for 6 da.

2. State at sight the interest on $240 for 60 da. at 6%; for 6 da.

3. State at sight the interest on $500 for 90 da. at 6%; for 120 da.; for 12 da.

Find by the 60-day method the interest on the following at 6%; at 4%; at 5%.

4. $200 for 60 da.; for 90 da.; for 15 da.

5. $900 for 10 da.; for 15 da.; for 30 da.

6. $222 for 9 mo.; for 1 yr. 4 mo.; for 10 mo. 21 da.

7. $650 for 1 yr. 6 mo.; for 4 mo. 20 da.; for 2 mo. 15 da.

Exact Interest. Interest for a time less than a year reckoned on a basis of 365 days to the year is called *exact interest.*

Exact interest is used by the United States Government and, when large sums of money are involved, by some state and city governments, and by many trust companies.

As exact interest is based on 365 da. to a year, neither the six per cent method nor the 60-day method can be employed in finding the exact interest. Why not?

Illustrative Example. Find the interest on $240 from June 1 to Dec. 3 of the same year, at 6%.

EXPLANATION

The time from June 1 to Dec. 3

$= (29 + 31 + 31 + 30 + 31 + 30 + 3)$ da. $= 185$ da., or $\frac{37}{73}$ yr.

```
     $240
      .06
   $14.40 = the interest for 1 yr.
       37
    10080
    4320
73)$532.800($7.298+ = the interest for 185 da.
```

That is, $7.30 = the interest, correct to the nearest $0.01.

FINDING EXACT INTEREST

Find the exact interest on:

1. $1,000,000 for 60 da. at 4%.
2. $2,000,000 from June 1 to Aug. 15 at 4%.
3. $2,500,000 from Apr. 1 to June 15 at $4\frac{1}{4}$%.
4. $1,500,000 from July 2 to Oct. 15 at $3\frac{1}{2}$%.
5. $2,000,000,000 from Sept. 20 to Dec. 15 at $4\frac{1}{4}$%.
6. $3,000,000,000 from June 15 to Sept. 1 at $4\frac{3}{4}$%.
7. Show that the common interest on $100 for 72 da. at 6% is the same as the exact interest for 73 da. on the same principal at the same rate.

CHAPTER IV

MEASURES

Refer to the tables at the end of the book, if you have forgotten any of the facts required in these exercises. Then learn these facts, as you will need them in solving problems of daily life.

LIQUID MEASURE

1. Write the table of liquid measure, using proper abbreviations.

2. A barrel ($31\frac{1}{2}$ gal.) of vinegar will fill how many 5-gallon kegs? how many gallons over?

3. When the *cup* or the *glass* is used as a unit of measure, 2 cups or 2 glasses are estimated to the pint. How much money is received from a gallon of milk sold at 10¢ a glass?

4. Estimating 12 tablespoonfuls to a cup, how many tablespoonfuls are there in a pint?

5. A gallon contains 231 cu. in. How many cubic inches are there in a quart? in a pint? in a cup?

6. Reduce 2 gal. 2 qt. to quarts; to pints; to gallons.

7. A gallon of soup is estimated to make 24 portions. Each order is what part of a pint?

8. A measure called a *barrel* is equal to $31\frac{1}{2}$ gal. and a measure called a *hogshead* is equal to 63 gal. How many barrels are there in a hogshead?

$\frac{8}{24} = \frac{1}{3}$ pt.

9. When half cream and half milk is sold at 30¢ a glass, this is how much more than cost when milk is 16¢ a quart and cream 70¢ a quart?

DRY MEASURE

1. Write the table of dry measure, using proper abbreviations.

2. How many quarts are there in a bushel? how many pints?

3. A 2-bushel bag of seed corn will make how many pint packages?

4. Reduce 1 bu. 3 pk. to pecks; to quarts; to bushels; to pints.

5. Estimating that 2 tablespoonfuls of sugar weigh an ounce, how many tablespoonfuls of sugar are there in a pound of sugar? How many teaspoonfuls, 1 tablespoonful being equal to 3 teaspoonfuls?

6. A bushel of potatoes means 60 lb. of potatoes. How many bushels are there in a shipment of 13,236 lb. of potatoes?

7. Onion sets weigh 28 lb. to the bushel. How many ounces of onion sets should be given to a quart?

8. A bushel of wheat means 60 lb. of wheat. What is the value of 30,630 lb. of wheat at $1.48 a bushel? at $2.06 per cwt.?

9. A bushel contains 2150.42 cu. in. How many cubic inches are there in a peck? in a quart? in a pint?

10. Is .8 of 2150.42 cu. in. nearly 1 cu. ft.? If the number of cubic feet in a bin is multiplied by .8, is the product nearly equal to the number of bushels it contains? Why?

11. A bushel of apples means, in many states, 50 lb. of apples. Four medium sized apples weigh a pound. What does a fruit dealer receive for a bushel of such apples if he sells them at 5¢ each? If he sells them at the rate of 2 for 5¢?

12. Three medium sized potatoes weigh a pound. A bushel (60 lb.) of potatoes of this size will last a family of five how many meals, allowing each member of the family two potatoes to a meal?

13. A bushel of peanuts means, in most places, 22 lb. of peanuts. How many ounces should be given for a quart in selling peanuts? If they were bought wholesale at $3.20 a bushel, what was the cost of a quart? What was the cost of a pound?

14. A bushel of potatoes will make about 150 orders of French fried potatoes. How much was received for a bushel of potatoes served in this way at 20¢ an order? Estimating one half of the amount received from these orders as profit, what is the profit? If the potatoes cost $1.50 a bushel, what was the cost of the potatoes in one order?

15. Mr. Morris sold a bushel of plums, one half in quart boxes and one half in pint boxes. He sold the quart boxes at 20¢ each and the pint boxes at 12¢ each. How much did he receive for the bushel of plums?

16. How many more cubic inches are there in a quart dry measure than in a quart liquid measure (see problem 9, page 103 and problem 5, page 102)?

17. Four persons went to a peach orchard and bought 3 bu. of peaches for canning. They divided them equally. How many pecks did each buy? The peaches cost $1.60 a bushel. How much did each pay?

MEASURES OF WEIGHT

1. How many ounces are there in a pound? What is the abbreviation for *pound* or *pounds?* What is the abbreviation for *ounce* or *ounces?*

2. How many ounces are there in $\frac{1}{2}$ lb.? in $\frac{1}{4}$ lb.? in $\frac{3}{4}$ lb.? in $1\frac{1}{2}$ lb.?

3. What part of a pound is 8 oz.? 4 oz.? 6 oz.? 12 oz.? 14 oz.?

4. Express each of the following as pounds:

1 lb. 4 oz. 2 lb. 8 oz. 3 lb. 12 oz. 1 lb. 10 oz. 1 lb. 14 oz.

5. A bushel of coarse salt weighs 56 lb. Express in pounds the weight of a quart of coarse salt; a barrel (5 bu.). Express the weight of a quart in ounces; in pounds and ounces.

6. A barrel of flour (196 lb.) will make how many 12-pound sacks and how many pounds over?

7. How many pounds are there in a short ton (T.)?

8. A strip of bacon weighing $6\frac{1}{2}$ lb. was sold for 28¢ a pound. The rind was removed from a piece of the same weight, which then weighed $5\frac{1}{2}$ lb. and sold sliced for 40¢ a pound. How much more money was received for the second piece than for the first?

9. A turkey whose live weight was 16 lb. 9 oz. weighed when dressed 11 lb. 14 oz. What was the loss in dressing?

10. Estimating 12 tablespoonfuls to a cup and $1\frac{1}{2}$ tablespoonfuls of milk to the ounce, what is the weight of a cup of milk? How many cups of milk will weigh a pound? A cup is what per cent of a pound?

11. In some states, grains, potatoes, and some other commodities are sold by the *hundredweight* (1 cwt.). Shipments on railroads are frequently charged for by the hundredweight. Find what potatoes are selling for per hundredweight when they are selling for $1.34 per bushel (60 lb.).

12. When potatoes are selling at $1.90 per 100 lb., what is the selling price of a pound of potatoes? of a bushel?

13. At $9.25 per 100 lb. what did a farmer receive for 2 steers, one of which weighed 960 lb. and the other 1036 lb.?

105r.

14. What profit did a dealer make on 1500 lb. of potatoes bought at $1.60 per 100 lb. and sold at 96¢ per $\frac{5}{8}$-bushel basket? (1 bu. of potatoes weighs 60 lb.)

15. What is the cost of 2560 lb. of baled hay at $25.50 a ton?

16. What did a farmer receive for 4270 lb. of wheat straw which he sold at $17 a ton?

17. A coal dealer delivered to Mr. Williamson's home six loads of coal which weighed as follows: 2060 lb., 1970 lb., 2080 lb., 2110 lb., 1970 lb., and 2050 lb. What was the cost of the coal at $15.50 a ton (2000 lb.)?

18. A dealer in Pennsylvania advertised pea coal at $10.50 per 2000 lb. A ton of coal in Pennsylvania means a *long ton* (2240 lb.). What was the dealer charging per long ton for this coal?

Three systems of weights are in common use: *avoirdupois*, used for the ordinary purposes of trade; *troy*, used in weighing gold, silver, platinum, and jewelry, except gems and precious stones; *apothecaries'*, used in prescribing dry medicines and in mixing them. All these weights have the *grain* in common. The avoirdupois pound contains 7000 grains, while the troy pound as well as the apothecaries' pound contains 5760 grains.

The abundance of literature current today dealing with general health conditions, care and prevention of diseases, especially of children, employs the terms used in apothecaries' weight with such frequency as to justify the inclusion of the table in a course of study. Attention should also be directed to the fact that gold, silver, etc., are not weighed by the weights which are used for commodities in general.

19. All gold and silver coins are .9 pure metal and .1 copper. The weight of a $5-gold piece is 129 grains. What is the weight of the pure gold in a $5-gold piece? What is the weight of the copper? What is the weight of the pure gold and of the copper in a $20-gold piece?

20. The standard silver dollar weighs 412.5 grains. What is the weight of the pure silver in a silver dollar? in a half dollar? in a quarter dollar?

21. From the following table of troy weight, tell how many pennyweights there are in an ounce, also how many grains there are in a pennyweight. Then from the table of apothecaries' weight, state the number of drams in an ounce; scruples in a dram; grains in a scruple.

Troy Weight	Apothecaries' Weight
1 pound (lb.) = 12 ounces (oz.)	1 pound (℔) = 12 ounces (℥)
= 240 pennyweights (pwt.)	= 96 drams (ʒ)
= 5760 grains (gr.)	= 288 scruples (℈)
	= 5760 grains (gr.)

22. How many 2-grain quinine pills can be made from an ounce of quinine?

23. The *carat* is frequently used to denote the purity (fineness) of gold, 24 carats denoting pure gold. How many grains of gold are there in a piece of jewelry 18 carats fine and weighing 4 pwt.?

24. A dealer received a shipment of 25 tons of western bran in 100-lb. sacks. He bought the bran at $34 a ton and retailed it at $2.40 a sack. What was his profit?

25. Between the ages of 11 yr. and 14 yr. a girl should gain 12 oz. a month. How many pounds should she gain in a year?

26. A wholesale dealer received a shipment of 280 bbl. (196 lb.) of flour packed in 140-lb. jute sacks. How many sacks did he receive? What did the flour cost him at $10.75 per barrel?

27. Estimate and test the weights of various objects until you can estimate closely the following weights:

1 oz. ½ lb. 1 lb. 2 lb. 5 lb. 10 lb.

MEASURES OF TIME

1. Write the table of time, using proper abbreviations.

2. State how many seconds there are in:

$\frac{1}{2}$ min. $\frac{1}{4}$ min. $\frac{3}{4}$ min. .3 min. .7 min. .9 min.

3. Express as a fractional part of a minute; as a decimal part:

6 sec. 18 sec. 12 sec. 45 sec. 48 sec.

4. The day begins at midnight. How many hours are there from midnight to noon? from noon to midnight? What does A.M. mean? P.M.?

5. Write, using proper abbreviations, 6 o'clock forenoon; 9.45 o'clock afternoon.

6. Write the time your school opens for the morning session; for the afternoon session. Write the time each session closes.

7. How long is your morning session? your afternoon session?

8. A full date is written thus: *February 22, 1732*. Why is this a well known date?

9. Write the date of your birth; the date on which you were 10 years old; the date on which you will be 20 years old.

10. Years divisible by 4 (except century years not divisible by 400) are *leap years*. Name the last four leap years; the next four.

11. Find out what 200 B.C. means; what 200 A D. means. How many years between these two dates?

12. Find your exact age today in years, months, and days.

13. At Richmond, Va., a partial eclipse of the sun began at 9.03 A.M. and ended at 10.02 A.M. What was the duration of this eclipse?

14. On Dec. 25 the sun rises at Boston at 7.38 A.M. and sets at 4.33 P.M. How long is it from sunrise to sunset at Boston on that day?

15. Find out what is meant by decade; fortnight; centennial year.

16. What length of time is referred to in the following statement: "A half-century and a half-decade passed and then in 1920 we see Lincoln in bronze facing Westminster Abbey." From what year is this period of time counted?

Multiply:

	hr.	min.	sec.		hr.	min.	sec.		hr.	min.	sec.
17.	2	30	20	18.	6	25	30	19.	8	3	6
			5				6				15

20. Bring to class a local time table and read from it the times at which trains leave your station and the times at which they arrive there.

21. Imagine that you are about to make a journey to some large city in this country far away from your home. Try to secure time tables which will enable you to trace out your route by train, and determine the time of your departure from your home station and arrival at your destination.

MEASURES OF LENGTH

1. Write the table of length, using proper abbreviations.

2. How many feet are there in a rod? how many yards?

3. How many yards are there in a mile? how many feet?

4. A yard is what per cent of a rod? A foot is what per cent of a rod?

5. A measure called a *chain* is used by surveyors. It is 4 rd.; how many yards are there in a chain? how many feet? how many inches?

6. A chain contains 100 *links*. How many inches are there in a link?

7. A unit of length called a *hand* (4 in.) is used to measure the height of horses. How many inches are there in the height of a horse which measures $15\frac{3}{4}$ hands?

8. Reduce to inches:

1.5 ft. $1\frac{3}{8}$ ft. $1\frac{3}{4}$ ft. 2.25 ft. 2 ft. 9 in. 1 yd. 6 in.

9. Reduce to feet, expressing the number of feet in each as a mixed number; as a whole number and a decimal:

18 in. 15 in. 30 in. $2\frac{1}{2}$ yd. 1 ft. 6 in. 2 ft. 9 in.

10. Reduce to yards:

18 in. 1 ft. 6 in. 27 in. 54 in. 2 rd. 2.5 rd.

11. A double roll of wall paper is 16 yd. long. How many strips 8 ft. 6 in. long can be cut from it and what is the length of the piece remaining?

12. How many peach trees must be bought for two rows if the trees are to be planted 16 ft. apart in the row and the end trees 8 ft. from the ends of each row, the rows being 80 ft. long?

13. The distance from New York to Washington by train is 228 mi. The airline used by aviators is 80% of this distance. What is the airline distance?

14. If a *geographic mile* is 6080 ft., how many common miles in a geographic mile? (Express correct to the nearest ten-thousandth.)

15. The speed of a vessel is measured in *knots per hour* (1 knot = 1 geographic mile). The fastest day's run which has been made by a steamship is 676 knots. Using the answer to problem 14, express this distance in common miles.

16. The freighter *Santa Cruz* made the run from New York to San Francisco via the Straits of Magellan, a distance of approximately 13,000 mi., in 47 da. 4 hr. What was the average speed per hour? Answer correct to the nearest .01 mi.

17. A measure called a *fathom* (6 ft.) is used to measure the depth of the ocean. Not far from the Great Bank of Newfoundland the ocean is 2640 fathoms deep. How many miles deep is it at that point?

18. In a 200-mi. race of pigeons from Orange, Va., to Lansdale, Pa., the winning bird flew the distance in 4 hr. 35 min. 45 sec. How many yards did it fly per minute correct to the nearest .01 yd.?

19. The *league* (3 geographic miles) is used to measure distances at sea. How many common miles are there in a league? See answer to problem 14.

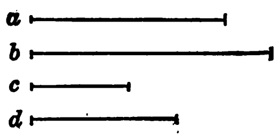
20. With a tape measure or a stout cord 50 ft. long, measure off a straight line 100 ft. long. Then step (*pace*) this distance. Knowing the number of your paces in 100 ft., how do you find the length of your pace? How long is your pace estimated to the nearest .01 of a foot?

21. Take 100 paces, then using a tape measure find the distance you paced. Knowing this distance, how do you find the length of your pace? How long is your pace, estimated to the nearest .01 of a foot? How does your answer compare with the answer to problem 20?

22. On the project of constructing 11,000 ft. of state road in Bucks Co., Pa., the lowest bid was $226,816. How much was that per foot, correct to the nearest cent? how much per mile?

MEASURING TO A SCALE

1. Measure lines *a*, *b*, *c*, and *d*. How long is each?



2. If line *a* represents 1 mi., we say that line *a* is drawn to a scale of 1 in. to 1 mi., and we write:

Scale: 1 in. = 1 mi.; or in practice: *1″ = 1 mi.*

3. When the scale is $1'' = 1$ mi., what distance is represented by *b*? *c*? *d*?

4. When the scale is $1'' = 10'$ ($10'$ means 10 ft.), what distance is represented by lines *a*? *b*? *c*? *d*?

5. Line *e* represents $100'$. Measure it to find the scale. That is, $1''$ of line represents what length on the ground?

e 

6. Draw a line to represent $2\frac{1}{2}$ mi.:

(*a*) Scale: $1'' = 1$ mi.
(*b*) Scale: $1'' = 2$ mi.
(*c*) Scale: $1'' = \frac{1}{2}$ mi.
(*d*) Scale: $1'' = 10$ mi.

7. Line *f* represents 360 mi.

f 

Draw a line as long as *f* and indicate a point on it such that the distance from this point to the nearer end of the line represents 90 mi. What length on *f* represents 180 mi.?

The scale of a map can be represented also by a fraction. Thus, if 2 in. on the map represents 150 ft. on the ground, the scale $= \frac{2}{150 \times 12} = \frac{1}{900}$, which means that a line on the map represents a line 900 times as long on the ground.

8. Study a map of your state in your geography and estimate distances from your home to several points in the state, using the scale on the map.

9. If there is a map of your county in your geography, estimate by the scale on the map distances from your home to various points in the county.

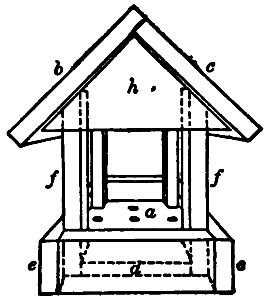
10. Draw a map of a garden 36 ft. long and 24 ft. wide to the scale of $\frac{1}{144}$.

BUILDING A BIRD HOUSE

REMARK. When it is realized that in the United States the loss to crops, fruit, etc., caused by insects is approximately $800,000,000 a year, and that birds are the chief destroyers of insects, we have one reason, but not the only one, for looking upon it as a duty to protect birds and to provide shelter for them as far as possible.

1. The nest shelter here shown[1] is designed for catbirds, brown thrashers, and song sparrows, and is intended to be placed in shrubbery. Study this shelter and the lumber diagram given on this page, and tell how many pieces of lumber must be used for the shelter.

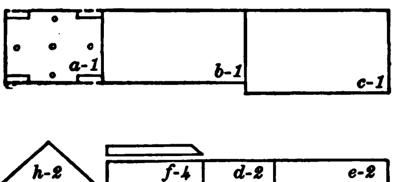
SUGGESTION. *a-1* means one piece marked *a* in the shelter; *h-2* means two pieces marked *h* in the shelter, and so on.



Nest shelter.

2. Where is the second piece marked *h* located in the shelter?

3. Where are the four pieces marked *f* located in the shelter? The two pieces marked *d*?



Scale 1/16″ = 1″

Lumber diagrams for nest shelter shown above. Thickness of boards ¾ inch.

4. On what scale is the lumber diagram drawn? Measure carefully each line on the diagram and determine the actual dimensions of each piece of lumber in the shelter.

5. Draw the lumber diagram to the scale of ⅛.

[1] Bulletin 609, U. S. Department of Agriculture.

MEASURING LINES, ARCS, AND ANGLES

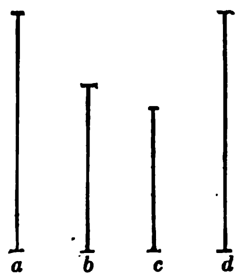
Straight Line. Any line such as line AB is called a *straight line*. A straight line has the same direction throughout its length.

The pupil should be provided with a ruler showing divisions up to sixteenths of an inch.

1. Estimate the length of the line AB, then measure it. If the error in your estimate was less than $\frac{1}{2}$ in., how much less? If it was less than $\frac{1}{4}$ in., how much less? If it was less than $\frac{1}{8}$ in., how much less?

2. Estimate the length of a group of straight lines, as *a*, *b*, *c*, and *d*. Test your estimates. Write your results in a table like the following:

Line	Estimated Length	Measured Length	Error in Estimate
a			
b			
c			
d			



3. If your error in estimating the length of line *a* (problem 2) was less than $\frac{1}{2}$ in., how much less? If it was less than $\frac{1}{4}$ in., how much less? If it was less than $\frac{1}{8}$ in., how much less? Answer the same questions for lines *b*, *c*, and *d*.

4. Estimate the length of four straight lines which you see in the schoolroom and can measure. Test, and then make a table like the one in problem 2.

5. A boy's height is marked on the blackboard. Let each of four pupils measure it and record the results. What was the greatest difference between any two measurements?

6. Suppose the measurements made for problem 5 were 4 ft. 9 in., 4 ft. $8\frac{1}{2}$ in., 4 ft. $8\frac{3}{4}$ in., and 4 ft. $9\frac{1}{4}$ in., what is the greatest difference between any two of them? What is the least difference between any two of them? Find the average of the four measurements.

Several measurements of the same dimension are apt to vary. Some of them may be too short, others may be too long; their average is usually more accurate than any one of them.

Angle. If two straight lines, as OA and OB, are drawn from the same point, as O, the *angle* AOB is formed, as in Figure 1.

FIG. 1.

The lines OA and OB are called the *sides* of the angle, and the point O is called the *vertex*.

The angle in Figure 1 is read *angle* AOB (written $\angle AOB$) or *angle* BOA (written $\angle BOA$), the letter at the vertex being placed between the other two letters. The angle may also be read *angle* O (written $\angle O$).

Equal Angles. On a thin sheet of paper trace $\angle A$, Figure 2, and show that if the vertex A is placed on F, Figure 3, and side

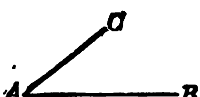


FIG. 2.

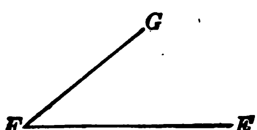


FIG. 3.

AB is made to fall along FE, side AC may be made to fall along side FG. Two such angles are said to be *equal*. Observe that the length of the sides of an angle does not affect its size.

Right Angle. If two straight lines as AB and CD meet (*intersect*), they form four angles. If the four angles are equal, each is called a *right angle* and the lines are said to be *perpendicular* to each other, or *drawn at right angles* to each other.

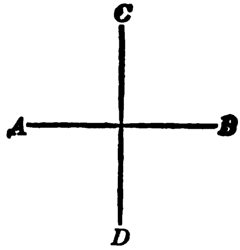


FIG. 4.

Thus, each of the angles in Figure 4 is a right angle.

Point out six right angles in your schoolroom.

Oblique Angles. Angle A, Figure 5, is less than a right angle; it is called an *acute angle.*

Angle B, Figure 6, is greater than a right angle; it is called an *obtuse angle. Oblique angle* is a name for either an acute angle or an obtuse angle.

FIG. 5.

FIG. 6.

On a sheet of paper, draw two lines which form an acute angle; an obtuse angle; a right angle.

Point out two acute angles in the schoolroom; two obtuse angles.

Plane Surface. A flat, smooth surface, like the surface of a blackboard, is called a *plane surface* or a *plane.*

Give two other examples of plane surface.

Curved Line. A line such as line AB is called a *curved line*, or a *curve.* No portion of a curved line is straight.

FIG. 7.

Circle. A closed curve such as $ABCD$ lying on a plane and such that all of its points are equally distant from a point within (the *center*) is called a *circle.*

A straight line, as OB, from the center to the circle is called a *radius.*

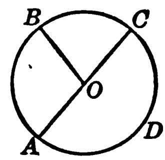


FIG. 8.

A straight line, as AOC, through the center of the circle terminating at both ends in the circle is called a *diameter.*

The length of the circle is called the *circumference.*

Any portion of the circumference, as BC, is called an *arc.*

An arc, such as ADC, equal to one half of a circle is called a *semicircle.* An arc equal to one fourth of a circle is called a *quadrant* (see Figure 9). Sometimes the surface inclosed by the curved line is called the circle, and the curved line the circumference.

1. In Figure 8, what radii are shown?

2. Name the diameter shown in Figure 8. How does it compare in length with a radius? A radius is what part of a diameter?

3. Name the arcs shown in Figure 8.

Degree of Arc. If two diameters of a circle are drawn at right angles to each other, they divide the circle into four equal parts. In the circle, Figure 9, they divide the circle into the four equal arcs: AB BC, CD, and DA (BC is read *arc* BC). What is each of these equal arcs called? If one of these equal arcs, as AB, is divided into 90 equal parts, each part is called one *degree* (1°). An arc is measured by finding how many degrees it contains.

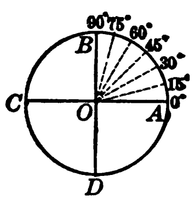


FIG. 9.

Thus, in Figure 9 arc AB is divided into six equal arcs; each of these arcs contains 15°.

1. How many degrees are there in a circle?

2. How many degrees are there in a semicircle? in a quadrant?

A Degree of Angle. An angle, as angle AOB, Figure 9, is also measured in degrees. Just as *arc* AB is said to contain 90 degrees, so the *right angle* AOB is said to contain 90 degrees. One ninetieth of a right angle is called an angle of *one degree* (1°). Angles are measured by finding how many degrees they contain.

Measuring and Constructing Angles. The pupil should have a *semicircular protractor* and should use a pencil with sharp point. To measure an angle, as angle AOD, the protractor is placed so that the straight edge falls on one side of the angle with the center at O. The line on the protractor which the other side of the angle falls along is then read. This reading gives the size of this angle.

Thus $\angle AOD$ is 20°. How many degrees in $\angle BOD$?

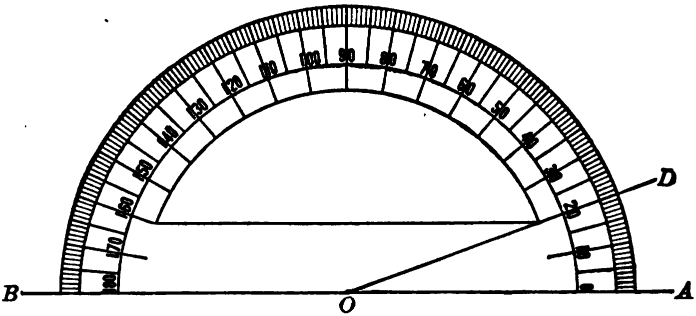


FIG. 10.

To construct an angle of any required size, as 30°, draw a straight line OA and place the straight edge of the protractor on the line OA so that the center rests at O. Count 30° from the point on OA where the curved edge touches OA and mark the point, say P. Connect O and P. The angle AOP contains 30°.

For very accurate measurements, as in surveying and astronomy, units less than a degree are used; as one sixtieth of a degree, called a *minute*, and one sixtieth of a minute, called a *second*.

TABLE OF ARC MEASURE

60 seconds (60″)	= 1 minute (1′)
60 minutes	= 1 degree (1°)
90 degrees	= 1 quadrant
360 degrees	= 1 circumference

TABLE OF ANGLE MEASURE

60 seconds (60″)	= 1 minute (1′)
60 minutes	= 1 degree (1°)
90 degrees	= 1 right angle

Drawing Circles. The pupil should be provided with a pair of compasses with sharp pencil point. If they are not available, a stiff piece of cardboard with holes, say 1 in. apart, may be used. A string with a piece of crayon attached may be used to draw circles on the blackboard.

1. Practice drawing circles with the means at your disposal.

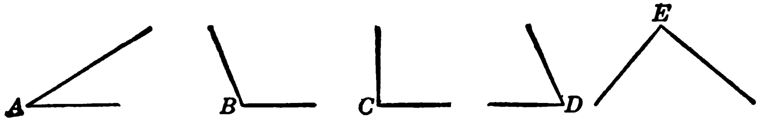
2. Draw a circle with radius 2 in.; 3 in.

3. Practice frequently until you become skillful in drawing circles on the blackboard with string and crayon.

4. Using a thin piece of cardboard, make a semicircular protractor. Then mark divisions on it 5° apart. To do this lay a piece of transparent paper on Figure 10, and draw long lines on it from *O*, 5° apart. These lines may then be used to lay off the divisions on the cardboard as required. Number the divisions as indicated in Figure 10.

MEASUREMENT AND CONSTRUCTION OF ANGLES AND ARCS

1. Draw angles that look like the following but with sides much longer. Estimate the number of degrees. Test your estimates by measuring each. Make a table like that on page 114.



2. With a protractor construct an angle of 30°; 90°; 45°: 60°; 120°; 150°; 175°.

3. Draw a circle with radius 2 in. Divide the circle into arcs of 15° each. How many arcs of 15° each does the circle contain?

4. Draw a triangle with one right angle. Measure the other two angles and find the sum of the three.

5. Draw a triangle with three acute angles. Measure the angles and find their sum.

6. Draw a triangle with one obtuse angle. Measure the other two angles and find the sum of all of them.

7. Compare the results of problems 4, 5, and 6.

8. By the use of a protractor and a circle with radius $2\frac{1}{2}$ in., draw an arc of 15°; 30°; 45°; 90°; 110°; 150°; 180°.

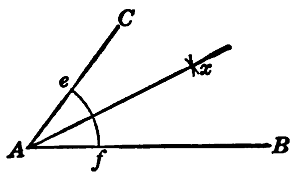
9. Measure the angles of a square and find their sum; of a rectangle.

10. Draw an irregular figure of 4 sides, measure the angles, and find their sum.

11. Compare the results in problems 9 and 10.

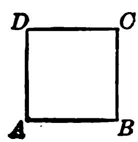
12. Given an angle, as $\angle BAC$. Required to divide $\angle BAC$ into two equal parts (*bisect* $\angle BAC$).

SUGGESTION. With A as a center and any convenient radius draw an arc cutting the sides of the angle at e and f. With e and f as centers and with equal radii draw arcs which will intersect at some point, as x. Now draw what line? Read the two angles formed. Measure them to see that they are equal.



13. Draw four angles of different sizes and bisect each.

Areas. You have already learned how to find the areas of squares and other rectangles, parallelograms, triangles, and trapezoids. It is necessary now only to review and apply what you have learned.



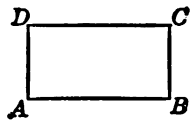
1. The area of a square is the square of the number of units in its length.

Thus, if $ABCD$ represents a square 4 ft. long, 4^2, or $16 =$ the number of square feet in $ABCD$.

2. **The area of a rectangle is equal to the product of its base and altitude.**

NOTE. By the product of the base and altitude, as used in the above principle, is meant the product of the *number of units* in the base and the *number of units* in the altitude, the base and altitude being expressed in terms of the same unit. Throughout this book when the product of two or more lines is spoken of, it is intended to have this meaning.

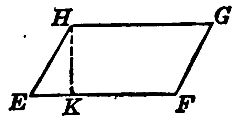
Thus, if $ABCD$ represents a rectangle whose base is 6 ft. and altitude 3 ft.,



6×3, or $18 =$ the area of $ABCD$.

3. **The area of a parallelogram is equal to the product of its base and altitude.**

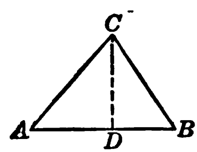
Thus, if $EFGH$ represents a parallelogram whose base (EF) is 6 ft. and altitude (HK) is 3 ft.,



6×3, or $18 =$ the area of $EFGH$.

4. **The area of a triangle is equal to one half the product of its base and altitude.**

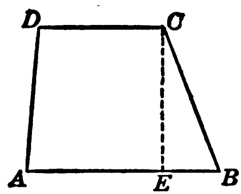
Thus, if ABC represents a triangle whose base (AB) is 6 ft. and altitude (CD) is 4 ft.,



$\frac{6 \times 4}{2}$, or $12 =$ the area of ABC.

5. **The area of a trapezoid is equal to one half the product of the altitude and the sum of the bases.**

Thus, if $ABCD$ represents a trapezoid whose altitude (CE) is 6 ft. and bases (AB) and (DC) are 8 ft. and 5 ft.,



$\frac{6 \times (8+5)}{2}$, or $39 =$ the area of $ABCD$.

SQUARE MEASURE

1. A square foot is a square a foot, or 12 in., long. How many square inches are there in a square 12 in. long? How many square inches are there in a square foot?

2. A square yard is a square a yard, or 3 ft., long. How many square feet are there in a square 3 ft. long? How many square feet are there in a square yard?

3. A square rod is a square a rod, or $5\frac{1}{2}$ yd., long. How many square yards are there in a square $5\frac{1}{2}$ yd. long? How many square yards are there in a square rod?

4. An acre is equivalent to a rectangle 20 rd. long and 8 rd. wide. How many square rods are there in an acre?

5. A square mile is a square a mile, or 320 rd., long. Find the number of square rods in a square mile. How many acres are there in a square mile?

6. Write the table of square measure, using proper abbreviations.

7. How many square yards are there in an acre? how many square feet?

8. How many square inches are there in 2 sq. ft. 6 sq. in.? 1 sq. yd.?

9. How many square feet are there in 1 sq. rd.? 288 sq. in.? 3 sq. yd. 3 sq. ft.? 2 sq. rd.?

10. How many square rods are there in 3 A. 1 sq. rd.? .6 A.? 60.5 sq. yd.? .42 A.?

11. Reduce 400 sq. in. to square feet and square inches.

12. Reduce 348 sq. rd. to acres and square rods.

13. Find the number of acres in a rectangular tract of land 363 ft. by 300 ft.

14. If you have a garden 12 ft. wide and 15 ft. long, how many square yards does it contain?

15. What will it cost to build a cement walk 3 ft. 6 in. wide and 40 ft. long at $2.70 a square yard?

16. A double roll of wall paper is 16 yd. long and 18 in. wide. How many square feet does it contain?

17. At $1.20 a square yard, find the cost of the linoleum required to cover a kitchen floor 14 ft. long and 12 ft. wide.

18. A house 36 ft. long and 30 ft. wide with porch 10 ft. wide extending along the length of the house is built on a lot $60' \times 100'$. If $\frac{3}{4}$ of the rest of the lot is used for a garden, and the remainder is lawn, how many square feet does the lawn contain?

19. The light area of a school building should not be less than .2 of the floor space. What is the least light area which a schoolroom should have if it is 36 ft. long and 30 ft. wide? How many windows $4' \times 9'$ are necessary to admit the proper amount of light?

20. How many square feet are there in the four walls and the floor of a room 24 ft. long, 21 ft. wide, and 9 ft. high? how many square yards?

21. How many square yards are there in the walls and ceiling of a room 27 ft. long, 24 ft. wide, and 9 ft. high?

22. What will it cost to plaster the walls and ceiling of a room having the dimensions of the one given in problem 21, estimating the cost to be $0.90 per square yard?

The unit of measure for lumber is the *board foot*, which is equivalent to a board a foot square and an inch thick. Lumber less than an inch thick is usually considered an inch thick in computing the number of board feet. Lumber men generally use the following rule for finding the number of board feet in a piece of lumber:

1x1x

Divide the product of the number of inches in the thickness, the number of inches in the width, and the number of feet in the length, by 12.

23. Find the number of board feet in a piece of lumber 3 in. thick, 4 in. wide, and 16 ft. long.

SUGGESTION. $\frac{3\times4\times16}{12}=?$

24. Find the number of board feet in 12 boards 16 ft. long and 12 in. wide.

SUGGESTION. When the length and the width only are given, consider the thickness as 1 in.

25. Find the number of board feet in 6 pieces 12 ft. long, 4 in. wide, and 3 in. thick.

In estimating the amount of flooring needed to cover a given area, contractors usually allow one fourth more than the area of the floor for 3″ or 4″ flooring, in order to provide for workage and waste.

Find the number of feet (board feet) of flooring $\frac{1}{2}''$ by 3″ required for floors of the following dimensions; remember what was said following problem 22 about computing the number of board feet when lumber is less than 1″ thick.

26. $16'\times12'$ 27. $18'\times15'$ 28. $18\frac{1}{2}'\times15\frac{3}{4}'$

29. At $60 per 1000 board feet (per M) find the cost of 24 pieces of lumber $16'\times6''\times3''$.

30. Allowing one third more than the area of the floor for 2″ or $2\frac{1}{2}''$ flooring, find the cost at $80 per M of the boards for a floor $24'\times18'$, if quartered oak flooring $\frac{1}{2}''\times2''$ is used.

31. The lumber mills at Eagle Gorge on Puget Sound recently cut a stick of timber which is believed to be the biggest piece of timber ever cut by a mill anywhere. It was 44 in. square and 80 ft. long. How many board feet of inch lumber did this stick contain if no allowance is made for waste in cutting?

Find the areas of these parallelograms:

	32.	33.	34.	35.	36.
Base:	18 ft.	16.5 ft.	5.5 yd.	8.25 rd.	10.8 rd.
Altitude:	6 ft.	8.25 ft.	16.5 ft.	27.5 yd.	6.25 rd.

Find the areas of these triangles:

	37.	38.	39.	40.	41.
Base:	16 in.	18 yd.	16.5 ft.	4.5 rd.	6.75 rd.
Altitude:	2 ft.	12 ft.	7 ft.	2.2 rd.	2.5 rd.

Find the areas of these trapezoids:

	Bases	*Altitude*		*Bases*	*Altitude*
42.	8 in. and 4 in.	6 in.	43.	8 rd. and 6.5 rd.	2 rd.
44.	12 ft. and 10 ft.	8 ft.	45.	148.6 ft. and 62.8 ft.	48 ft.

If A represents the area, l the number of units in the length, and w the number of units in the width of a rectangle, we may write:

$$A = l \times w$$

46. Find A when $l=6$ and $w=4$.

47. Find l when $A=18$ and $w=2$.

48. Find w when $A=24$ and $l=6$.

If A represents the area, b the number of units in the base, and a the number of units in the altitude of a parallelogram, we may write:

$$A = b \times a$$

49. Find A when $b=12$ and $a=6$.

50. Find b when $A=40$ and $a=4$.

51. Find a when $A=24$ and $b=8$.

If A represents the area, b the number of units in the base, and a the number of units in the altitude of a triangle, we may write:

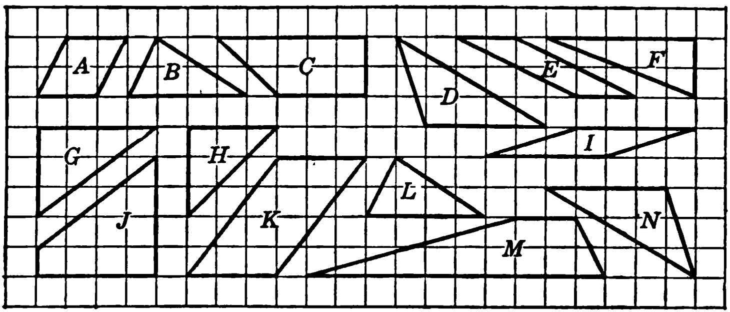
$$A = \frac{b \times a}{2}$$

52. Find A when $b=6$ and $a=4$.

53. Find b when $A=30$ and $a=6$

54. Find a when $A=18$ and $b=9$.

55. Which of these figures are parallelograms? triangles? trapezoids?



Estimate the area of each of these figures, if the side of each small square represents 1 rd. Find the areas, using the methods you have learned for finding the areas for such figures. Check your estimates with these results.

Locating Places on the Surface of the Earth. In order to locate places on the surface of the earth, an imaginary circle is drawn around the earth through Greenwich, England, and the north and south poles. Halfway, or 90° from either pole, another imaginary circle, called the *equator*, is drawn east and west around the earth. These two circles divide each other into two equal parts of 180° each.

Meridian. The half circle supposed to be drawn from pole to pole through a certain place is called the *meridian* of the place. It is divided by the equator into two equal parts.

Prime Meridian. The meridian of Greenwich is called the *prime meridian*, and its opposite meridian, or the other half of

the circle of which the prime meridian is one half, is the *180th meridian.*

Latitude. The distance of a place from the equator, measured in degrees upon the meridian of the place, is called the *latitude* of the place, and is north or south according as the place is north or south of the equator.

Longitude. The distance in degrees on the equator from the prime meridian to the meridian of a place is called the *longitude* of the place, and is east or west according as the place is east or west of the prime meridian.

As you study latitude and longitude you should consult your geography. You should learn to locate a point on the map if you know its latitude and longitude. You should also learn to estimate the latitude and longitude of various places.

LATITUDE AND LONGITUDE

1. What places have no latitude? no longitude?

2. What places have the same latitude? the same longitude?

3. What is the greatest latitude a place may have? the greatest longitude?

4. How do you find the difference in longitude between two places on the same side of the prime meridian?

5. How do you find the difference in longitude between two places on opposite sides of the prime meridian? A certain place is 120° east of the prime meridian and another is 75° west of the prime meridian. What is the difference in degrees, by the shorter arc, between their meridians?

6. The length of the equator is approximately 25,000 miles; what is the length of a degree of longitude in miles at the equator? Does the length of a degree of longitude vary as one passes from the equator toward the poles?

7. The captain of a ship finds he is in longitude 66° 30′ W. How many degrees is he from the meridian of Boston, which is approximately 71° 4′ W.?

8. A ship in longitude 6° W. gets a wireless from another ship in longitude 4° E. What is the difference of their longitudes?

9. A ship in longitude 8° E. sails west 24°. In what longitude is it then?

10. A ship in longitude 10° W. sails east 20°. What longitude is it in then?

11. Find on a map of the United States several pairs of cities that differ little in longitude; little in latitude.

12. Find out the latitude and longitude of the largest city in your state; the approximate latitude and longitude of your home.

13. The latitude of the United States mint at New Orleans is 29° 57′ 46″ N. If the latitude of another place on the same meridian is 30° 20′ N., what is their difference in latitude? In what direction is this place from New Orleans?

Longitude and Time. In what direction does the earth rotate on its axis? What figure does any point on the earth's surface describe during a rotation? How many degrees are there in this circle?

If any fixed point is used to represent the sun, and a globe, held so as to represent the earth, is rotated from west to east, it can be seen that when the sun time is 12 o'clock at a place on any meridian, the sun time will be *past* 12 o'clock at all places east, and *before* 12 o'clock at all places west.

If any point on the earth moves in 24 hours through 360°, in 1 hour that point turns through $\frac{1}{24}$ of 360°, or 15°. In 1 minute that point turns through $\frac{1}{60}$ of 15°, or 15′. In 1 second that point turns through $\frac{1}{60}$ of 15′, or 15″.

From this we see that:

A difference in longitude of 15° corresponds to a difference in time of 1 hr.
A difference in longitude of 15′ corresponds to a difference in time of 1 min.
A difference in longitude of 15″ corresponds to a difference in time of 1 sec.

From this table you can see that:

(1) If the number of degrees in the difference of longitude is divided by 15, the quotient is the number of hours in the difference in time.

(2) If the number of minutes in the difference of longitude is divided by 15, the quotient is the number of minutes in the difference in time.

(3) If the number of seconds in the difference of longitude is divided by 15, the quotient is the number of seconds in the difference in time.

From this it is clear that:

(1) $\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.

(2) 15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their longitude.

LONGITUDE AND TIME

1. When it is sunrise at 45° E., how long before it will be sunrise at 30° E.? at 15° E.? at the prime meridian? at 15° W.? at 30° W.?

2. If two places differ by 75° of longitude, by how much do they differ in time? If they differ by 5 hours in time, by how many degrees do they differ in longitude?

3. If two places differ by 35° in longitude, by how much do they differ in time? If they differ by 2 hours and 20 minutes in time, by how much do they differ in longitude?

4. The longitude of Chicago is 87° 34′ 8″ W., and of Boston 71° 3′ 30″ W. When it is 10 A.M. at Boston, what is the time at Chicago?

87° 34′ 8″ W.	is the longitude of Chicago.
71° 3′ 30″ W.	is the longitude of Boston.
16° 30′ 38″	is the difference of longitude.

$\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.

$$15\overline{)16\quad 30\quad 38}$$
$$1\quad 6\quad 2.5$$

∴ 1 hr. 6 min. 2.5 sec. is the difference of time, correct to the nearest tenth of a second.

hr.	min.	sec.	
10	0	0	A.M. is the time at Boston.
1	6	2.5	is the difference of time.
8	53	57.5	A.M. is the time at Chicago.

The difference of time is subtracted from the time of Boston, since Chicago is west from Boston, and, therefore, has earlier time than Boston.

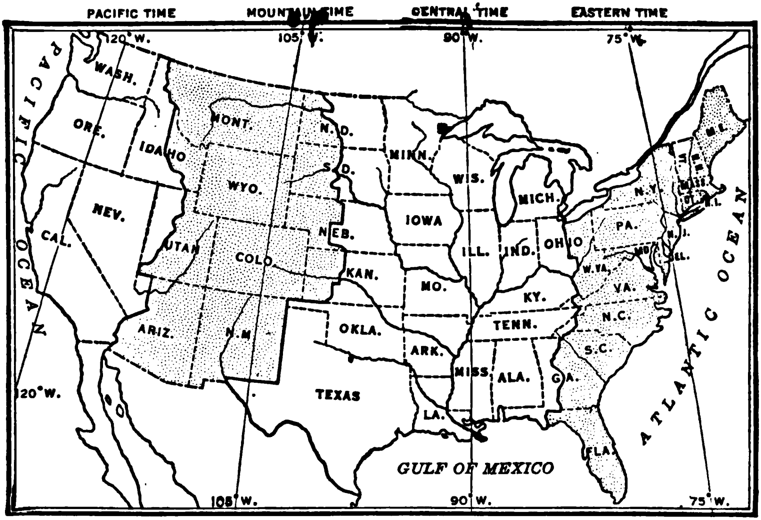
5. Find the difference in longitude between the first city in the first table and the first city in the second table; between each of the other cities in the first table and the corresponding city in the second table.

TABLE I — TABLE II

PLACE	LONGITUDE	PLACE	LONGITUDE
1. Boston	71° 7′ 21″ W.	1. London	9′ 17″ W.
2. St. Louis	90° 15′ 16″ W.	2. New York	74° 3″ W.
3. Paris	2° 20′ 45″ E.	3. Chicago	87° 34′ 8″ W.
4. San Francisco	122° 24′ 15″ W.	4. Manila	120° 58′ E.
5. Philadelphia	75° 9′ 5″ W.	5. Rome	12° 27′ 14″ E.
6. New Haven	72° 55′ 34″ W.	6. St. Paul	93° 5′ W.

6. Find the difference in time between each city in the first table and the corresponding city in the second table.

Standard Time. In the United States, when the sun is due south of a place it is noon at that place. Clocks set at 12 M. when the sun is due south show local sun time, which is the kind of time we have studied up to this point. It is readily seen that only clocks upon the same meridian will show the same sun time, or solar time. If a person were to go 1° west or east of any given meridian, his watch would be 4 minutes fast or slow.



To avoid this inconvenience, a system of *standard time* has been adopted. The United States has been divided into four standard time belts running north and south (see map). The most eastern belt is the Eastern standard time belt, and its standard time, *i.e.* the time for all places lying within this belt, is the correct solar time for the meridian of longitude of 75° W. The next is the Central time belt, and its standard time for the entire belt is correct sun time for the meridian of 90° W. Name each of the other belts. What meridian determines the standard time of each?

These boundary lines may be changed from time to time by Congress or by order of the Interstate Commerce Commission, to meet certain local conditions or to suit the convenience of the railroads.

In addition to these four meridians, the meridian of 150° W. has been established the time meridian of Alaska standard time.

By how many degrees of longitude are these standard time meridians separated? What is the difference in time between any standard time belt and the adjoining time belt?

How does the local sun time on the eastern margin of each belt compare with the standard time at that place? How does this difference vary as one moves westward toward the meridian from which the belt gets its standard time? How do local time and standard time compare at the meridian? Answer similar questions for the western half of the belt.

STANDARD TIME

1. When it is noon in the Eastern belt, what is the time in each of the other belts?

2. When it is noon by Mountain time, what is the time in each of the other belts?

3. In traveling from Philadelphia to San Francisco, would a traveler find his watch too slow or too fast, and by how many hours?

4. A traveler who left San Francisco at noon Sunday, Pacific time, arrived in New York at noon Friday, Eastern time. How long did it require to make the journey?

5. What standard time do you use?

6. At what standard time does your school begin? What is the standard time then in San Francisco? in Salt Lake City? in New Orleans? in Philadelphia? in Kansas City? in Chicago? in Boston?

7. What is your longitude? What is the difference between your standard time and your local time?

8. If an intercollegiate football game was called at 2 P.M. standard time, at Palo Alto, California, what was the standard time in Columbus, Ohio? in New York? in Chicago? in Boston? where you live?

9. President Harding was inaugurated at 12:30 P.M. in Washington. What standard time was this in New Orleans? in Seattle? in Portland, Maine? in Minneapolis? in Denver?

Division of Land. In fixing the boundaries of farm lands as first laid out in the United States, that is, in the eastern states, little regard was paid to regularity of boundary; but in surveying the public lands of the United States, Congress adopted the simple and accurate method of dividing these lands by a series of parallel lines six miles apart running north and south and east and west. These parallel lines divide the land into *townships*, which are approximately six miles square.

Range and Tier. A series of townships extending north and south is called a *range*; a series extending east and west is called a *tier*. Ranges are numbered east and west from a *principal meridian;* tiers are numbered north and south from the *base line*. A township is designated by the number of the tier and the number of the range in which it is located.

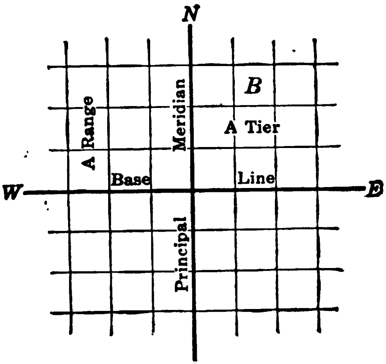


FIG. 1.

Thus, township marked B, Fig. 1, is designated T. 3 N., R. 2 E., read *township three north, range two east,* which means that B is in the third tier north of the base line and in the second range east of the principal meridian.

Sections. A township is divided into *sections* approximately[1] 1 mile square and numbered as shown in Figure 2, beginning at the northeast corner.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 2.

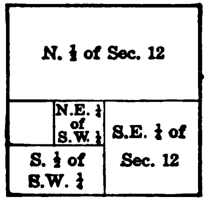


FIG. 3.

A section is divided into quarter-sections, and the township, section, and quarter-section corners are permanently marked.

Quarter-sections may be subdivided, if desired, into half-quarter-sections and quarter-quarter-sections.

DIVISION OF LAND

1. How many square miles are there in a township?

2. How many acres are there in a section? in a half-section? in a quarter-section? in a quarter-quarter-section?

In examples 3–8 make drawings similar to Figure 1, and locate with respect to the principal meridian and the base line:

3. T. 2 N., R. 3 W.
4. T. 2 N., R. 2 E.
5. T. 4 N., R. 1 W.
6. T. 1 N., R. 3 E.
7. T. 2 S., R. 2 W.
8. T. 3 S., R. 2 E.
9. Read N.W. ¼, Sec. 12, T. 3 N., R. 1 W.
10. Read S. ½, Sec. 30, T. 2 N., R. 4 W.
11. Read W. ½, Sec. 10, T. 2 S., R. 3 W.

[1] Townships and sections would be exact squares if it were not for the fact that the meridians converge at the poles of the earth.

Make a drawing similar to Figure 3, and locate each of the following tracts of land; give the number of acres in the tract:

12. S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$.
13. W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$.
14. S.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$.
15. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$.

16. How many rods of fence are required to enclose S.E. $\frac{1}{4}$ of Sec. 12 (see Figure 3)?

17. How many rods of fence are required to enclose N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ of Sec. 12 (see Figure 3)?

Volumes. The number of cubic units that a solid contains is called its *volume.*

As you have already learned how to find the volumes of rectangular solids, it is necessary now only to review and to apply what you have learned.

1. **The volume of a cube is equal to the cube of its length.**

NOTE. See note, page 121.

Thus, if the cube represented by Figure 1 is 4 ft. long, 4^3, or $64 =$ the number of cubic feet in the cube, or its volume.

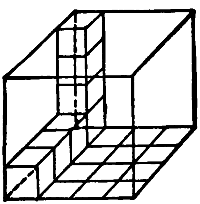


FIG. 1.

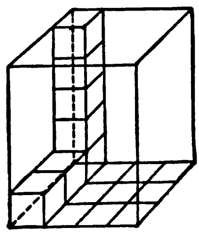


FIG. 2.

2. **The volume of a rectangular solid is equal to the product of its three dimensions.**

Thus, if the rectangular solid represented by Figure 2 is 4 ft. long, 3 ft. wide, and 5 ft. high,

$5 \times 3 \times 4$, or $60 =$ the number of cubic feet in the rectangular solid, or its volume.

CUBIC MEASURE

1. How many cubic inches are there in a cube 2 in. long? 3 in. long? 4 in. long?

2. How many cubic inches are there in a cube 12 in. long? How many cubic inches are there in a cubic foot?

3. How many cubic feet are there in a cube 3 ft. long? How many cubic feet are there in a cubic yard?

4. Write the table of cubic measure, using proper abbreviations.

5. Show that a pan in the form of a rectangular solid $11'' \times 7'' \times 3''$ holds a gallon (231 cu. in.).

6. The United States Container Act in effect Nov. 1, 1917, provides that the standard basket or other container for small fruit shall have the following capacities: $\frac{1}{2}$ pt. = 16.8 cu. in.; 1 pt. = 33.6 cu. in.; 1 qt. = 67.2 cu. in. Are these standards based on the bushel (2150.4 cu. in.)?

7. A cord of wood is $8' \times 4' \times 4'$. How many cubic feet does it contain? How many cords in a pile of wood $32' \times 4' \times 4'$?

8. Estimating .8 bu. of wheat to the cubic foot, how many bushels will a bin hold that is 4 ft. long, 4 ft. wide, and 6 ft. deep?

9. If V denotes the volume, l the number of units in the length, w the number of units in the width, and h the number of units in the height of a rectangular solid, we may write:

$$V = l \times w \times h$$

(*a*) Find V when $l=10$, $w=8$, and $h=6$.
(*b*) Find l when $V=60$, $w=4$, and $h=3$.
(*c*) Find w when $V=105$, $l=7$, and $h=3$.
(*d*) Find h when $V=200$, $l=10$, and $w=4$.

10. How many cubic feet must a bin contain to hold 240 bu. of wheat, estimating 1 cu. ft. equal to .8 bu.?

11. Estimating 1 cu. ft. equal to .8 bu., how high must a bin be made to hold 240 bu. of wheat, if it is to be built on floor space 10 ft. long and 5 ft. wide?

12. Susquehanna granite is 2.704 times as heavy as water, which weighs 62.5 lb. per cubic foot. What is the weight of a block of granite $4' \times 2\frac{1}{2}' \times 1'$?

13. The unit of measure for ordinary stone walls is the *perch*, which is a wall 16.5 ft. long, 1.5 ft. wide, and 1 ft. high. How many cubic feet does a perch contain?

14. How many perches are there in a pile of stone 33 ft. long, 4 ft. wide, and 6 ft. high?

15. At $2.50 a perch, find the cost of building the walls of a cellar 50 ft. long and 36 ft. wide if the walls are 18 in. thick and are built to the height of 10 ft., making no allowance for corners or openings.

SUGGESTION. Consider the walls as the equivalent of a wall whose length is $2 \times (50 \text{ ft.} + 36 \text{ ft.})$, width 1.5 ft., and height 10 ft.

Concrete work is usually estimated by the cubic yard, except such as is less than 6 in. in thickness, which is usually estimated by the square foot.

16. At $9 a cubic yard, find the cost of building the foundation walls for a factory 90 ft. long and 60 ft. wide if the walls are 27 in. thick and are to be built 9 ft. high, making no deductions for corners.

17. A gallon contains 231 cu. in. How many gallons are there in a cubic foot, correct to the nearest .01 of a gallon?

18. How many gallons will a tank hold which is 11 ft. long, 7 ft. wide, and 3 ft. deep? (Cancel.)

1 perch 24.75 cu. ft.

PROJECTS

POULTRY RAISING

John's father keeps four breeds of chickens on his poultry farm: Plymouth Rocks, Wyandottes, Rhode Island Reds, and Leghorns. He gave John 25 young birds of each breed and asked him to keep an account of the eggs produced and food consumed per bird of each breed. Here is John's account for two years.

Breed	First (Pullet) Year		Second Year	
	Production (Eggs per Bird)	Food Consumed (Lb. per Bird)	Production (Eggs per Bird)	Food Consumed (Lb. per Bird)
Plymouth Rocks	155	89.8	119	88.6
Wyandottes	144	80.3	115	80.4
Rhode Island Reds	151	86.6	117	86.5
Leghorns	170	76.2	138	79.9

1. How many dozen eggs did the Plymouth Rocks produce in the first year? in the second year? Answer these questions for each of the other breeds.

2. John sold the eggs produced by the Leghorns the first year at an average of $0.522 a dozen and those produced by the other breeds at an average of $0.46. How much did he receive for the eggs produced by the 25 hens of each breed the first year?

3. He sold the eggs produced by the Leghorns the second year at an average of $0.54 a dozen and those produced by the other

breeds at an average of \$0.485 a dozen. How much did he receive for the eggs produced by the 25 hens of each breed the second year?

4. The cost of the feed averaged \$3 per 100 lb. What did the feed cost for the Plymouth Rocks for the first year? for the second year? Answer the same questions for each of the other breeds.

5. Using the answers to problems 2, 3, and 4, find how much more John received for the eggs produced by the 25 birds of each breed during the first year than he paid for the feed they consumed; how much more the second year.

6. Using the answers to problem 5, find how much more John received from the sale of the eggs produced by his flock the first year than he paid for the feed they consumed; how much more the second year; how much more both years.

7. Suppose John had decided to sell 20 of each of the breeds after the birds had stopped laying at the end of the pullet year, at which time the average weight of each Leghorn is 3.5 lb. and of each of the other breeds 5.5 lb.; and suppose that the Leghorns could be sold at 25¢ a pound and the other breeds at 30¢: find which group of 20 birds would yield him the best return, considering the money received for the eggs produced by each group of 20 birds and the cost of their feed.

8. From a study of this project, which breed of hens would you advise a person to keep who is going into the poultry business?

9. From 600 Leghorns John's father made the same profit per bird for their pullet year that John did, but he deducted \$1.50 from this profit to account for labor, depreciation, and interest on investment. What was his net profit from the 600 birds?

DAIRYING

As the value of milk as a food for grown people as well as for children is becoming better understood, and consumers find how much more cheaply they can live by a liberal consumption of dairy products, interest in dairying has correspondingly increased. Scientific methods are being rapidly extended in selecting profitable dairy cows, in feeding for milk production, and in preparing and marketing dairy products. Prior to 1890, it was the common belief that a certain weight of milk from one cow would make as much butter or cheese as an equal weight from any other cow, so that at creameries and cheese factories each patron was paid in proportion to the weight of the milk delivered. It is now well known that the weight of butter or cheese that can be made from a given quantity of milk depends upon the fat present in the milk. For this reason milk and cream are weighed and tested at most creameries and factories, and patrons are paid according to the weight of the fat in the milk delivered.

1. Butter fat is the principal constituent of butter; it is the pure oil contained in milk, cream, or butter. As determined from many samples and as given by the U. S. Department of Agriculture, the composition of butter is as follows: butter fat, 82.41%; water, 13.9%; salt, 2.51%; curd, 1.18%. What is the weight of the butter fat in 1000 lb. of butter? What is the weight of the water? of the salt? of the curd?

2. This table shows the number of pounds of milk produced in one year by each of five cows, and the per cent of butter fat it contained. Each cow produced how many pounds of butter fat?

Name	Breed	Production	Test	Butter Fat
		Pounds of Milk	% Butter Fat	Pound
Pride	Ayrshire	10,260	3.86 %	
Topsy	Jersey	9124	5.36 %	
Rose	Guernsey	7533	5.25 %	
Flora	Holstein	11,873	3.81 %	
Lottie	Native	9032	4.14 %	

3. Dot, a graded Holstein, produced in one year 13,971 lb. of milk, which yielded 592.5 lb. of butter fat. Find what per cent of butter fat this milk yielded, correct to the nearest .01%.

4. Authorities say that the average American family's food expenditures

	NOW ARE	SHOULD BE
Meat and fish	35 %	12 %
Milk and its products	20 %	44 %
Bread and cereals	15 %	13 %
Vegetables and fruit	13 %	17 %
Eggs	6 %	6 %
Sugar	5 %	3 %
Miscellaneous	6 %	5 %

When $10 is spent for food, how much of it is now spent for each of the above items? How much should be spent?

5. A certain milk dealer in the East recently quoted these prices for milk bought from farmers: 8¢ a quart for milk testing 4% butter fat; 8.1¢ for milk testing 4.1%; 8.2¢ for milk testing 4.2%, and so on; 7.9¢ for milk testing 3.9%; 7.8¢ for milk testing 3.8%, and so on. At these prices what did he pay for 100 qt. of milk, testing as follows:

a. 4.3%? *b.* 4.7%? *c.* 4.9%? *d.* 5%? *e.* 3.8%?
f. 3.6%? *g.* 5.1%? *h.* 4.5%? *i.* 5.2%? *j.* 3.7%?

6. When dairymen sell cream to factories, they are paid not for the actual weight of the cream they deliver, but for the butter fat it contains. Show that 2000 lb. of cream testing 35% butter fat contains as much fat as 3500 lb. testing 20% butter fat.

7. Show that the dairyman (problem 6) thus loses 1500 lb. of skim milk by selling cream which tests 20% butter fat instead of 35%. Find his loss, considering that for feeding calves, pigs, and chickens, skim milk is worth $0.45 per 100 lb.

8. To find the money value of skim milk, it is common to *estimate skim milk as worth half as much per hundred pounds as corn is worth per bushel.* At this estimate, what is 180 lb. of skim milk worth when corn is worth $0.80 a bushel? $0.90 a bushel?

9. The Wisconsin Registry of Production is an agency intended to promote dairying. To be placed on the register a cow must have averaged a pound of butter fat a day for a year. Here is a summary of the Registry of Production records of 441 cows for a period of one year. Supply the missing numbers in the column of "Average Test"; the first number, already found, expresses the per cent which 421.8 is of 11,885, correct to the nearest .01%.

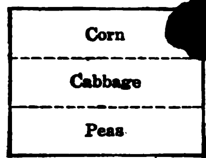
Breed	Number of Cows	Average Yearly Production		Average Test
		Milk (Pounds)	Fat (Pounds)	
Pure-bred Holstein	86	11,885	421.8	3.55
Grade Holstein	170	11,675	409.0	
Pure-bred Guernsey	22	8,215	421.1	
Grade Guernsey	82	8,835	401.5	
Pure-bred Jersey	6	7,807	406.1	
Grade Jersey	24	8,095	405.4	
Pure-bred Ayrshire	2	10,477	410.4	
Grade Shorthorn	2	8,842	390.9	
Native	27	9,793	404.3	

10. From the table in problem 9, name the breeds in order of average yearly milk production, beginning with the one which showed the highest average. Do the same for average yearly production of butter fat.

11. Aggie, a six-year-old grade Holstein, is the highest record cow in the above list of 421 cows. In this year, she produced 16,916 lb. of milk averaging 3.95% butter fat. How many pounds of butter fat did she produce?

GARDEN PROJECT

1. The figure at the right represents John's garden. Measure carefully each boundary line, and from the scale find the length of the garden; the width; the number of square feet it contains.



Scale 1" = 36'

2. John divides his garden into three equal parts, one part for sweet corn, one for cabbage, and one for peas. How many square yards did he allow for each?

3. He planted the corn in rows 3 ft. apart, running the length of the garden, each outer row $1\frac{1}{2}$ ft. from the edge and the hills 2 ft. apart in the row, each end hill 1 ft. from the end of the row. Each hill produced an average of 3 ears of corn. How many dozen ears did John grow in the garden?

4. John sold the sweet corn at 40¢ per dozen. His expenses for fertilizer and seed for this part of his garden were $2. How much more than this amount did he receive for the corn?

5. Next to the corn John planted cabbage in rows 2 ft. apart, the outer row next the corn $1\frac{1}{2}$ ft. from the ground allotted to corn, and the row next the peas $1\frac{1}{2}$ ft. from the ground allotted to peas. He set out the cabbage plants 1 ft. apart in the row, and each end plant 1 ft. from the end of the row. How many heads of cabbage did John raise in his garden, provided that each plant produced a head of cabbage?

6. John sold the cabbages at the rate of 2 heads for 15¢, except 8 heads which he sold at 10¢ each. The plants cost 20¢ a dozen, which was his only expense for this part of his garden. What did this part pay John for his time?

7. In the remaining third of the garden John planted peas in rows 1 ft. apart, each outer row $1\frac{1}{2}$ ft. from the edge. How many rows of peas did he plant?

8. John sold the peas at 30¢ a half peck, receiving for them \$9.45. How many bushels of peas did he raise? How many pecks did he raise to the row?

9. How much money did John receive for the produce which he raised in his garden?

10. If John had planted the entire garden in sweet corn, as in problem 3, and if it had produced as there stated, how much money would he have had left from the sale of the corn at 40¢ a dozen after paying \$6 for fertilizer and seed?

11. If John had planted the entire garden in cabbages placing the plants as stated in problem 5 with each outer row $1\frac{1}{2}$ ft. from the edge, what would his garden then have yielded him had he sold the cabbages at the rate of 2 heads for 15¢, the plants costing him 20¢ a dozen? Would his garden have paid him better than to have planted it all in sweet corn? Would it have required more of his time?

12. If John had planted his whole garden in peas, the rows 1 ft. apart and each end row $1\frac{1}{2}$ ft. from the edge, how many rows of peas would he have had? If he had sold the peas at the price stated in problem 8, how much would he have received for them? Would this have paid him better than to have had the whole garden in either corn or cabbage?

13. Make a drawing of John's garden to the scale of $1'' = 4'$, and draw lines to show the rows of corn, the rows of cabbage, and the rows of peas, placing the lines correctly in the drawing.

14. If you have ever planted a home garden and cared for it for a season, tell the class what your garden produced and what you did with the yield.

GENERAL REVIEW

1. Write the following numbers in a column, in the order of their value, with the least written first; find their sum:

.005 .0025 .375 .008 .000875 .075 .5 .00625

2. Paul's father earns $2720 a year, working 50 weeks of 6 days each and 8 hours a day. What are his wages per hour?

3. Name the quotients at sight; check:

$2\frac{1}{2}\overline{)5}$ $2\frac{1}{2}\overline{)7\frac{1}{2}}$ $2\frac{1}{2}\overline{)12\frac{1}{2}}$ $2\frac{1}{2}\overline{)10}$ $2\frac{1}{2}\overline{)15}$ $2\frac{1}{2}\overline{)25}$

Find the quotients and check:

4. .1728÷.072 5. 288÷.036 6. 1.035÷.15
7. 86.4÷.48 8. 61.5÷.075 9. 9.52÷5.6

10. What would you have to pay for each of these articles?

Catalogue Price	Catalogue Price
(a) $4.00, discount 20%	(e) $2.40, discount 25%
(b) $2.40, discount 10%	(f) $7.20, discount 12½%
(c) $7.50, discount 33⅓%	(g) $3.60, discount 12½%
(d) $15.00, discount 25%	(h) $9.60, discount 37½%

11. Frank earned $15. He put in the bank 25% of the first $5 he earned, 40% of the next $5 he earned, and 50% of the third $5 he earned. How much did Frank put in the bank?

12. Find the cost of shipping by freight (first-class) 275 lb. of electrical goods from Chicago to Seattle at $3.40 per 100 lb.

13. Anna put $50 in the bank for a year. The bank paid her 4% interest. How much did she have at the end of the year?

14. What part of an acre does a lot contain if it is in the form of a square and can be inclosed with 16 rd. of fencing?

15. A dealer who bought carrots at $2.50 per 3-bushel barrel retailed them at 10¢ per $\frac{1}{4}$ peck. How much did he gain on each barrel? Can you tell the class how to work this problem without using any numbers?

16. To my savings of $12.50, I added 50¢ a week for 7 weeks; then I spent for a raincoat $\frac{1}{2}$ of what I had saved, also $1.25 for a cap. How much had I then?

17. In an advertisement was the following statement: "All-wool blankets at $13.50, one third less than they have been." What was the price before reduction? What was the reduction?

18. It is estimated that each of the 30,000,000 wage earners in the United States loses on an average about 9 days each year on account of sickness. Estimating the average wages of these workers at $0.40 an hour for an 8-hour working day, what is the money loss to them in wages in one year?

19. A boy shoveled snow one afternoon from 2.45 till 5.15 at 30¢ an hour. How much money did he earn?

20. Allowing $2\frac{1}{2}$ oz. of sugar per day for each member of a family, how long should 5 lb. of sugar last a family of four?

21. A grocer bought 6 barrels of sugar, net weight, 362 lb., 353 lb., 352 lb., 349 lb., 358 lb., and 374 lb., respectively. He paid $7.85 per 100 lb. for the sugar and retailed it at 10¢ a pound. What was his profit on the 6 barrels?

22. An Army and Navy store advertised 16,000 all-wool U. S. army blankets for sale at $4.40 each. The advertisement stated the blankets were worth $12 each. At this estimate, what reduction was offered on the 16,000 blankets? what per cent of reduction?

23. If a worker in a shoe factory has his day increased from 8 hr. to 10 hr., his working day is increased by what per cent?

24. For \$3 a nurseryman offers a half dozen of each of 5 varieties of pot-grown strawberry plants; how much is that apiece? For \$10 he offers 25 of each of 5 varieties; how much is that per plant?

25. Mr. Hahn had a 35-year-old apple orchard which had yielded an average of 125 bu. a year. One spring recently he sprayed his trees for control of insect pests and diseases. The crop that year was 2250 bu., 95% of which were perfect fruit. The latter yield was how many times the former average yield? He sold the perfect fruit at \$1.20 a bushel. What did he receive for it?

26. A dealer gained 50% of cost when he sold oranges at 60¢ a dozen. What per cent of cost did he gain on those of the same lot which he sold at the rate of 3 for 25¢?

27. A dealer marked goods at \$1.20 a yard, but he sold them at a discount of 20% and still made a profit of 20% of the cost. Find the cost of the goods per yard?

28. In five years recently, property worth \$1,416,375,000 was destroyed by fire in the United States. This is the equivalent of the cost of how many new houses at \$5000 each? Of this loss \$56,650,915 was due to defective chimneys; what per cent of the loss was due to defective chimneys, correct to the nearest 0.1%?

29. A quart of onion sets will plant a row 100 ft. long. How many quarts will plant 10 rows each 25 ft. long?

30. Mr. Jones planted early sweet corn on May 5. It was ready for use in 80 days. On what date was it ready for use?

31. Food experts allow 2 oz. of fat a day for each grown person. This is an allowance of how many pounds per year (365 da.)?

32. An intensive study of success and failure as made by the savings bank section of the American Bankers' Association has furnished the following facts for every 100 men representing an average group of Americans starting out in life at the age of 25. Read these facts.

AGE	RICH	WELL-TO-DO	LIVING ON EARNINGS	NOT SELF-SUPPORTING	DEAD
35	10	10	40	35	5
45	1	3	65	15	16
55	1	3	46	30	20
65	1	4	5	54	36
75	1	2	0	34	63

Read the per cent that are rich, well-to-do, and so on, at the different ages given.

33. In a certain city, carpenters' wages were reduced from $\$1.12\frac{1}{2}$ to 90¢ an hour. What was the per cent of reduction?

34. The United States Department of Agriculture has adopted 85% to 90% as the standard for seed sweet corn germination. James planted 300 grains of sweet corn. What is the least number of grains which he should expect to germinate if the seed is of standard quality and conditions favorable? the greatest number?

35. The *Arizonian*, a steamship running from New York to San Francisco, saves 26.8 days by going through the Panama Canal instead of rounding Cape Horn. The daily operating expenses are $450. The canal tolls are $7891.20. How much money does the vessel save by going through the canal? (From *Lessons to Teach Thrift*, issued by U. S. Treasury Department.)

36. The owners of a factory occupying a site 500 ft. long and 400 ft. wide offered it for sale at $1.75 a square foot (that is, per square foot of the site occupied), and stated that the price asked for the factory was only $\frac{1}{4}$ of its real value. What was the price asked, and what did the owners consider the real value of the property?

37. If you know the marked price of a bicycle and know that the dealer is offering it at a discount of 10%, how can you find quickly the discount?

38. After allowing 20% of the selling price for overhead charges, what profit does a dealer make on an overcoat bought for $40 and sold for $60? His profit is what per cent of the selling price less overhead charges?

39. If a tradesman's profits are 30% above cost and he allows a discount of $1 on a bill of $26, what per cent of cost is his profit on this sale?

40. What must the marked price of shirts be apiece that cost $18 a dozen so that the dealer may make a profit of 33⅓% of cost and still allow a discount of 20% of the marked price?

41. If the money paid by the consumer for butter is divided as indicated in the "Per Cent" column of the following table, fill in the missing numbers of the "Cents" column, supposing that the consumer pays 60¢ a pound for butter. Observe that 40.62 is 67.7% of 60.

	Per Cent	Cents		Per Cent	Cents
Farmer	67.7	40.62	Receiver		
Hauling	4.3		Jobber	5.0	
Creamery	6.7		Broker		
Railway	2.3		Packaging	2.9	
Storage	0.5		Retailer	9.9	
Shrinkage	0.7				

42. It is estimated that 100 bu. of corn, by weight, placed in the crib November 30, will have shrunk to 92 bu. by June 30. What is the per cent of shrinkage for the seven months?

43. A farmer had 250 bu. of corn for sale November 30 for which he was offered 70¢ a bushel. He refused the offer and kept the corn until June 30, at which time he sold it for 80¢ a bushel. Did he gain or lose by holding the corn to the latter date, making the allowance for shrinkage stated in problem 42?

44. If a represents a certain number, what represents a number 5 more than a? 5 less than a? 5 times a? a divided by 5?

45. An agent charged $16.80 for selling a consignment of butter at a commission of 8%. How much should he return to the person for whom he sold the butter, after taking out his commission and paying storage amounting to $6.75?

46. Read the following, supplying the missing numbers at sight. The number in parenthesis following each statement is the number of errors a class of fifty high school graduates made in an attempt to name these missing numbers. What per cent of the answers given were wrong in each case? Thus, for 1 *gal.* =··· *qt.*, the class made 6 errors out of 50 possible correct results; therefore, 12% of the answers given were wrong.

1 bu.	=··· pk.	(0)	1 pk.	=··· qt.	(19)
1 gal.	=··· qt.	(6)	1 cu. yd.	=··· cu. ft.	(9)
1 ton	=··· lb.	(7)	1 qt.	=··· pt.	(0)
1 ft.	=··· in.	(0)	1 score	=··· units	(11)
1 rd.	=··· yd.	(15)	1 pt.	=··· gills	(12)
1 doz.	=··· units	(2)	1 quire	=··· sheets	(22)
1 lb.	=··· oz.	(1)	1 sq. rd.	=··· sq. yd.	(36)
1¢	=··· mills	(4)	1 yr. (common)	=··· da.	(2)
1 mi.	=··· rd.	(25)	1 cu. ft.	=··· cu. in.	(15)
1 yd.	=··· ft.	(0)	1 sq. yd.	=··· sq. ft.	(9)
1 sq. ft.	=··· sq. in.	(9)	1 gross	=··· units	(12)
1 acre	=··· sq. rd.	(21)	1 leap year	=··· da.	(6)

Freezing point (on the thermometer in common use) =··· degrees (31).

PART II—CHAPTER I

REVIEW

REVIEW OF INTEGERS, FRACTIONS, AND DECIMALS

1. The population of continental United States is 105,710,620 (census 1920). Read this number correct to the nearest million; correct to the nearest hundred thousand; correct to the nearest ten thousand; correct to the nearest thousand.

2. Make twenty examples in addition, each consisting of nine numbers of three figures each. Try to get the correct answers to as many of these examples as you can in 8 minutes. Check results.

3. Make twenty examples in subtraction, with the minuend in each a number of eight or nine figures, and the subtrahend a number of eight figures. Try to get the correct differences to as many of these examples as you can in 4 minutes. Check results.

4. With the figures 6, 7, 8, and 9, twenty-four different numbers can be written, each figure being used but once in writing each number. Write these numbers. Using 69 as a multiplier and the first twelve of them as multiplicands, also 87 as a multiplier and the last twelve as multiplicands, find as many correct products as you can in 6 minutes. Check results. Keep a list of these examples and drill on them from time to time until you can do them rapidly and accurately.

5. Using the products obtained in problem 4 as dividends and using 68 as a divisor for the first twelve and 79 as a divisor for the last twelve, find as many of the twenty-four quotients and remainders as possible in 8 minutes. Check results.

6. The product of two factors is 1728; one of them is 144 and the other is a. Find the value of a.

7. Illustrate how the dividend is found if the divisor, quotient, and remainder are given.

What is the best record you can make in working these 60 examples in fractions?

	a	b	c	d
8.	$\frac{2}{5}+\frac{1}{5}$	$\frac{3}{5}-\frac{1}{5}$	$4\frac{2}{3}\times 6$	$\frac{9}{10}\div 3$
9.	$4\frac{2}{5}\div 2$	$\frac{3}{8}\times 1\frac{1}{2}$	$\frac{3}{5}+\frac{4}{5}$	$\frac{1}{2}-\frac{3}{16}$
10.	$5\times 2\frac{1}{5}$	$11\frac{3}{4}\div 5$	$\frac{1}{6}+\frac{3}{8}$	$\frac{3}{4}-\frac{7}{12}$
11.	$\frac{3}{8}-\frac{1}{5}$	$\frac{2}{5}+\frac{5}{6}$	$\frac{1}{4}\div 3$	$\frac{3}{4}\times\frac{8}{9}$
12.	$\frac{3}{4}\times\frac{5}{9}$	$6\div\frac{2}{3}$	$\frac{5}{12}+\frac{7}{12}$	$\frac{5}{8}+\frac{7}{10}$
13.	$\frac{11}{16}-\frac{3}{16}$	$4\times 2\frac{5}{6}$	$15\div 1\frac{1}{2}$	$8\frac{5}{8}\times\frac{3}{5}$
14.	$2\frac{1}{2}\div\frac{1}{4}$	$\frac{3}{4}+\frac{1}{8}$	$\frac{5}{12}-\frac{3}{8}$	$2\frac{2}{3}-\frac{1}{2}$
15.	$2\frac{3}{4}\times 3\frac{1}{2}$	$\frac{5}{8}+\frac{7}{8}$	$7\frac{7}{8}-1\frac{1}{8}$	$3\frac{3}{16}\times 8$
16.	$87\frac{1}{2}\div\frac{7}{8}$	$\frac{2}{3}+\frac{8}{9}$	$\frac{5}{6}-\frac{3}{16}$	$1\frac{3}{4}\div 1\frac{1}{6}$
17.	$13\frac{3}{4}-7$	$\frac{1}{4}+\frac{5}{12}$	$5\times\frac{3}{10}$	$\frac{5}{8}\div 1\frac{2}{3}$
18.	$\frac{3}{4}+\frac{11}{12}$	$3\frac{1}{2}-\frac{5}{8}$	$6\times 54\frac{3}{4}$	$\frac{5}{6}+\frac{13}{15}$
19.	$4\times\frac{2}{9}$	$48\frac{1}{2}\times 144\frac{1}{4}$	$6\frac{2}{3}\div 3$	$\frac{3}{8}+\frac{3}{5}$
20.	$\frac{2}{3}\times 6$	$3\frac{1}{3}-1\frac{1}{2}$	$\frac{7}{10}+\frac{3}{15}$	$18\times\frac{5}{6}$
21.	$\frac{3}{8}\div\frac{9}{16}$	$2\frac{2}{5}\times 78$	$\frac{5}{8}\times\frac{3}{10}$	$9\frac{1}{2}\times\frac{1}{8}$
22.	$87\frac{1}{2}\div 12\frac{1}{2}$	$500\div 62\frac{1}{2}$	$35\frac{1}{3}\times 58\frac{1}{4}$	$634\frac{1}{2}\div 12$

23. Which of the following statements are true? Explain why you think each is true or not true.

a. $\frac{1}{8}+\frac{1}{8}=\frac{2}{16}$ *b.* $\frac{2.5}{4}=\frac{25}{40}$ *c.* $\frac{37\frac{1}{2}}{62\frac{1}{2}}=\frac{3}{5}$

d. $\frac{7}{10}-\frac{3}{5}=\frac{4}{5}$ *e.* $\frac{3}{2.5}=\frac{30}{25}$ *f.* $\frac{2\times\cancel{5}}{3\times\cancel{5}}=\frac{2}{3}$

g. $\frac{2+\cancel{3}}{7+\cancel{3}}=\frac{2}{7}$ *h.* $\frac{2.5}{4.5}=\frac{25}{45}$ *i.* $\frac{\frac{1}{4}}{2}=\frac{4\times\frac{1}{4}}{4\times 2}$

24. Write the correct answer to each part of problem 23, expressing the answer in lowest terms or as a mixed number.

25. Check the following: $24 \times 12\frac{1}{2} = 3 \times 100$. This illustrates that dividing one factor and multiplying another by the same number does not change their product. Give five other illustrations.

26. Divide the lesser of the two fractions $\frac{4}{5}$ and $\frac{2}{3}$ by the greater and multiply the quotient by their sum.

Add; then subtract. Check each result.

	a	*b*	*c*	*d*	*e*	
27.	.075	2.756	4.0	.7	$.02\frac{1}{2}$	($.02\frac{1}{2} = .025$)
	.07	1.5	3.054	.308	.0075	
28.	$.03\frac{3}{4}$	$.07\frac{3}{4}$	$.087\frac{1}{4}$	1.5	$3.3\frac{1}{3}$	($3.3\frac{1}{3} = 3.33\frac{1}{3}$)
	.0015	$.001\frac{1}{2}$	$.05\frac{1}{2}$	$.07\frac{3}{4}$	$0.66\frac{2}{3}$	

29. Multiply at sight each of the following by 10; then divide each by 10. Multiply and then divide each by 100.

2.5 3.75 .45 .035 85 .001

Supply the missing number in each of the following; check results;

	Dividend	Divisor	Quotient		Dividend	Divisor	Quotient
30.	.625	125	?	31.	37.5	?	30
32.	?	125	700	33.	.875	12.5	?
34.	3.75	?	.5	35.	?	4.2	21
36.	.021	?	.5	37.	.021	.42	?

38. One man can assemble 2 machines per day. How many machines can 19 men assemble in 13 days?

39. A man receives 48¢ per hour. How much would he earn in a year if he works 8 hours per day, 6 days per week, and 50 weeks per year?

40. A job requiring 398 hours' work is divided equally among 36 men. How many whole hours must each man work and how much overtime would 1 man have to work to finish the job?

41. How many rails each 30 ft. long will be required to lay 2 miles of double track? If rails weighing 100 lb. per foot are used, what will be the cost of the rails at $28 per ton?

42. Which would be the cheaper and how much, to employ a man at 79¢ an hour who takes 52 hr. to do a piece of work, or to employ a man at 68¢ an hour who takes 76 hr. to do it?

43. The national wealth of the United States is estimated at $400,000,000,000; that of the United Kingdom, $120,000,000,000; France, $92,500,000,000; Germany, $83,000,000,000; Italy, $35,500,000,000; Belgium, $12,000,000,000; Japan, $23,500,000,000. At this estimate, the wealth of the United States is how much more than that of the other six countries combined?

44. If a line 1 in. long is taken to represent $10,000,000,000, what is the length of the line that represents the wealth of the United States as given in problem 43? What length of line represents the wealth of the United Kingdom? Answer the same question for each of the other countries. Arrange these lines in a graph.

45. If a farmer has 800 bu. of corn to deliver and his hired man can make two trips a day hauling 50 bu. at a load, what does it cost him per bushel to deliver the corn, estimating the wages of man with team at $6 a day?

46. Reduce $\frac{1}{16}$ to a decimal.

47. If you wish to draw a line .9 of an inch long, and have a rule marked off in eighths of an inch, can you draw the line of the desired length with this rule? If you draw a line $\frac{7}{8}$ of an inch long, will it be as much as $\frac{1}{32}$ of an inch shorter than a line .9 of an inch long?

48. If railroad ties are placed 18 in. apart from center to center, how far will 20,000 ties reach?

49. Express decimally the difference between .9 and $\frac{7}{8}$.

50. It was recently estimated that in one of the leading dairy states five of every eighteen cows were being kept at a loss. At this estimate, what part of them were being kept at a loss? If there are 23,747,000 cows in the United States, how many are being kept at a loss?

51. Mr. Mason owns property fronting 150 feet on Spruce Street. One year he paid $3\frac{1}{8}$¢ a foot for having the portion of Spruce Street in front of his property oiled. The next year he paid $5\frac{7}{8}$¢ a foot. How much did he pay both years? How much more did he pay the second year than the first?

52. Light travels 186,330 miles per second from the sun to the earth, a distance of 92,897,400 miles. Find to the nearest second how long light takes to travel this distance.

53. Ordinary collars are made in $\frac{1}{2}''$ sizes. My neck is 12.7″ round. What is the nearest size in collars that the shopkeeper can sell me? What size must I buy? Why?

54. The center of a hole is $\frac{23}{32}$ in. from one edge of a brass plate $\frac{63}{64}$ in. wide. How far is the center of the hole from the opposite edge of the plate?

55. A man who gets $\frac{3}{8}$ of a dollar an hour receives $\frac{7}{8}$ of a dollar for doing a piece of work. How long did it take him to complete the job?

56. The length of the line drawn from center to center of two circles, respectively $1\frac{7}{8}$ in. and $1\frac{5}{8}$ in. in diameter, is $3\frac{3}{16}$ in. How much of this line lies outside of the circles?

57. Which would be the cheaper, to employ a man at $55\frac{1}{2}$¢ an hour who takes $6\frac{3}{4}$ hr. to do a piece of work or to employ a man at $45\frac{1}{2}$¢ an hour who takes $7\frac{1}{4}$ hr. to do the same work?

58. If a barrel contains $45\frac{1}{2}$ gal. of oil, and $24\frac{3}{4}$ gal. are taken out at one time and $11\frac{3}{8}$ gal. at another time, how many gallons are left?

59. A cubic inch of steel weighs .283 lb. What is the weight of a steel shaft containing 356 cu. in.?

End.

REVIEW OF THE THREE PROBLEMS OF PERCENTAGE

1. Express each of the following as a decimal in hundredths, and as a common fraction in lowest terms:

4% $87\frac{1}{2}$% 5% $6\frac{1}{4}$% $14\frac{2}{7}$% $8\frac{1}{3}$% $16\frac{2}{3}$%

2. Name $37\frac{1}{2}$% of: 8¢; 24¢; 32%; 64 ft.

3. Name $87\frac{1}{2}$% of each of the amounts in problem 2; $62\frac{1}{2}$% of each; $6\frac{1}{4}$% of each.

4. Name 100%, then 200%, then 250% of $4; 50¢; 20 bu.; 16 lb.; 100%.

5. Write the following as decimals. Thus for the first write .0075.

$\frac{3}{4}$%	$2\frac{1}{2}$%	7.5%	$3\frac{1}{4}$%	$\frac{1}{2}$%	4.8%	3.65%
6.45%	$6\frac{1}{2}$%	$4\frac{3}{4}$%	$3\frac{1}{2}$%	6.75%	.75%	2.5%

Find:

	a	*b*	*c*
6.	$\frac{3}{4}$% of $200	$2\frac{1}{2}$% of 160 bu.	7.5% of $680
7.	$\frac{1}{2}$% of $684	4.8% of 325 in.	3.65% of $500
8.	$6\frac{1}{2}$% of $250	$4\frac{3}{4}$% of 240 mi.	$3\frac{1}{2}$% of $920
9.	.75% of $5000	2.5% of 320%	$\frac{1}{4}$% of $2400

10. Select the equivalents in the following. Prove your answers.

50%	$\frac{1}{2}$%	$\frac{1}{2}$	5%	.5	$\frac{1}{20}$
$\frac{1}{5}$	$\frac{3}{4}$	.05	$\frac{1}{200}$	75%	.20

11. If Edward sold 75% of his crop of peas, what part of it did he sell? If he sold 3 bu. 3 pk., how many bushels did he raise?

12. State what the number is if 20% of it is:

4	$2\frac{1}{2}$	$\frac{1}{5}$	100	$\frac{1}{2}$	6	1.5	.2

13. State what the number is if $37\frac{1}{2}$% of it is:

3	6	9	$\frac{3}{4}$	75	.6	7.5	18

14. What decimal may be used for 6%?

6% of a number is 120 may be written *.06* $\times$ *a number* = *120.*

If .06 $\times$ a number = 120, what is the number?

Suggestion. If the product of two factors is given together with one of the factors, think how the other is found.

Find the amount of which:

	a	*b*	*c*	*d*
15.	\$275 is 5%	\$424 is 4%	\$864 is 6%	\$9.63 is 3%
16.	\$575 is $2\frac{1}{2}$%	\$9.50 is $4\frac{3}{4}$%	\$52.50 is $3\frac{1}{2}$%	\$51 is $4\frac{1}{4}$%

17. The Crescent basket ball team played 24 games one season and won 15 of them. What part of them did it win? what per cent of them?

State what part, then what per cent of:

	a	*b*	*c*	*d*
18.	4 is 2	10 is 1	20 is 5	100 is 6
19.	12 is 8	16 is 6	24 is 21	24 is 2

20. What per cent of 3620 is 199.1?

Suggestions. (1) 1 % of 3620 = .01 $\times$ 3620, or 36.2
199.1 $\div$ 36.2 = ?

(2) Some number, n, $\times$ 3620 = 199.1
Therefore, the number, n, = 199.1 $\div$ 3620

Try both of the above methods. Which do you prefer?

What per cent of:

	a	*b*	*c*
21.	\$275 is \$16.50	682 A. is 40.92 A.	\$248 is \$11.16
22.	\$2645 is \$105.80	2700 mi. is 129.60 mi.	\$5680 is \$298.20

Supply the right number for x:

23.	20% of $40 = x$	20% of $x = 8$	$x\%$ of $40 = 8$
24.	$12\frac{1}{2}\%$ of $96 = x$	$12\frac{1}{2}\%$ of $x = 12$	$x\%$ of $96 = 12$
25.	$16\frac{2}{3}\%$ of $30 = x$	$16\frac{2}{3}\%$ of $x = 5$	$x\%$ of $30 = 5$

26. What is 500% of 1¢? of \$1? of 5¢?

27. What per cent of \$100 is \$1? 50¢? \$0.25?

28. What per cent of 5¢ is $2\frac{1}{2}$¢? 5¢? 10¢? 1.25¢?

29. 4 times any number is what per cent of it?

30. .045 times any number is what per cent of it?

31. In a recent period of seven years, the cost of house furnishings increased 176%. What was the cost at the end of this period of goods which at the beginning of the period would have cost \$100?

32. Mr. Conant spends 5% of his income for bread; Mr. Crosby spends only $2\frac{1}{2}\%$ of his income thus. The former gets \$30 a week, the latter \$250 a month. Bread increases 22% in price. What is the increased expenditure for bread in each case?

33. It is estimated that breakage in packing and transporting causes an annual loss of 2,000,000,000 eggs in the United States each year. The annual production is estimated at 24,000,000,000. The loss is what per cent of the production? Estimating eggs at \$0.30 a dozen, what is the money loss?

34. The price of meat advanced 20% above the price of the preceding year, and the third year decreased 20% from the price for the second year. The price the third year was what per cent of the price the first year?

7

35. Prices in 1914 were $8\frac{1}{2}\%$ above those of fourteen years earlier. In 1918 prices were 39% above those of 1914. What was the percentage increase from 1900 to 1918?

36. The world's petroleum output in 1918 was approximately 514 million barrels. Of this the United States produced 355 million barrels. What percentage of the whole did the rest of the world produce, correct to the nearest tenth?

37. A person bought a house for $1730. This was equivalent to paying what price four years later after the price of such houses had advanced 112%?

38. The average length of the school year in the United States is about 160 da. The average attendance for each child enrolled in school is 120 da. The average attendance is what per cent of the school year? Has your school a better record than this?

39. In seven years recently, stove coal advanced in New York City from $7.07 per ton of 2000 lb. to $11.54. What was the average yearly per cent of advance, correct to the nearest .01%?

40. Wood ashes contain 5% potash, 1.5% phosphoric acid, and 32.5% lime. How many pounds of each are there in 2 tons of wood ashes?

41. A country increases its production of wheat per acre by 4%, and it increases its acreage of wheat by 4%. What is its increased per cent in production of wheat? If prices increase 40%, what is the increase per cent in the total selling price of wheat?

42. In a recent year, 7625 new books were published, 671 of which were books of fiction. What per cent of the total were books of fiction, correct to the nearest .01%?

43. Fifty persons just out of high school were given 25 questions on the tables of measures. They gave 750 wrong answers. What per cent of the answers given were correct?

REVIEW OF TRADE DISCOUNT

1. On purchases of War Department surplus canned meats, a discount of 10% was allowed on all orders to the amount of from \$2501 to \$4000. What discount was allowed on an order of \$3250?

2. One dealer marked an overcoat \$60 and later offered it at a discount of 25%. Another dealer marked the same kind of overcoat \$50 and later offered it at a discount of $12\frac{1}{2}$%. Which was the better offer to the customer and how much?

3. Cotton towels, the regular price of which was 25¢ each, were advertised at \$2 a dozen. What rate of discount was this from the regular price?

4. A department store advertised dress satin at \$1.75, the regular price of which was \$2.25 a yard. What per cent reduction was offered?

5. At a special sale, hard water soap was offered at 75¢ per dozen cakes. The regular price was 10¢ a cake. What per cent reduction was offered from the regular price?

6. At the close of the winter season, a certain dealer advertised wool suits at \$27, announcing that this was at a discount of 46% from the regular price. What was the regular price?

7. Mr. Jordan bought a bill of goods amounting to \$264 subject to a discount of $12\frac{1}{2}$% and an additional discount of 2% for cash payment. Mr. Jordan paid cash for the goods. How much did he pay?

8. On June 1, Ferry & Co. bought a bill of goods amounting to \$560 payable in 60 da., subject to a discount of 1% if paid in 30 da., or $2\frac{1}{2}$% if paid in 10 da. When was the bill due? How much would settle the bill June 25? June 8?

9. On a bill of goods listed at \$540 I am offered a single discount of 45%, or successive discounts of $33\frac{1}{3}$%, 10%, and 5%. Which is the better offer and by how much?

REVIEW OF PROFIT AND LOSS

1. Thread bought at 60¢ per dozen spools was retailed at 8¢ a spool. The dealer estimated the overhead charges at 1¢ a spool. At this estimate, his profit was what per cent of the selling price? what per cent of the cost (including overhead charges)?

2. What should a grocer charge for 12 oz. of cheese costing \$0.24 a pound, in order to make a profit of $33\frac{1}{3}\%$ of the cost? His profit is what per cent of the selling price?

3. If goods are bought at a discount of 20% and 20% from the list price and sold at the list price, the profit is what per cent of the cost? what per cent of the selling price?

4. If a dealer made a profit of 75% of cost by selling apples at 35¢ per half peck, what did they cost per bushel?

5. Goods which were marked to gain 60% of cost, were sold at a discount of 25% of the marked price. What per cent of the cost was gained? what per cent of the selling price?

6. A grain dealer bought 100 bu. of corn Dec. 1 at \$0.60 a bushel delivered. He sold it July 1 at \$0.80 a bushel, by which time it had shrunk 8%. What per cent of the cost did he gain?

7. A grocer gained $66\frac{2}{3}\%$ of the cost of oranges by retailing them at 50¢ a dozen. What per cent of cost would he make by selling them at 5¢ each?

8. In one week James bought 540 daily papers at 3 for 4¢. He sold them at 2¢ each with the exception of 20, which were not returnable. What per cent of the cost did he make?

9. A manufacturer clears $12\frac{1}{2}\%$ on goods costing him \$6.40 to make; the retailer makes a further $37\frac{1}{2}\%$ on the cost to him. At what price will the retailer sell the goods?

10. A firm sold two machines for \$360 each, gaining 20% on the first and losing 20% on the second. Did the firm gain or lose and how much?

REVIEW OF COMMISSION

1. Edith sells lace on a commission of 40%. In one month she sold 80 yd. at 60¢ a yard, 26 yd. at 50¢ a yard, and 42 yd. at 40¢ a yard. Her expenses for the month were $12.80. How much did she make above expenses?

2. A poultryman shipped to a commission merchant 600 doz. eggs, which were sold at $0.52 a dozen. The agent deducted a commission of 15% and $6.25 for other expenses. How much money did the poultryman receive?

3. My agent purchased for me 101,388 lb. of corn at $0.80 a bushel (56 lb.). He prepaid freight amounting to 8¢ per hundredweight and charged a commission of 2%. Find the total cost of the corn.

4. I paid $8000 for a house, which I rent for $60 a month. My agent charges me 5% commission for collecting the rent. My expenses on the property were $240 last year. What per cent of the cost of the house was my net yearly income?

5. A commission merchant sent a dealer $287.20 as the net proceeds of a sale of poultry amounting to $320, after he had deducted his commission and $7.20 for storage. What rate of commission did the agent charge?

6. A lawyer collected for his client 80% of a debt, and charged a commission of 8% of the sum collected. He paid over to his client $736. How much money did he collect? How much commission did he retain? How much of the debt remained uncollected?

7. Mr. Endicott sells coffee and tea in rural districts. He receives a salary of $80 a month and 10% commission on all sales in excess of $600 for the month, and an additional 5% on sales above $1000. What were his earnings for a month in which his sales were $826? for a month in which his sales were $1120?

REVIEW OF INTEREST

1. Find the interest on $250 from April 15 to June 14 at 6%.

SUGGESTION. Count the exact number of days from April 15 to June 14. In making this count exclude April 15 but include June 14. Reckon 1 day's interest as $\frac{1}{360}$ of a year's interest.

Find the interest on:

2. $346 from June 1 to August 30 at 6%; at 5%.
3. $465 from Aug. 15 to Dec. 13 at $4\frac{1}{2}$%; at 5%.
4. $346 from May 4 to Oct. 1 at $5\frac{1}{2}$%; at 3.75%.
5. $672 from March 25 to Dec. 20 at 4.8%; at 5.5%.
6. $126.48 from May 1 to Aug. 2 at 6%; at 6.5%.
7. $300 for 1 yr. 6 mo. at $2\frac{1}{2}$% semiannually.
8. $700 for 9 mo. at $1\frac{1}{2}$% quarterly.
9. $550 for 1 yr. 3 mo. at $1\frac{3}{4}$% quarterly.
10. $1500 for 1 yr. 9 mo. at 1% quarterly.
11. $262.50 from May 20, 1924, to Jan. 15, 1925, at 6%.
12. $624.75 from July 15, 1927, to March 11, 1928, at 4%.

13. Find the exact interest on $1250 from March 1 to April 30 at 4%.

SUGGESTION. Reckon 1 day's interest at $\frac{1}{365}$ of 1 year's interest.

14. Find the exact interest on $2500 from Sept. 1 to Nov. 30 at 3.75%.

15. Find the exact interest on $10,000 from April 1 to June 15 at $3\frac{1}{2}$%.

16. In the equation $I = Prt$,

(1) Let $P = 400$, $r = .06$, and $t = 3$; find I
(2) Let $I = 72$, $r = .06$, and $t = 3$; find P
(3) Let $P = 400$, $I = 72$, and $t = 3$; find r
(4) Let $P = 400$, $I = 72$, and $r = .06$; find t

CHAPTER II

BANKING AND NEGOTIABLE PAPERS

Business of a Bank. A bank is an institution whose chief business it is to care for and lend money.

Deposits are money put into banks for safekeeping. Some banks pay a small rate of interest on part or all of these deposits; others do not.

All banks lend most of their deposits, keeping in their safes only as much as the laws require and experience shows the depositors or the creditors of the bank may need. Banks make most of their profits by lending their deposits.

The First National Bank
OF EBENSBURG
DEPOSITED FOR ACCOUNT OF
M. D. Kittell
Ebensburg June 2, 1921
All checks are credited subject to payment.
Checks—State name of CITY or TOWN

	DOLLARS	CENTS
BILLS	160	
COINS	22	75
CHECKS		
Boston	227	48
Philadelphia	332	16
	742	39

DEPOSIT SLIP

Opening an Account at a Bank. Many people find it convenient to pay bills by means of checks. In order to do this, a person must open an account with a *commercial bank* or *bank of deposit.* Such banks do not pay interest on small accounts, and the depositor makes use of them entirely for convenience and safety.

When a person makes his first deposit, *i.e.*, when he opens his account, he delivers his deposit to the receiving teller or cashier of the bank, and is given a pass book in which the amount of the deposit has been entered, and also

a blank check book. Each depositor must file his signature with the bank, in order that the latter may tell whether checks signed with his name are genuine or not.

Deposit Slip. Every deposit which a person makes in a commercial bank must be accompanied by a deposit slip. Read the deposit slip which Mr. Kittell made out (see page 166), and tell what each item means. What was the total amount of his deposit? Supposing that he had remaining from previous deposits in the bank $228.56, what is the entire amount which he has on deposit June 2?

Try to get a deposit slip at your local bank; study its form to make sure that you can properly make one out if you are sent to the bank to make a deposit.

A Check. Mr. Kittell owes A. D. Cromwell a bill of $100. In order to pay this bill he draws the following check. Read this check.

No. 18232		
Date June 6, 1921		
To A. D. Cromwell		
For Groceries		
Bal. deposited	228	56
June 2	742	39
Total	970	95
Amt. of check	100	00
Bal. forward	870	95

Ebensburg, Pa., June 6, 1921 No. 18232

THE FIRST NATIONAL BANK 60–931

Pay to the order of A. D. Cromwell $100 $\frac{00}{100}$

One Hundred ———— $\frac{00}{100}$ Dollars

M. D. Kittell

Meaning of a Check. A check is a written order by which a bank is directed by a depositor to pay out money belonging to the depositor.

The part to the left of the perforated line of a check is called the *stub*. This remains in the check book when the check is torn out; it serves as a memorandum of payments made and shows the balance on hand in the bank after each check has been drawn. The stub should be filled out before the check is written. Why?

The *drawer*, or *maker*, of a check is the person who signs it. Who is the maker of the check, page 167? The *payee* is the person, company, or institution to whom the check is payable. Who is the payee in this check? The *face* of a check is the amount named therein. What is the face of this check? On or after what date is this check payable? Which of the persons named must be a depositor in the First National Bank of Ebensburg, Pa.?

Mr. Cromwell takes Mr. Kittell's check to the First National Bank, Ebensburg, writes his name across the back of the check, and presents it to this bank for payment. If the *paying teller* (the man who pays out money) does not know Mr. Cromwell personally, it will be necessary for some one to identify him. He may receive the cash or have the amount credited to his account (if he has an account there, and so desires). The amount of the check is charged against Mr. Kittell's account and the cancelled check is eventually returned to him.

Mr. Cromwell might have this check cashed at any other bank where he is known, or have it credited to his account in any other bank where he has an account.

It is the best business practice for the holder of a check to present it for payment, or forward it for presentment, on the day that he receives it, or on the first business day following. Find out the reasons for this.

Indorsement. A check is usually made payable *to the order of* the payee; it may then be transferred by him to another. This is done by the payee's writing his own name across the back of the check; this process is called *indorsing* the check. The person

receiving the check then becomes the holder, and may again transfer it by indorsement to another person, and so on.

INDORSEMENT IN BLANK

A. D. Cromwell

1. An indorsement containing the name of the indorser only is called an *indorsement in blank*.

INDORSEMENT IN FULL

Pay to the order of
S. K. Stubbs
A. D. Cromwell

2. An indorsement directing payment to be made to the order of a certain person is an *indorsement in full*, or a *special indorsement*.

Who must first indorse the check on page 167? If indorsed payable to the order of S. K. Stubbs, who must then indorse it?

Certified Check. To certify a check, the bank stamps "certified" across the face of it, and deducts the amount of the check from the maker's account in the bank. On presentation at some other bank, the payee may receive cash at once or have the amount credited to his account; otherwise he may have to wait until the bank where he presents the check can ascertain whether the maker has an account equal to the face of the check.

PROBLEMS IN BANKING

Make out deposit slips for the following. In the case of checks deposited supply name of some bank or place.

1. R. L. Jones deposited with the Indiana National Bank of Indianapolis, Ind., on April 25, 1921: bills, $325; coin, $21.05; checks, $18.75, $32.50, $110.25.

2. F. D. Farr deposited with the Continental and Commercial Bank of Chicago, Ill., on Nov. 2, 1921: bills, $312; gold, $5; silver, $25.75; checks, $40.50, $52.90, $86.18, $19.45.

3. E. C. White deposited with the First National Bank of Cleveland, Cleveland, O., on Jan. 2, 1922: bills, $428; coin, $10.90; checks, $115.30, $92.18, $6.49, $51.75.

4. J. W. Davis deposited with the National State Bank of Camden, Camden, N. J., on Feb. 8, 1922: bills, $674; silver, $34.80; gold, $10; checks, $78.65, $24.18, $167.25, $248.50.

5. Suppose you have money deposited in the nearest bank; write the check which you would draw to pay W. S. Delp $5.28. Who must be the first indorser of this check?

6. Suppose that A. D. Hughes draws a check in your favor for $10 on the Continental & Commercial Bank of Chicago, and you have Edward Brooks cash the check for you; write the check and your indorsement in full.

7. On June 1, W. P. Bolton had a balance in the bank of $622.96. Since that time he has drawn checks on his deposit for $32.10, $91.18, and $46.25, and has deposited $125, $32.65 and $98.42. What is his present balance?

8. E. M. Kaufmann had a balance of $386.10 on Oct. 1. Since that time he has drawn checks on his deposit for $18.36, $75.32, $19.08, and $58.25, and has deposited $48.15, $125.75, and $56.82. What is his present balance?

9. Show the indorsement on a check made payable to R. D. Conrad and indorsed in full by him to E. L. Mack.

10. When a bank pays a check it cancels the check. Find out what this means. At certain times each year, or when requested to do so, banks return cancelled checks to the drawer. Ask someone who draws checks on your home bank to show you a cancelled check. Why should the drawer carefully preserve such checks?

11. If a depositor wishes to withdraw cash for his own use from his bank deposit, he may write *Cash, Myself,* or *Self* in place of the name of the payee, and then sign his name to the check. Suppose you have money deposited in your home bank, write the check that you could use to withdraw $2 for your own use.

Savings Banks. Money deposited in a savings bank may not be withdrawn by means of checks. It is placed there for purposes of safety and profit, as savings banks pay a certain rate of interest for the use of these deposits. How do savings banks get the money with which to pay interest to their depositors? Savings banks usually pay interest on their deposits quarterly or semi-annually.

Depositors in savings banks are given bank books which must be presented when deposits are made and when money is drawn out; they should also be presented at the end of each interest period, so that the amount of interest may be entered on the book.

The practices prevailing among savings banks governing the length of the interest period, withdrawals, and deposits are so varied that no fixed rules can be laid down. Find out the methods followed by savings banks in your state.

Interest on Savings. If a boy deposits $10 in a savings bank which pays 4% interest each year but pays the interest semi-annually, then there is due at the end of six months 2% of $10, or $0.20 interest on this deposit. This interest is added to the deposit, so that $10.20 is on interest for the next six months at 2%, and will earn $0.204 interest. (Check each interest here given.)

At the end of the year the $10 deposited amounts to $10.404, which is the amount that will draw interest for the next six months; at the expiration of that time interest will again be added to the amount that was on interest during the six months to form a new principal for the following six months, etc. In this case we say that the boy is receiving *compound interest* on the deposit. Suppose his deposit had been $25, find the amount at the end of the year. Change the deposit to $100 and the rate of interest per year to 3%, and find the amount at the end of the year.

Meaning of Compound Interest. If interest is added to the principal at the end of each interest period, to form a new principal for the next interest period, the entire interest, that is, the difference between the final amount and the original principal, is the *compound interest* on the original principal.

Compound Amount. The final amount when compound interest is reckoned is called the *compound amount.*

Methods of Finding Compound Interest. Compound interest may be found by the method similar to that used in finding simple interest, as in the illustration below; or it may be found by use of the table on page 173. Learn to work problems by both methods. Check carefully all work given below.

Example. Find the compound interest on \$5000 for $1\frac{1}{2}$ yr. at 4% per annum, compounded semiannually.

\$5000	= the principal
.02	
\$100	= interest for 1st 6 mo.
5000	
\$5100	= amt. on int. for 2nd 6 mo.
.02	
\$102	= int. for 2nd 6 mo.
5100	
\$5202	= amt. on int. for 3rd 6 mo.
.02	
\$104.04	= int. for 3rd 6 mo.
5202.00	
\$5306.04	= amt. at end of $1\frac{1}{2}$ yr.
5000.00	
\$306.04	= the compound int. for $1\frac{1}{2}$ yr.

Using the table on page 173, we find that the compound amount of \$1 for 3 periods (in this case each period is one-half a year) at 2 % is \$1.06121. Multiplying this amount by 5000, we have \$5306.05. Hence the interest by this method is \$306.05, the difference of 1 cent in the interest as found by this method and the interest as found by the first method being due to the fact that \$1.06121 is the compound amount of \$1 for 3 periods at 2 %, expressed correct only to the fifth decimal place, while the exact amount is \$1.061208.

From the above solution find the compound interest on \$5000 for 1 yr. at 4% per annum, compounded semiannually. What is the compound amount? What is the compound amount for $1\frac{1}{2}$ yr.?

TABLE OF COMPOUND INTEREST

Amount of $1 for each of 20 Periods at Rates from ¾% to 6%

Periods	¾ Per Cent	1 Per Cent	1¼ Per Cent	1½ Per Cent	2 Per Cent
1	1.00750	1.01000	1.01250	1.01500	1.02000
2	1.01506	1.02010	1.02516	1.03023	1.04040
3	1.02267	1.03030	1.03797	1.04568	1.06121
4	1.03034	1.04060	1.05095	1.06136	1.08243
5	1.03807	1.05101	1.06408	1.07728	1.10408
6	1.04585	1.06152	1.07738	1.09344	1.12616
7	1.05370	1.07214	1.09085	1.10985	1.14869
8	1.06160	1.08286	1.10449	1.12649	1.17166
9	1.06956	1.09369	1.11829	1.14339	1.19509
10	1.07758	1.10462	1.13227	1.16054	1.21899
11	1.08566	1.11567	1.14642	1.17795	1.24337
12	1.09381	1.12683	1.16075	1.19562	1.26824
13	1.10301	1.13809	1.17526	1.21355	1.29361
14	1.11028	1.14947	1.18995	1.23176	1.31948
15	1.11860	1.16097	1.20483	1.25023	1.34587
16	1.12699	1.17258	1.21989	1.26899	1.37279
17	1.13544	1.18430	1.23514	1.28802	1.40024
18	1.14396	1.19615	1.25048	1.30734	1.42825
19	1.15254	1.20811	1.26611	1.32695	1.45681
20	1.16118	1.22019	1.28193	1.34686	1.48595

TABLE OF COMPOUND INTEREST — *Continued*

Periods	2½ Per Cent	3 Per Cent	3½ Per Cent	4 Per Cent	5 Per Cent	6 Per Cent
1	1.02500	1.03000	1.03500	1.04000	1.05000	1.06000
2	1.05063	1.06090	1.07123	1.08160	1.10250	1.12360
3	1.07689	1.09273	1.10872	1.12486	1.15763	1.19102
4	1.10381	1.12551	1.14752	1.16986	1.21551	1.26248
5	1.13141	1.15927	1.18769	1.21665	1.27628	1.33823
6	1.15969	1.19405	1.22926	1.26532	1.34010	1.41852
7	1.18869	1.22987	1.27228	1.31593	1.40710	1.50363
8	1.21840	1.26677	1.31681	1.36857	1.47745	1.59385
9	1.24886	1.30477	1.36290	1.42331	1.55133	1.68948
10	1.28009	1.34392	1.41060	1.48024	1.62890	1.79085
11	1.31209	1.38423	1.45997	1.53945	1.71034	1.89830
12	1.34489	1.42576	1.51107	1.60103	1.79586	2.01220
13	1.37851	1.46853	1.56396	1.66507	1.88565	2.13293
14	1.41277	1.51259	1.61870	1.73168	1.97993	2.26090
15	1.44830	1.55797	1.67535	1.80094	2.07893	2.39656
16	1.48451	1.60471	1.73399	1.87298	2.18288	2.54035
17	1.52162	1.65285	1.79468	1.94790	2.29202	2.69277
18	1.55966	1.70243	1.85749	2.02582	2.40662	2.85434
19	1.59865	1.75351	1.92250	2.10685	2.52695	3.02560
20	1.63862	1.80611	1.98979	2.19112	2.65330	3.20714

FINDING COMPOUND INTEREST

Find the compound interest without using the table:

	Principal	*Rate*	*Compounded*	*Time*
1.	$100	3%	annually	2 yr.
2.	$100	4%	annually	3 yr.
3.	$200	4%	semiannually	2 yr.
4.	$200	4%	quarterly	1 yr. 3 mo.

Find the compound interest of the following, using the table:

	Principal	*Rate*	*Compounded*	*Time*
5.	$1000	3%	annually	4 yr.

SUGGESTION. $1.12551 = the compound amount of $1 for 4 yr. at 3%.

6.	$20	5%	annually	10 yr.
7.	$1000	6%	annually	5 yr.
8.	$1000	6%	semiannually	5 yr.

SUGGESTION. 5 yr. = 10 semiannual periods. The semiannual rate = 3%.

9.	$1000	6%	quarterly	5 yr.

SUGGESTION. 5 yr. = 20 quarterly periods. The quarterly rate = $1\frac{1}{2}$%.

10.	$2000	5%	semiannually	3 yr. 6 mo.
11.	$750	3%	semiannually	5 yr. 6 mo.
12.	$5000	4%	quarterly	2 yr. 9 mo.

13. A war savings stamp cost $4.12 in January. Five years after purchase it is worth $5. Show that $5 is just about the amount of $4.12 placed on compound interest at 4% per annum, compounded quarterly.

14. If John places $4.12 in a savings bank which pays 3% per annum, interest compounded quarterly, how much less will this amount to at the end of five years than the amount received by his sister from $4.12 invested at the same time in a war savings stamp?

How Savings Grow. The Savings Division of the United States Treasury Department has been endeavoring for some time to check any tendency of our people toward wastefulness and extravagance. In its endeavor to do this, it has carried on a campaign for teaching thrift, making special appeals to children by showing them how savings grow. Thus, they are told that in thirty years, the daily saving of a dime will amount to more than $1000 if invested at 4% compound interest; that a weekly deposit of 25¢ invested at 4% compounded semiannually will amount to $71.88 in five years.

It is a part of the business of our banks also to encourage thrift and to teach economy. In no better way can they render service to the community than by giving the people, especially the children, a definite plan for saving their money. Many bankers have already recognized this fact and are offering attractive banking facilities even for children. Thus, general comment was made in the newspapers recently on the fact that the First Trust and Savings Bank of Canton, Ohio, had organized a distinct banking department for boys, which they announced was probably the first of its kind.

PROBLEMS IN THRIFT

The Savings Division of the Third Federal Reserve Bank District of Philadelphia sent out the following table in an "Outline for Teaching Thrift to School Children:"

WEEKLY SAVINGS COMPOUNDED EVERY 6 MONTHS AT 4 PER CENT

Weekly Savings	*1 Yr.*	*2 Yr.*	*3 Yr.*	*4 Yr.*	*5 Yr.*
$ .25	$13.26	$27.06	$41.41	$56.34	$71.88
.50	26.52	54.12	82.82	112.69	143.76
1.00	53.05	108.24	165.65	225.38	287.53

1. Read the above table and tell what it means.

2. Esther saved and invested 25¢ a week. Using the table, find how much she will have at the end of 1 yr; at the end of 2 yr.; 3 yr.; 4 yr.; 5 yr.

3. Using the table, find how much Paul will have at the end of 4 years if his weekly savings are 75¢; if his weekly savings are $1.25.

4. Using the table, find the accumulated savings of your class at the end of 4 years if each member saves 25¢ weekly.

5. Henry belongs to a "Five-Cent Club." The plan of this club is to save 5¢ the first week of January, 10¢ the second, 15¢ the third, and so on through the year. How much does Henry save in this way in a year (52 wk.) if he pays all his dues?

6. Henry's father belongs to a "Ten-Cent Club." The plan of this club is to save 10¢ the first week of January, 20¢ the second, 30¢ the third, and so on through the year. How much does he save in this way in a year (52 wk.) if he pays all his dues?

Savings Bank Accounts. The following is copied from a bank book and shows a person's deposits and withdrawals, also the interest compounded semiannually at 3% per annum, and the balance on hand after each entry. Read it through carefully.

Date	Deposited		Drawn Out		Interest		Balance	
1921								
Nov. 13	400						400	
1922								
Jan. 1							400	
Feb. 10			60				340	
Apr. 26	25						365	
June 14	10						375	
July 1			15		5	10	365	10
Sept. 21			34				331	10
Nov. 27	40						371	10
Dec. 5			100				271	10
1923								
Jan. 1					4	07	275	17
May 7			10				265	17
July 1					3	98	269	15

SAVINGS BANK ACCOUNTS

Interest for a period is usually reckoned on the smallest sum on deposit in the bank during that interest period. What was the smallest sum in the bank account, page 176, between Jan. 1, 1922, and July 1, 1922? Find the interest on this sum at 3% per annum for the interest period. Does your answer correspond to the interest credited on the account July 1? Prove that the interest for each of the other periods has been correctly reckoned. What was the bank balance July 1, 1923? As stated before, practices vary greatly in different states.

Arrange the following as in the statement on page 176:

1. Deposits: Dec. 15, 1922, $50; Apr. 20, 1923, $75; Oct. 28, 1923, $80. Find the balance due Jan. 1, 1924, interest compounded semiannually Jan. 1 and July 1, at 4% per annum.

2. Deposits: Oct. 16, 1923, $200; March 2, 1924, $125; Nov. 6, 1924, $250. Withdrawals: Nov. 13, 1923, $100; Dec. 20, 1924, $225. Find the balance due Jan. 1, 1925, interest compounded semiannually, Jan. 1 and July 1, at 3% per annum.

3. Deposits: Feb. 13, 1923, $300; Nov. 20, 1923, $100; Jan. 6, 1924, $275; Dec. 2, 1924, $50; Feb. 6, 1925, $25. Withdrawals: June 23, 1923, $75; Sept. 16, 1923, $150; Apr. 21, 1924, $100; June 11, 1924, $60. Find the balance due Jan. 1, 1926, interest compounded semiannually Jan. 1 and July 1, at 3% per annum.

4. Deposits: Nov. 30, 1923, $600; Apr. 20, 1924, $100; Aug. 10, 1924, $100; Oct. 3, 1924, $150; Jan. 25, 1925, $200. Withdrawals: Dec. 20, 1923, $200; Apr. 29, 1924, $50; Sept. 5, 1924, $300; June 20, 1925, $175. Find the balance due July 1, 1925, interest compounded quarterly Jan. 1, Apr. 1, July 1, and Oct. 1, at 3% per annum.

Postal Savings System. This system, which has been established by the United States government, provides a savings bank in connection with the post office. These savings draw a low rate of interest, but, on the other hand, the depositor has the security of the national government for repayment.

Any person ten years old or over may open a postal savings account at his home post office. He may deposit any number of dollars at any time, provided he does not deposit more than $100 in any one month or have a total balance to his credit at any one time of more than $2500 exclusive of accumulated interest. The depositor receives postal savings certificates covering the amount of each deposit.

These certificates are issued in fixed denominations of $1, $2, $5, $10, $20, $50, $100, $200, and $500, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. Savings certificates cannot be transferred or negotiated and will be payable only to the one to whom issued.

Interest is paid on postal savings accounts at the rate of 2% per annum for each full year that the money is on deposit beginning the first day of the month following the one in which the money is deposited. A depositor may at any time withdraw the whole or any part of his deposits with any interest due by surrendering savings certificates, properly indorsed, for the amount desired.

Savings Cards and Stamps. A postal savings account cannot be opened for less than $1. Amounts less than $1 may be saved for deposit by the purchase of 10-cent postal savings cards and adhesive 10-cent postal savings stamps. A postal savings card with nine 10-cent savings stamps affixed will be accepted as a deposit of $1 either in opening an account or in adding to an existing account.

To the Teacher. Further information about the Postal Savings System may be obtained by applying at any post office or by addressing the Third Assistant Postmaster General, Division of Postal Savings, Washington, D. C.

POSTAL SAVINGS SYSTEM

1. If I open a postal savings account and deposit $3, what certificates will be given me?

2. What will be given you at the post office if you deposit in postal savings $1.50?

3. How much money may one person deposit in one month in postal savings?

4. If I open a postal savings account Nov. 11, when does interest begin on my first deposit?

5. What is the greatest amount, exclusive of accumulated interest, which you may have on deposit at any one time?

6. If I add a deposit of $25 to my postal savings account July 15 of the current year, when does interest begin thereon? What is the rate of interest? When may I draw the first year's interest on this deposit? How much interest will there be?

7. If I add a deposit of $50 to my postal savings account Feb. 20 of the current year, when does interest begin thereon? When may I draw my first year's interest on this deposit, and how much will it be?

8. A savings certificate for $10, one for $20, and one for $50 were issued to me Nov. 20 of last year. On what date each year may I receive interest on these, and how much will it be?

9. After having opened a postal savings account, what is the least amount that I may deposit? Explain how I may save 20¢ for deposit.

10. If I open a postal savings account Feb. 20 of the current year and deposit $78, what certificates will be given me?

11. If you have two certificates of $1 each, three of $2 each, four of $5 each, and one of $10, what certificates would you surrender to withdraw $28 from your deposits?

Promissory Note. If G. L. Wise buys from R. E. Curtin 67 bu. of corn at $0.75 per bushel and cannot conveniently pay for it for 60 days, he may give Mr. Curtin a paper like the following called a *promissory note.*

$50 25/100	PHILADELPHIA, PA.,	May 7, 1923
Sixty days	after date, I	promise to pay to
the order of	R. E. Curtin	
Fifty ——————————— 25/100 Dollars		
at THE FRANKLIN NATIONAL BANK OF PHILADELPHIA.		
For value received		
Due July 6, 1923.		G. L. Wise

Meaning of a Promissory Note. A promissory note is a written promise made by a person (the *maker*) to pay to a certain person (the *payee*), or to his order, or to bearer, a specified sum of money at a certain time or on demand.

Who is the maker of the above note? the payee? What is the face of the note? What is the date when Mr. Wise must pay the note? Prove that this date is 60 days from the time the note was drawn. Where is Mr. Wise expected to pay this note? In whose possession, probably, is this note from May 7 to July 6?

Time Note. A note may be made payable *a certain number of days after date, a certain number of months after date,* or *upon a certain date.* Such a note is called a *time note.* The day on which a note becomes due is called the *day of maturity.*

A note payable a certain number of days after date is due at the expiration of that number of days, not counting the date. A note payable a certain number of months after date is due so many calendar months after date.

Demand Note. A note may be made payable *on demand*. This means that the note is due whenever the holder demands payment.

A promissory note may be made payable to *the order of the payee*, to the *payee or bearer*, or to the *payee* only. It is indorsed or otherwise transferred in the same manner as a check.

Interest-bearing Notes. A note may be written so as to include the payment of interest. The note shown below is an interest-bearing note; that on page 180 is non-interest-bearing.

$150 $\frac{00}{100}$ CHICAGO, *April 7,* 19*22*

Ninety days after date *I* promise to pay to

the order of *William F. Dale*

One hundred fifty $\frac{00}{100}$ Dollars

at *The Continental and Commercial National Bank*

Value received *with interest at 6%*

No. *964* Due *July 6, 1922* *J. M. Rossiter*

Name the maker of the above note; the payee; the date of maturity. What is the *maturity value* of the note, that is, the face of the note plus the interest thereon for ninety days?

When no rate is mentioned in an interest-bearing note (that is, if "at 6 %" were omitted from the above note), it bears interest at the legal rate of the state in which it is payable.

When a non-interest-bearing note is not paid when due, it will bear interest at the legal rate from the day it is due to the day it is paid.

Borrowing Money from a Bank. Many business houses, and sometimes individuals, have very heavy expenses at certain seasons of the year and very large returns from sales at other seasons

They find it good business policy to borrow the money necessary to carry them through the period of heavy expenses, knowing that they can readily pay it back in a few months. For such loans, business people generally apply to a bank, paying the bank a specified rate of interest for the use of the money. Where do banks obtain the money loaned in this way? Does a bank charge the person who borrows from it a higher or a lower rate of interest than it pays its depositors? Why?

Security. If James Burns asks for a loan of $500 from the bank with which he does business, he will be required, before the loan is made, to sign a promissory note for the amount, and usually to give some *security* for the payment of the loan when it is due. The security is usually provided in one of three ways:

(1) James Burns, having obtained the consent of some responsible person, makes the note payable to him. That person then indorses the note, by which he makes himself responsible for the payment thereof if James Burns fails to pay it when due.

(2) James Burns signs a note, called a *collateral note,* and leaves some valuable *collateral* (such as government bonds or certificates of stock worth more than the face of the note) with the bank, and with the collateral a signed authorization to the bank to sell the collateral to pay the note when due if it remains unpaid.

(3) James Burns signs a note and then obtains an additional signature to it of some one financially responsible, who by signing the note makes himself liable for its payment, provided that the note is written "*I promise to pay*" or "*We, or either of us, promise to pay,*" or in any like wording. Such a note is called a *joint and several note.*

PROMISSORY NOTES

1. Can you give any reason why the maker of a promissory note should carefully preserve it after he has paid it?

2. When is a promissory note due that is dated April 1, and payable in 2 mo.? in 60 da.? in 3 mo.? in 90 da.?

3. Write a promissory note, face $5, with yourself as maker and one of your classmates as payee. Date the note October 1 of the current year, make it payable in 60 da., and find when it is due.

4. If R. E. Curtin wishes to transfer the note, page 180, to H. C. Hoover, in what ways may he write the indorsement?

5. Write two forms of a promissory note, face $100, which show that Samuel Evans is security for the payment of the note which is made by Charles Lee and payable in 60 da. at your home bank. Date the note appropriately and supply the name of the payee. See (1) and (3), page 182.

6. Cut from a sheet of paper a piece the shape of the note, page 181. On it write a promissory note for $120, payable in 90 da. at your home bank with interest at 6%. Use your own name as that of the maker and the name of some classmate as that of the payee. Date the note appropriately and find its maturity value.

7. How much interest is due at maturity on a note for $200 dated Indianapolis, Ind., July 2, of the current year, and payable in 90 da. with interest at the legal rate (see table, page 188)?

8. Suppose E. M. Book buys a horse from George Hensel for $250 payable in 6 mo. Write the joint and several note which Mr. Book would give Mr. Hensel if James D. Reilly signs the note with Mr. Book. Date the note Dec. 15 of the current year. Make it payable at your nearest bank with interest at 5%, and find the maturity value of the note. Supposing the makers of the note live in your home town, where should the note be dated?

9. One of the members of the class should obtain, if possible, a blank form of collateral note and bring it to school for the members of the class to read what is printed thereon.

10. If G. L. Wise is unable to write (see note page 180) find out how he may sign the note.

Drafts. You have already learned that bills may be paid by checks, postal money orders, and express money orders. Bills may also be paid by bank *drafts.* A draft is a written order by which one party directs another party to pay a specified sum of money. A *bank draft* is a draft drawn by one bank upon another bank in which the former has a deposit. It is in reality a check drawn upon one bank by another. Nearly all banks keep deposits in other banks in the leading commercial cities. A draft made by one bank upon another is often called a cashier's check.

WEST CHESTER, PA., March 26, 1921 No. 6003

FARMERS AND MECHANICS TRUST COMPANY

Pay to the order of F. H. Starkey $5.00

Five ———————————— no/100 Dollars

TO THE FIRST NATIONAL BANK
NEW YORK CITY (1-65)

Norris S. Ingram
Asst. Treasurer

COUNTERSIGNED
J. Comly Hall
Vice-President

Purpose of a Bank Draft. The purpose of the bank draft shown above may be explained thus: F. H. Starkey of West Chester, Pa., desiring to send $5 to George L. Buck of Boston, purchases the draft of the Farmers and Mechanics Trust Company of West Chester, Pa., indorses it "pay to the order of George L. Buck, F. H. Starkey," and sends it to George L. Buck, who also indorses it and presents it for payment at a bank in Boston. This Boston bank cashes the draft and collects it from the First National Bank, New York City.

Making Collections by Draft. A draft used to secure the payment of a debt is called a *commercial draft.* The following is a common way of using a draft for this purpose:

Suppose A. Pupil of Cleveland, O., owes The Arithmetic Company of Boston \$250.75. A draft like the following is prepared by The Arithmetic Company (the *drawer*) and deposited with the Colonial Bank in Boston (the *payee*) for collection. The Boston bank sends the draft, with a letter of explanation, to a Cleveland bank for collection. A. Pupil (the *drawee*) of Cleveland, Ohio, is notified by the Cleveland bank, and if he acknowledges the debt and pays it, the Cleveland bank notifies the Boston bank and either forwards a check to, or credits the Boston bank with, the amount of the draft, less the fee for collection. The Arithmetic Company is then credited by the Boston bank with the amount received, called the *proceeds*.

If the drawee refuses to honor a draft it is returned to the drawer with the reason given for refusal to do so.

$\$250\frac{75}{100}$ BOSTON, MASS., *April 14*, 19*24*

At sight pay to the order of

The Colonial Bank of Boston

Two hundred fifty ———————— $\frac{75}{100}$ Dollars

To *A. Pupil, Cleveland, Ohio*

The Arithmetic Company
Per *M. B. D.*

Time Drafts. If the draft on this page read "30 days after sight" (or "60 days" or "90 days"), instead of "at sight," it would be called a *time draft* and would be payable 30 da. (or 60 days or 90 days) after *acceptance*. A time draft is often used in business, as it gives the debtor time to make arrangements to pay it.

The practice of making collections by drafts varies in different localities. Visit a local bank and ask to have its manner of handling drafts explained.

Trade Acceptance. Merchants and the manufacturers of goods were, in the past, frequently accustomed to carry the accounts of their customers for 30, 60, 90 days, or longer, or they accepted notes in payment, but in recent times there has come into quite wide use the *trade acceptance,* which is really a formal acknowledgment of obligation for goods purchased, with promise to pay at some definite time.

If the Old Colony Shoe Company of Boston sells E. B. Blackburn & Company of Rochester, N. Y., a bill of goods payable in, say, 90 days, it may send a trade acceptance, as shown below, for E. B. Blackburn & Company to sign, as follows:

TRADE ACCEPTANCE

No. 121 $500 84/100

Boston, July 3, 1922

Ninety days after date pay to the order of

Old Colony Shoe Company

Five Hundred and ——— 84/100 Dollars

THE OBLIGATION OF THE ACCEPTOR HEREOF ARISES OUT OF THE PURCHASE OF GOODS FROM THE DRAWER

To E. B. Blackburn & Co.
Rochester, N. Y.

Old Colony Shoe Company
J. F. Leland, Treasurer

ACCEPTED 1922 Payable at Freeman's Bank Boston, Mass. By E. B. Blackburn & Co.

This is returned to the Old Colony Shoe Company, which may then present it at its bank of deposit in Boston. The Old Colony Shoe Company gets its money immediately instead of waiting 90 days. The bank holds the trade acceptance until the date of maturity and then collects it from E. B. Blackburn & Company.

DRAFTS

1. In the sight draft, page 185, who directs that payment be made? Who is ordered to pay? When is the draft due? Who

is expected to pay it when it is due? Name the drawer; the drawee; the payee. Who receives the draft when it is paid?

2. You wish to remit $10 to J. P. Hull of Augusta, Maine. To do this you purchase a draft for this amount at your local bank, which has deposits in the First National Bank, New York City. Write the draft and explain how it is used in making this remittance.

3. Write the sight draft which you could use to collect from E. C. Bruce, Detroit, Mich. a debt of $50. If you were to deposit this draft with your local bank, explain the steps that would be taken to collect the draft.

4. On what date is a draft due dated Feb. 15 of the current year, payable thirty days after sight, and accepted Feb. 18?

Bank Discount. Banks make the greater part of their profits by lending money left with them for safe keeping, which from day to day is considerably in excess of the amount needed to pay checks of depositors. Banks usually loan money on demand notes or on notes payable in 30 da., 60 da., 90 da., or 6 mo. The interest on the latter notes is usually paid in advance and when so paid is called *bank discount.* The time for which a note is discounted is called the *term of discount,* and the difference between the maturity value of the note and the discount, is called the *proceeds.*

In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, *the day of discount* as well as *the day of maturity* is counted in reckoning the term of discount.

Meaning of the Table. The table on page 188 gives for each state and territory of the United States and for the District of Columbia the legal rate and the maximum contract rate.

By maximum contract rate is meant the highest rate of interest that the law allows to be contracted for and collected.

States and Territories	Interest Laws		States and Territories	Interest Laws	
	Legal Rate	Maximum Contract Rate		Legal Rate	Maximum Contract Rate
Alabama	8	8	Montana	8	12
Arkansas	6 to 10	6 to 10	Nebraska	7	10
Arizona	6	10	Nevada	12	12
California	7	Any	New Hampshire	6	6
Colorado	8	Any	New Jersey	6	6
Connecticut	6	6	New Mexico	6	12
Delaware	6	6	New York	6	6
Dist. of Columbia	6	10	North Carolina	6	6
Florida	8	10	North Dakota	6	10
Georgia	7	8	Ohio	6	8
Idaho	7	12	Oklahoma	6	10
Illinois	5	7	Oregon	6	10
Indiana	6	8	Pennsylvania	6	6
Iowa	6	8	Rhode Island	6	Any
Kansas	6	10	South Carolina	7	8
Kentucky	6	6	South Dakota	7	12
Louisiana	5	8	Tennessee	6	6
Maine	6	Any	Texas	6	10
Maryland	6	6	Utah	8	12
Massachusetts	6	Any	Vermont	6	
Michigan	5	7	Virginia	6	6
Minnesota	6	10	Washington	6	12
Mississippi	6	8	West Virginia	6	6
Missouri	6	8	Wisconsin	6	10
			Wyoming	8	12

Illustrative Problems. 1. A note dated January 15, 1922, payable in 6 months, was discounted at a Pennsylvania bank February 2, 1922. Find the discount and the proceeds if the face of the note was $150.

Explanation: 6 months after January 15, 1922, or July 15, 1922, was the date of maturity.

February 2 was the date of discount.

∴ 164 days = the term of discount.

6 %, the legal rate in Pennsylvania, is the rate of discount.

$0.027\frac{1}{3}$ = the bank discount on $1 for 164 days.

150 × $0.027\frac{1}{3}$, or $4.10 = the required bank discount.

$150 − $4.10, or $145.90 = the proceeds.

2. Find the proceeds of a note of $250 dated Sept. 1, 1922, payable in 6 mo. with interest at 5%, if this note was discounted at a New Hampshire bank, Dec. 31, 1922.

When an interest-bearing note is discounted by a bank, the bank discount is computed on the amount of the note at maturity and for the time between the date of discount and the date of maturity.

EXPLANATION:	$250.00	= the face of the note.
	6.25	= the interest for 6 mo. at 5%.
	$256.25	= the amount of the note at maturity.
	2.56	= the bank discount for 60 da. at 6%.
	$253.69	= the proceeds.

PROBLEMS IN BANK DISCOUNT

Find the proceeds of the following notes:

	DATE	TIME	FACE	DISCOUNTED
1.	June 1	4 mo.	$600	June 4, in New York
2.	May 15	90 da.	$200	May 16, in Ohio
3.	Mar. 12	60 da.	$250	Mar. 13, in Pennsylvania
4.	Jan. 15	4 mo.	$225	Jan. 16, in your state
5.	Feb. 1	90 da.	$150	Feb. 1, in Michigan
6.	Oct. 20	30 da.	$500	Oct. 20, in Nebraska
7.	Sept. 1	3 mo.	$200	Sept. 1, in Massachusetts

Find the proceeds of the following notes:

	FACE	DATE	TIME	INTEREST	DATE OF DISCOUNT	RATE OF DISCOUNT
8.	$200	Feb. 1	6 mo.	5 %	June 1	6%
9.	$350	June 1	6 mo.	$4\frac{1}{2}$%	Oct. 15	6%
10.	~~$500~~	~~Mar. 1~~	~~9 mo.~~	~~$4\frac{1}{2}$%~~	June 15	6%

11. Find the proceeds of a note dated March 10, payable in 60 da., if the face of the note was $600 and it was discounted March 10 at a New Jersey bank.

12. Find the proceeds of a note of $268 dated May 2, payable in 30 da., if it is discounted at an Illinois bank May 10.

Partial Payments. When a note is paid in part, such payment is called a *partial payment,* and it is receipted on the back of the note. This acknowledgment is called an *indorsement.*

The following is the rule most generally used for determining the balance due on a note with indorsements:

United States Rule. The rule adopted by the Supreme Court of the United States, known as the "United States Rule of Partial Payments," has been adopted as the legal rule by nearly all the states.

Find the amount of the given principal to the time when the payment, or the sum of the payments, is equal to or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before.

New Hampshire, Vermont, and Connecticut have their own rules. If you live in one of these states, find out what these rules are and solve these problems by them.

Illustrative Problem. Find the balance due Dec. 15, 1923, on the following note:

$300.00	Camden, N. J., June 1, 1922
Nine months	after date I promise to
pay to the order of	J. J. Kane
Three hundred ————————	Dollars

At **THE NATIONAL STATE BANK OF CAMDEN**
CAMDEN, NEW JERSEY

For value received, *interest at 6%*

M. J. Miller

INDORSEMENTS

Received on the within note

1923, Mar. 1, $120

J. J. Kane

1923, Sept. 15, $5

J. J. Kane

1923, Nov. 1, $50

J. J. Kane

SOLUTION BY UNITED STATES RULE

Principal, June 1, 1922,	$300.00
Interest on principal from June 1, 1922, to Mar. 1, 1923 (273 da.),	13.65
Amount, Mar. 1, 1923,	$313.65
First payment,	120.00
New principal, Mar. 1, 1923,	$193.65
Interest on principal from Mar. 1, 1923, to Sept. 15, 1923 (198 da.), $6.39.	
(Interest due, Sept. 15, 1923, exceeds payment then made and no settlement is made.)	
Interest on principal from Mar. 1, 1923, to Nov. 1, 1923 (245 da.),	7.91
Amount, Nov. 1, 1923,	$201.56
Payments applied, Nov. 1, 1923 ($5 and $50),	55.00
New principal, Nov. 1, 1923,	$146.56
Interest on new principal from Nov. 1, 1923, to Dec. 15, 1923 (44 da.),	1.07
Amount due, Dec. 15, 1923,	$147.63

Check each interest given in this solution. Show that the interest due Sept. 15, 1923, exceeds the payment then made.

Find by the United States rule the balance due on the date of settlement:

1. On a note dated Albany, N. Y., Sept. 1, 1921, for $500 with interest at the legal rate, containing the following indorsements:

July 1, 1922, $50. Mar. 4, 1923, $50.
Aug. 31, 1922, $100. Aug. 31, 1923, $5.
Settled, Mar. 1, 1924.

2. On a note dated Harrisburg, Penn., June 1, 1920, for $1000 with interest at 4½%, containing the following indorsements:

Apr. 20, 1921, $200. Oct. 2, 1922, $50.
July 16, 1921, $100. Jan. 31, 1923, $200.
Settled, Apr. 1, 1923.

3. On a note dated Boston, Mass., Mar. 1, 1921, for $400 with interest at 4%, containing the following indorsements:

Aug. 13, 1921, $200. Feb. 28, 1922, $100.
Oct. 13, 1922, $50.
Settled, Mar. 1, 1923.

4. On a note dated Richmond, Va., Aug. 1, 1921, for $600 with interest at 4½%, containing the following indorsements:

Nov. 20, 1921, $200. Dec. 15, 1922, $200.
May 31, 1922, $10. Jan. 18, 1923, $100.
Settled, Mar. 3, 1923.

5. On a note dated Columbus, O., March 1, 1921, for $600 with interest at the legal rate, on which payments of $100 each were made at regular intervals of 6 months until March 1, 1924, when the balance due was paid.

SUGGESTION. Regard 6 months as 6 calendar months, or half a year.

6. In problem 5, the sum of the interests paid was how much less than the interest on $600 for 3 yr. at the legal rate in Ohio?

CHAPTER III

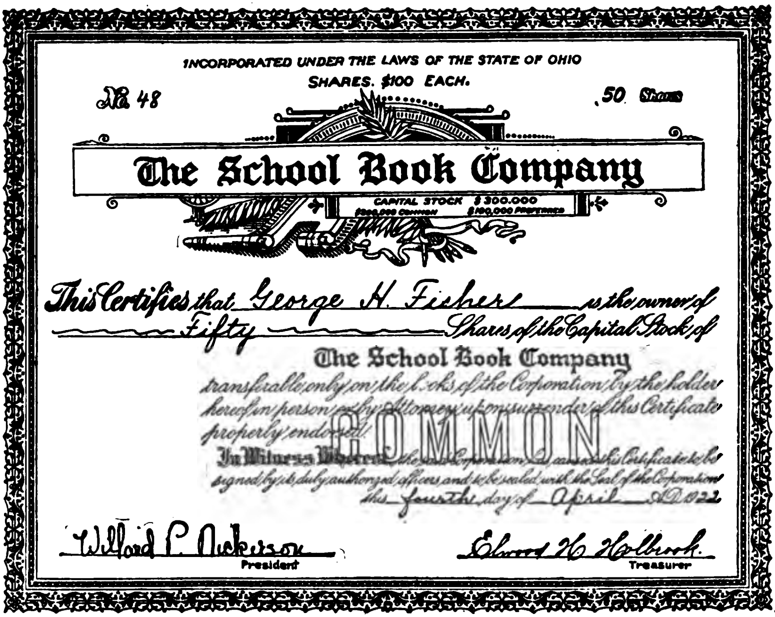
INVESTMENTS, TAXES, AND INSURANCE

A Corporation. The Pennsylvania railroad and its equipment represent property to the amount of nearly $500,000,000, called *capital stock*, which is divided into equal parts called *shares* of $50 (*par value*) each. More than 135,000 persons, called *stockholders*, are joint owners of this capital stock, or in other words, of the railroad property itself. In order that a great enterprise such as that represented by this railroad can be more efficiently carried on, the owners are formed into an association called a *corporation*, which is authorized by law to conduct business as one person. The powers granted a corporation and the restrictions under which it operates are set forth in a document granted it by the state in which it is incorporated, called the *charter* of the corporation. The stockholders elect a few of their number, called *directors*, to carry on the business of the company, and these directors choose the officers, — generally a president, vice president, secretary, and treasurer.

Par Value. The original value of a share of stock, called the *par value*, is usually $50 or $100, though it may be an amount different from either of these, as $1 or $10. Some shares of stock are of no par value.

Market Value. Stocks, like any other property, may be bought and sold. The price at which stock sells is called its *market value*. If stock, the par value of which is, say, $100, is selling at more than $100 a share, it is said to be *above par;* if it is selling for less than $100, it is said to be *below par*.

Certificate of Stock. Each shareholder of a corporation receives a *certificate of stock,* much like the following. Read this one.


INCORPORATED UNDER THE LAWS OF THE STATE OF OHIO
SHARES. $100 EACH.

No. 48 — 50 Shares

The School Book Company

CAPITAL STOCK $300,000
$200,000 COMMON $100,000 PREFERRED

This Certifies that George H. Fisher is the owner of Fifty Shares of the Capital Stock of

The School Book Company

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation this fourth day of April A.D. 1922

Willard P. Dickerson, President — Elwood H. Holbrook, Treasurer

Stock Exchange. The leading stocks are usually bought and sold through *brokers* in a place called a *stock exchange,* which is merely an auction room in a large city where brokers having stocks to sell endeavor to dispose of them to other brokers who have orders to buy such stocks. Brokers usually make the following charges, called *commission* or *brokerage,* for buying or selling stocks:

Shares selling under $10	$7\frac{1}{2}$ cents.
Shares selling between $10 and $125	15 cents.
Shares selling at $125 and over	20 cents.

Minimum commission, $1.00.

Quotations. Stocks are quoted at the market value of a share as follows:

STOCK QUOTATIONS IN THE NEW YORK MARKET, MARCH 19, 1921

Atchison	81½	Southern Pacific	74½
Mexican Petroleum	148¾	Union Pacific	118

"Atchison 81½" means that on March 19, 1921, a number of shares of this stock was sold in the New York market at $81½ per share.

The daily newspapers give the market prices of leading stocks. Bring to class a newspaper containing market reports of sales of these stocks and read the market prices of several of them.

Dividend. Any portion of the net profits of a corporation which is divided among the stockholders is called a *dividend.* It is usually a certain per cent of the par value of the stock, though sometimes a corporation declares dividends of a certain amount of money per share, as 15¢, or $1.50.

Preferred Stock and Common Stock. There are sometimes two kinds of stock of a corporation: *preferred stock,* which generally pays dividends and at a fixed rate, and *common stock,* upon which a dividend may be paid after all preferred dividends are paid and all other obligations met. The amount of the dividends upon the common stock may vary with the earnings of the corporation. Stocks sometimes pay no dividends.

PROBLEMS IN STOCKS

In solving all problems in this book involving purchase or sale of stock, apply the rates of commission given on page 194.

1. Find the cost of 10 shares of Union Pacific railroad stock quoted at 118¼.

EXPLANATION:

$118.25	= the market value of 1 share
10	
1182.50	= the market value of 10 shares
1.50	= the brokerage on 10 shares
$1184.00	= the cost of 10 shares.

Find the cost of the following:

2. 50 shares of Pennsylvania railroad stock at $35\frac{3}{4}$.

3. 20 shares of Southern Pacific railroad stock at $74\frac{1}{2}$.

4. 25 shares of United States Steel stock (common) at $81\frac{1}{4}$.

5. 5 shares of United States Steel stock (preferred) at $109\frac{3}{4}$

6. 20 shares of Chandler Motors stock at $80\frac{1}{4}$.

Compare the prices quoted in Examples 2–6 with the prices prevailing today. Find out whether during the last six months the trend of prices for stocks has been up or down.

Find the proceeds from the sale of the following:

7. 40 shares of Northern Pacific stock quoted at $78\frac{3}{4}$

SUGGESTION. Instead of adding the brokerage as in problem 1, subtract the brokerage.

8. 20 shares of Otis Elevator stock at $125\frac{3}{4}$.

9. 100 shares of Superior Oil Corporation stock at $8\frac{3}{4}$.

10. 50 shares of Bethlehem Steel stock (preferred) at $101\frac{1}{2}$.

11. How much did the person receive who sold the stock mentioned in problem 2? in each of the problems 3 to 6?

7

12. How much is gained by buying 25 shares of Amalgamated Copper Co. stock at $60\frac{1}{2}$ and selling it at $92\frac{3}{4}$, brokerage on the sale and purchase?

13. What profit is made by buying 1300 shares of Chicago Great Western railroad stock at 17 and selling it at $18\frac{1}{8}$, brokerage on the sale and purchase?

14. What profit is made by buying 20 shares of railroad stock at $98\frac{1}{2}$ and selling it at $112\frac{1}{2}$, brokerage on the sale and purchase?

15. Find the annual income from 20 shares of Union Pacific railroad stock, par $100, which pays a quarterly dividend of $2\frac{1}{2}\%$.

SUGGESTION. $4 \times 2\frac{1}{2}\%$ of \$100 = the annual income from each share.

Find the annual income on each of the following:

16. 50 shares of United Gas Improvement stock, par $50, which pays a quarterly dividend of 1%.

17. 100 shares of Utah Copper Company stock, par $10, during a year in which it paid a quarterly dividend of 15%.

18. Bring to school the names of two corporations in your vicinity such as railroad companies, banks, or manufacturing companies. Find out what is the par value of the common stock each has issued, its market value, and the dividend it pays; also whether or not it has any outstanding preferred stock.

19. What rate per cent of income from his investment does a man receive on 6% stock, par value $50, if he purchased it when its market value was $56¾ per share? Answer correct to the nearest .01 per cent.

Explanation

6% of $50, or $3 = the income per share.
$0.15 = the brokerage per share.
$56.75 + $0.15, or $56.90 = the cost per share.
$3 = the income on $56.90 invested.
$0.569 = the income per share, if the rate were 1%
3 ÷ .569 = 5.272+.
Therefore, 5.27% = the required rate.

20. What rate of interest, correct to the nearest .01 per cent, does 6% stock, par $100, pay if purchased at 124¼, brokerage 15¢?

21. What rate per cent of income from his investment does a man receive who buys stock at $35 per share if it pays an annual dividend of $2 per share? Answer correct to the nearest .01 per cent.

22. How much is gained by buying 50 shares of mining stock at 60½ and selling it at 92¾, brokerage on the sale and purchase?

23. If the capital stock of a corporation is $500,000, divided into shares of $50 par, how many shares are there of this stock? How many shares of it do ten men own if their combined holdings are 51% of the stock?

24. If a company with a capital stock of $75,000 distributes $3000 in dividends, how much does it pay on each share of its stock, par value $50? What rate of dividend does it pay on its capital stock?

25. Mr. Carter receives every three months from the Union Pacific Railroad Company a check for $50 as his dividend at $2\frac{1}{2}$% per quarter on his holdings of the stock of this railroad. The par value of the stock is $100. How many shares does he own?

26. When a company with a capital of $500,000 declares a dividend of 6%, how much money is to be distributed as a dividend among its stockholders? What dividend is paid on each share if the par value of the stock is $50? What is Mr. Bailey's dividend on 75 shares of this stock?

27. Mr. Baker's income from railroad stock is $240 a year. It pays an annual dividend of $5 per share. How many shares does he own? What will he receive for this stock if he sells it at $96\frac{1}{4}$, brokerage 15¢ per share?

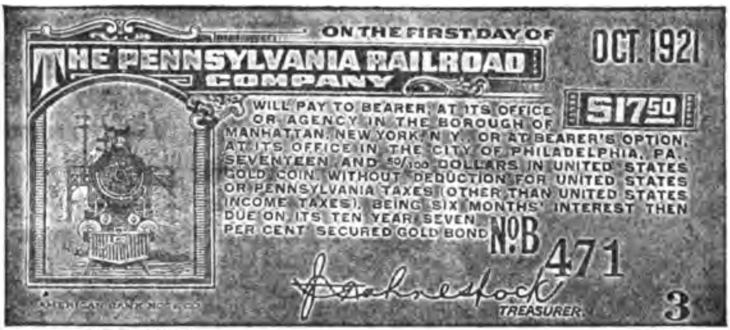
28. If a mining company with a capital of $1,000,000 pays in one year two dividends of 5¢ each per share, and two dividends of 10¢ each per share, how much dividend does it pay in that year, the par value of its stock being $5 per share? The dividends for this year are what per cent of the capital stock?

29. Mr. Clark wishes to invest sufficient money in 6% stock, par value $50, to provide an annual income of $600 for a dependent. If the market value of the stock is $48\frac{1}{2}$, how much must he invest, brokerage 15¢ per share?

Bonds. Governments, national, state, and local, as well as private corporations need to borrow money. For example, to help to meet the expenses of the World War, the United States sold Liberty Loan Bonds and Victory Notes, receiving for them more than $21,000,000,000. Any purchaser of these bonds has merely loaned his money to the government. As evidence of this loan he received one or more of the bonds, which are the government's promise to pay the amount specified therein together with interest at a stated rate every six months until the loans are repaid.

Meaning of a Bond. A *bond* is a written obligation of a government or a corporation to pay a specified sum of money at a certain time together with interest at regular intervals at a fixed rate per annum. Bonds issued by private corporations are usually secured by a *mortgage,* which is an agreement by which the holder of the bonds may sell the property of the corporation if the interest on the bonds is not paid when due, or if they are not paid at maturity

A Coupon Bond. A coupon bond is a bond made payable to the bearer and, when sold, is delivered to the purchaser without indorsement or assignment being necessary. It has attached to it interest coupons payable to the bearer at specified times. When interest is due, a coupon is cut off and presented to a bank for payment.



One of the Coupons (Enlarged) from the Bond on page 199

A Registered Bond. A *registered bond* is a bond issued without interest coupons, filled out in the name of the registered owner, made payable to him or his assigns, and registered in his name.

Interest on registered bonds is sent to the registered owner or to his attorney as it becomes due.

Bonds may sell above or below their face value. They are usually named from the rate of interest they pay and the year in which they are payable. Thus, "U. S. 4s reg., 1925" means U. S. Government bonds registered, bearing 4% interest and payable in 1925.

Commission. The commission charged by a broker for buying or selling bonds is usually $\frac{1}{8}$% of the face value of the bonds.

What to Consider in Purchasing Bonds. It is estimated that more than $500,000,000 is lost each year by investments in worthless stocks and bonds of corporations which agents and promoters

seem to have little trouble in selling to the public. Before purchasing bonds, one should be reasonably sure that:

(1) The property which secures the bond is ample to protect its face value, even under adverse business conditions.

(2) The character of the business is dependable, its management efficient, and its possibilities such that the regular payment of interest is reasonably assured.

(3) It is marketable. Some bonds may represent good values but be so little known that their prospective purchasers are limited. Bonds listed on leading stock exchanges are usually more readily sold than those not so listed.

As the market value of stocks and bonds varies greatly, due to several causes, and as there are so many worthless stocks offered for sale, inexperienced persons, and others as well, should buy these securities from none but reliable brokers or bankers; they should never buy those which promise returns considerably in excess of prevailing rates of interest.

Stocks and Bonds Compared as Investments. The stockholders of a corporation are the owners of it, while the bondholders are creditors of the corporation; hence interest on bonds, at a fixed rate, must be paid before any dividend can be declared on the stock, which is only done in case the net earnings justify the payment of a dividend. This is the main reason why bonds of a corporation are a safer investment than the stocks of the same corporation.

INVESTMENTS IN BONDS

1. Andrew bought a $100 Liberty Loan Bond, the interest on which is due June 15 and Dec. 15 of each year until the bond is due, the interest rate being $3\frac{1}{2}\%$ per annum. How much interest does he receive June 15? Dec. 15? His bond is a coupon bond; what must he do to get the interest?

2. Andrew bought the bond mentioned in problem 1, June 15, 1917, when it was issued. It is a 15–30 year bond (which means that the government may redeem it in fifteen years after it was issued, but that its date of maturity is thirty years after June 15, 1917). On what date may the government redeem the bond? On what date is it due?

3. If Andrew holds this bond for 15 years from the date of purchase, how much interest will he have received on it by that time? How much if he holds it to the date of maturity?

4. What interest does the United States government pay in one year on First Liberty Loan Bonds amounting to $1,989,455,550, the interest rate being $3\frac{1}{2}\%$ per annum?

5. April 1, 1920, the Pennsylvania Railroad Company sold $50,000,000 of ten-year 7% bonds. When will these bonds be due?

6. The interest on the bonds mentioned in problem 5 is payable April 1 and October 1 each year until the bonds are paid. How much interest must the Pennsylvania Railroad Company pay on these bonds each 6 months? how much in 10 years?

7. Find out from the daily market reports of sales of bonds, the market quotation on the bonds mentioned in problem 5, and what a purchaser of $10,000 of these bonds would have to pay for this amount today, including brokerage.

8. June 1, 1920, the Hershey Chocolate Corporation issued $10,000,000 of $7\frac{1}{2}\%$ ten-year coupon bonds in denominations of $1000, $500, and $100, interest payable June 1 and December 1. Mrs. Hartman bought a $1000 bond of this issue at $97\frac{1}{2}$. What did she pay for it including brokerage? What semiannual interest does she receive on the bond?

9. What is the semiannual interest on the whole issue of the bonds mentioned in problem 8? What semiannual interest does the holder of a $500 and two $100 bonds of this issue receive?

10. Has your state, city, or school district issued any bonds which are not yet due? If it has, for what purpose were they issued? When will they be due? What rate of interest do they pay? Do you consider state bonds a safe investment? Give the reasons for your answer.

Building and Loan Associations. More than four million persons in the United States are members of associations, called *building and loan associations*, of which there are nearly 7500 in this country. These associations are private corporations, the object of which is to encourage systematic savings on the part of members and to aid them in becoming owners of homes for themselves.

The Board of Directors issues stock at certain intervals, say, each year. Every member of the association is a holder of one or more shares, and is obligated to pay dues of, say, $1 monthly on each share held. When the sum of these payments together with accumulated interest and profits is equal to the par value of the stock, say, $200 per share, this amount is paid the stockholder less any amount to which he may be indebted to the association for loans from it.

PROBLEMS IN BUILDING AND LOAN ASSOCIATIONS

1. At $1 per month per share, what are the monthly dues on 5 shares which I hold in a building and loan association? What are my dues for 1 yr.?

2. If I pay dues on 10 shares at $1 per month per share, how much do I pay in dues in 12 yr.?

3. If each of my 10 shares (see problem 2) is worth $200 at the end of 12 yr. (which is about the time required for a $200 share, dues $1 per month, to mature), how much more is my stock worth then than the amount of dues which I paid on it in the 12 years?

4. In case a member neglects to pay his dues on or before a specified day of each month, he is fined. If I neglect to pay on time my dues on 10 shares in January, April, September, and December, how much fine must I pay for the four months at 2¢ per share per month?

5. If Mr. Webb holds 10 shares in a building and loan association and borrows from it $800, his monthly payments after receiving the loan consist of his monthly dues and the interest for one month on his loan. If his monthly dues are $1 per share, how much does he pay to the association each month if the rate of interest on the money borrowed is 6% per annum?

6. Mr. Brown held 40 shares in a building and loan association, dues $1 per month. After paying dues for 6 years he purchased a $6000 home on money borrowed from the association at 4.8% per annum. What were his monthly payments to the association after the loan?

7. Six years after Mr. Brown borrowed the $6000 (see problem 6), the stock which he owned was worth $200 a share. How much more was it worth then than the dues and interest which he paid to the association in the twelve years?

8. When Mr. Brown's stock matured (see problems 6 and 7), the association paid him a sum of money equal to the value of his stock less the amount of his loan; how much money did it pay him?

9. Not all building and loan associations operate on the same plan. Find out whether or not there is a building and loan association where you live and, if there is, on what plan it is operated; also how many shares you would have to take out and your monthly dues in order that your stock may be worth $1000 when it matures.

10. Give reasons why building and loan associations are beneficial to the neighborhood.

Meaning of Taxes. A class of 25 pupils decided to carry flags and wear badges at a play festival. A committee of the class was appointed to find out what the flags and badges agreed upon would cost and to determine what each member should pay. They also appointed a member to collect the money. The committee found that the flags and badges which they wanted would cost $8.75 and that each member should pay 35¢, which was agreed upon. Here we say that the members of the class were *taxed* 35¢ each to meet this expense; or that, when he paid his part, each paid a *tax* of 35¢.

Necessity for Taxes. Taxes, as generally understood, are money paid to meet the expenses of the government, local, state, or national.

Thus, *local taxes*, which are those paid to counties, townships, cities, and towns, are used to meet the expenses for building and maintaining roads, bridges, public schools, and other public institutions; also for fire and police protection; for the salaries of certain officials; and for various other purposes.

State taxes, which are those paid to the state, are expended for salaries of state officials, for building and upkeep of state institutions, and as appropriations to aid in building roads, and maintaining schools and various other public institutions; they may be expended by the state for various other purposes as, in some states, for mothers' pensions, teachers' pensions, compensation to disabled soldiers and workmen, etc. Does your state pay out any money for mothers' pensions, or teachers' pensions, or as compensation to disabled soldiers and workmen?

National taxes, which are paid to the Treasurer of the United States, are used to maintain the army and navy, to pay the salaries of United States officials, interest on the national debt, as well as the debt in whole or in part as it comes due, pensions to soldiers, and appropriations made by Congress for a great variety of purposes.

There are also license fees paid by merchants, inheritance taxes imposed by the state and nation, gunners' license fees, dog licenses, automobile licenses, etc. Find out what kind of licenses must be paid where you live. Find out also what is the fee for these licenses, and to whom the fee is paid.

Kinds of Property. Most local and state taxes are paid by the owners of property. For the purposes of taxation, property is divided into *real estate,* as lands, houses, and all other immovable property, and *personal property,* as money, live stock, and other movable property.

Assessed Valuation of Property. Officers, called *assessors,* are elected or appointed to place a valuation on property subject to tax. This valuation is called the *assessed valuation* of the property. Some properties are assessed at about their real value while others of the same kind may be assessed at considerably less than their real value. From the total assessed valuation of the property upon which tax is to be raised and the amount of tax to be raised, the amount of tax which should be paid for each dollar of the assessed valuation can be determined.

Thus, if the assessed valuation of the taxable property, real and personal, in a certain town is $9,000,000, and a tax of $234,000 is *levied* to meet the expenses of the town for the year, the amount which should be paid on each dollar of the assessed valuation is $234,000 ÷ 9,000,000, or $0.026.

Tax Rate. If $0.026 is fixed as the amount of tax to be paid for a certain year on each dollar of the assessed valuation of the property upon which the tax is to be raised, 2.6¢ is called the *tax rate.* Instead of speaking of the tax rate as 2.6¢, we may say that the tax rate is 26 mills, or $2.60 per $100, or $26 per $1000, or 2.6%.

Most states levy upon the voters in the state a personal tax known as a *poll tax,* which is a small fixed sum. Do all the voters in your state pay a poll tax?

COMPUTING TAXES, TAX RATES, AND ASSESSED VALUES

1. When is the property where you live assessed?
2. Who assesses it? How is this done?
3. Who collects the taxes where you live? When? Where?

4. Are poll taxes levied where you live? Upon whom? When and where are they paid?

5. Try to obtain a tax notice or a tax receipt and show it to the class.

6. An owner of real estate usually pays at least a county tax, a school tax, and a road tax, and, in some states, a state tax. Find out just what taxes are paid by an owner of real estate in your community. What was the rate of taxation for each tax last year? Find out the assessed valuation of a piece of real estate. How are the tax rates applied to it to compute the total tax?

7. For what purposes are the school taxes used?

8. If the rate of school tax in a certain school district is 6 mills on $1, what is the school tax of Mr. Waters whose property is assessed at $12,000?

9. If the rate of town tax is $1.26 per $100, what is Mr. Jones' town tax on his public garage assessed at $15,600?

10. In a certain city the assessed value of the real estate in a recent year was $17,000,000 and that of the personal property, $5,000,000. The tax rate for all purposes was $28 per $1000. What was the total tax?

11. In a certain city the market value of the real estate in a recent year was $13,000,000 and that of the personal property $6,250,000. The property was assessed at 40% of its real value and a tax of $4 per $1000 levied. What was the amount of tax levied?

12. When the tax rate is 9½ mills per $1, what is the tax rate per $100? per $1000? What is the rate expressed as a per cent?

13. When the tax rate is $1.25 per $100, what is the tax rate per $1? per $1000? expressed as a per cent?

14. Mr. Williams paid a tax of $93.75. What was the assessed value of the property, the tax rate being $0.75 per $100?

15. When the tax rate is increased from 10 mills to $14\frac{1}{2}$ mills, how much will the tax on Mr. Nelson's property be increased if its assessed value is $7800? What is the per cent of increase in Mr. Nelson's taxes?

When the assessed valuation and the tax to be raised are as follows, find the rate of tax per $1; per $100; per $1000:

	Assessed Valuation	*Tax*		*Assessed Valuation*	*Tax*
16.	$3,000,000	$24,000	17.	$85,000,000	$1,360,000
18.	$12,500,000	$137,500	19.	$125,500,000	$2,133,500

20. Find a farmer's total taxes, exclusive of state tax, for a year on a farm of 75 acres assessed at $40 an acre and personal property amounting to $1200, if the rate of county tax levied is $4\frac{1}{2}$ mills, the rate of school tax $7\frac{1}{2}$ mills, and the rate of road tax 6 mills. What taxes does a farmer pay in your state?

21. The assessed valuation of the property of a certain town is $420,000, upon which a tax of $3150 is to be raised. What must be the rate of taxation and how much tax must a man pay whose property, real and personal, is assessed at $10,000?

22. Mr. Ray's county tax this year is based on the following valuation: real estate, $7500; personal property, $750. What is his county tax for this year, the rate being 3 mills on $1?

23. If Mr. Ray does not pay his county tax by Sept. 1 (see problem 22), 1% of the tax will be added as a monthly penalty. If he neglects to pay his tax until Dec. 1, how much penalty will he have to pay? Would it have been more economical for Mr. Ray to have borrowed the money at 6% to pay his tax?

24. William pays a tax of $3 on his dog Rover. How much money would William have to put at interest at 6% to provide a fund to pay this annual tax?

United States Government Receipts and Expenditures. The expenses of the United States Government, while varying from year to year, are, nevertheless, enormous for any year. Recently the Treasurer of the United States paid out in a single year over $23,000,000,000. For another year the expenditure for the War Department amounted to over $9,000,000,000. The vast sum necessary for the support of the national government is raised chiefly by: (1) a tax, called *customs*, or *tariff duties*, levied on articles brought into this country; (2) a tax, called *internal revenue tax*, levied on incomes and certain profits, and on beverages, products manufactured from tobacco, etc.

Classification of Articles Imported. Articles imported are classified as follows:

(1) Articles subject to *ad valorem duty*, which is a certain per cent of the value of articles at the place of purchase; as rugs, on which the duty is 50% ad valorem.

(2) Articles subject to *specific duty*, which is reckoned at so much per pound, per bushel, etc., of the articles imported; as oats, on which there is a specific duty of 6¢ per bushel.

(3) Articles subject to both ad valorem and specific duties; as perfumery, on which there is an ad valorem duty of 50% and a specific duty of 40¢ per pound.

(4) Articles on which there is no duty (those on the *free list*), as coffee.

Tariff laws are changed from time to time so that articles classed at one time subject to one duty or both duties or no duty may be placed at another time in another class.

Port of Entry. The United States is divided into districts for collection of duties. In each of these districts one or more cities are designated as *ports of entry*, in each of which there is a United States customhouse where articles imported are examined and duties paid.

Amounts less than 50¢ are rejected and those equal to or greater than 50¢ are considered $1 in the total sum upon which duty is reckoned.

UNITED STATES CUSTOMS

1. In a recent year articles representing 65.01% of the value of our imports for consumption were on the free list. What was the value, correct to the nearest dollar, of the articles imported that year subject to duty, the value of the imports for consumption being $5,238,621,668?

Find the duty on the following:

	Imports	*Duty*
2.	8 tons (2240 lb.) of maple sugar	3¢ per pound
3.	Children's dress goods costing $875	35% ad valorem
4.	648 lb. ground spice	1¢ per pound
5.	3 tons (2240 lb.) of grape fruit	1½¢ per pound
6.	Toys costing $2150	35% ad valorem
7.	200 tons of hay	$2 per ton
8.	5 gross of steel pens	8¢ per gross
9.	Cut glassware costing $6500	45% ad valorem
10.	Watch cases valued at $2750	30% ad valorem

11. What is the duty on 365 lb. of chemicals costing $1.80 per pound, on which there is a duty of 30% ad valorem and a specific duty of 5¢ per pound?

12. The duty on automobiles valued at $2000 or more is 45% ad valorem; on others 30%. Find the duty on 2 automobiles, one valued at $5000 and the other at $1800.

13. Find the duty on an importation of goods from Paris, dutiable value 1850 francs ($0.193), at 60% ad valorem.

14. Find the duty on 500 doz. Cuban pineapples on which there is a duty of $5 per 1000.

15. Try to find out whether or not the duty as stated in each of the problems of this exercise is still levied.

Income Tax. Every citizen of the United States, wherever resident, and every resident alien individual having a net income for the year of $1000 or over if single (or if married and not living with husband or wife), or $2000 or over if married and living with husband or wife, must file a return of such income with the Collector of Internal Revenue for the district in which the individual resides or where he has his principal place of business.

A *normal income tax* of 4% of the net income subject to such tax is levied on the first $4000 above a personal exemption from taxation of $1000 for a single person. In the case of the head of a family or a married person (living with husband or wife) the exemption is $2500 if the net income subject to tax does not exceed $5000, but if it does exceed this amount, the exemption is $2000. A further exemption is allowed of $400 for each person (other than husband or wife) dependent upon and receiving his chief support from the taxpayer, if such dependent is under 18 years of age or incapable of self-support because mentally or physically defective. 8% is levied upon the remainder of said income subject to normal tax.

Some incomes are exempt from taxation, as compensation paid its officers, by a state or a sub-division of a state.

A *surtax,* which is a tax additional to normal tax, is imposed upon the total taxable net income from all sources in excess of $6000. Since the great majority of individuals who file reports have incomes less than $6000, the application of surtax is of interest to only a limited number of taxpayers, and hence will not be considered in this treatment.

INTERNAL REVENUE RECEIPTS

1. Mr. Fields is unmarried and has no dependents. If his net income for a certain year is $2500, what is his income tax for that year?

Suggestion. $1000 of his net income is exempt from taxation.

2. Mr. Baldwin is married and has no dependents. If his net income is $3500, what is his income tax for that year?

3. Mr. Martin is married and has two dependent children under eighteen years of age. His net income for a certain year was \$3800. What was his income tax for that year? What would be his income tax for that year if he were also the main support of his parents, who are physically incapacitated?

4. Mr. Andrews is unmarried and has no dependents. His gross taxable income from all sources for a certain year was \$6500. To find his net income he was allowed to make the following deductions: business expenses, \$4200; gifts to charity, \$50; interest paid, \$100; taxes paid, \$50; losses, \$300; bad debts, \$100; depreciation of property used in producing income, \$200. What was his income tax for that year?

5. The manufacturer of tobacco products must pay a revenue tax of \$0.00140625 on each $\frac{1}{8}$-ounce package of tobacco which he manufactures for sale. What is his revenue tax on 1 pound of such packages?

6. What revenue tax must a manufacturer of tobacco products pay on 200 packages of manufactured tobacco weighing 3 ounces each, the revenue tax on each package being \$0.03375?

7. In a recent year the internal revenue tax on tobacco, snuff, etc., was \$206,003,091.84; on beverages (non-alcoholic), such as soft drink, mineral water, etc., \$7,182,219.25; admission to theatres, concerts, cabarets, etc., \$50,919,608.42. What revenue did the government derive from these three objects of taxation in that year?

8. Assuming that the tax on soft drink, mineral water, etc., was \$7,000,000 instead of the amount given in problem 7, and taking \$350,000,000 as the amount spent by the people of the United States for soft drink, mineral water, etc., in that year, as shown by the government report, what per cent of the amount spent was paid as revenue tax?

Nature of Insurance. A boys' club had a cabin built at a cost of \$1000. Wishing to avoid the expense of repairing or rebuilding it in case it should be damaged or destroyed by fire, a committee was appointed to learn how this might be done. This committee found that an *insurance company* would *insure* the cabin against damage or loss by fire provided the club would pay the insurance company a certain sum, called the *premium*. The insurance company would issue a written contract, called a *policy*, agreeing, in consideration of the payment of this premium, (1) that if the cabin were damaged by fire, they would repair the damage or pay the club sufficient money to do so; (2) that if the cabin were destroyed by fire they would rebuild it or pay the amount for which it was insured.

There are several other kinds of property insurance beside fire insurance. Find out what they are.

Rate of Premium. The *rate of premium* is usually the amount charged for insuring property to the amount of \$100 for one year.

The rate of premium may be expressed as a certain per cent of the insured value. Rates for 2 yr., 3 yr., 4 yr., and 5 yr. are generally $1\frac{3}{4}$, $2\frac{1}{2}$, $3\frac{1}{4}$, and 4 times the annual rate.

The rate of premium varies according to the risk involved. The risk depends upon three things: (1) the character of the building, (2) the use to which it is put, and (3) its location. A frame building with a shingle roof used as a public garage in the country without fire protection would carry a higher rate than the same building used for the same purpose with good fire protection. The same building used as a grocery store with good fire protection would carry a lower rate. This frame building used for the same purpose as a modern fireproof building and having the same fire protection would carry a higher rate than the fireproof building. The distance from a fire department sometimes causes the rate to vary.

Term of Insurance. The length of time for which insurance is taken out, or the *term of insurance,* is usually one year or three years or five years. *Short rates* are rates for less than a period.

Visit a fire insurance agent and find out how rates vary where you live. If possible procure a blank policy and bring it to class.

FIRE INSURANCE

1. Find out the amount, the rate, and the term of the insurance on your house.

2. Secure, if you can, the same information as in problem 1 for your school building; a public garage; the city hall; the court house in your county; a department store; a photographer's studio; a hardware store; any other building in which you are interested.

3. A barn was insured for $3500 for 3 years. Find the premium, the annual rate being 55¢.

SUGGESTION. A 3-year rate is how many times the annual rate?

4. Mr. Reeves had his house insured for $2800 at 8¢ per $100 for 1 year, and the contents of the house insured for $1500 at 12¢ per $100 for 1 year. What amount of premium did he pay? What would a 5-year policy cost him?

5. If a store building is insured for $8000 at 75¢ per $100 for 1 year, what is the annual premium? the premium for 3 years? the premium for 5 years?

6. After harvest time, Mr. Worth placed $2500 additional insurance on his hay. At the end of 90 days he sold the hay and cancelled the policy. What amount should the company return if the annual premium charged was 55¢ per $100 and the company retains 40% of the annual premium as the short rate for 90 days?

7. What is the annual rate per $100 insurance when a building is insured for $3000 and the annual premium is $22.50?

8. Mr. Taylor has his garage, valued at \$8000, insured for 80% of its value at $1\frac{1}{2}\%$ per year. What is the amount of this annual premium?

9. Mr. Hooper took out a policy on his farm property for the following amounts: \$4500 on dwelling; \$2500 on contents; \$3000 on barn; \$7000 on contents of barn, divided as follows: \$1500 on implements; \$2500 on crops; \$3000 on cattle; \$1000 on tenant house. Find the 3-year premium on this policy if the annual rate on dwelling and contents is 20¢ per \$100; on barn and contents, 55¢ per \$100; on tenant house, 25¢ per \$100.

10. Mr. McCall took out a 1-year policy for \$10,000 on his garage building at an annual rate of \$3.11 per \$100. He also insured the contents of the garage on the same policy for \$5000, the annual rate on which was \$3.43 per \$100. What is the amount of his annual premium? How much would he save in five years if he took out a 5-year policy instead of renewing his policy each year for the 5-year period?

11. A factory building worth \$64,000 is insured for 80% of its value at $1\frac{2}{3}\%$. The machinery and stock worth \$45,600 are insured for $\frac{7}{8}$ of their value at 2%. The building and contents are completely destroyed by fire. Find the net loss to the insurance company; find the net loss to the owner.

12. One insurance company has offered me a 3-year policy of \$6500 at $1\frac{1}{2}\%$ on my home; another company offered me a policy renewed annually for the same period at 55¢ on \$100 for a year. Which is the better offer and by how much?

13. I have my motor car, which is valued at \$1800, insured with the Union Pacific Insurance Company for 90% of its value, against fire, theft, and collision. The fire rate is 1%, the theft rate $\frac{3}{4}\%$, and the collision rate $2\frac{1}{2}\%$. Find the amount of premium paid to this company for my motor insurance.

14. A farmer insured his wheat crop of 110 acres against damage or destruction by hail in each of three different hail insurance companies at \$10 per acre. The rate of premium charged by each company was 10%. What protection per acre against total loss by hail is the farmer guaranteed? Find the total amount of premium paid by the farmer.

15. During the growing season the farmer's crop (see problem 14) was partially destroyed by hail, the companies' adjusters estimating the damage at 60% over the entire area. Find the total amount of insurance received by the farmer from the three companies. If this crop averaged 15 bushels to the acre when threshed and the wheat is sold at \$1.40 per bushel, find the farmer's return per acre from the 110 acres.

16. A grain merchant in Chicago ordered his agent at Des Moines to buy 10,000 bu. of wheat at \$1.75 per bushel, 4000 bu. of oats at 60¢ per bushel, and 3200 bu. of corn at 80¢ per bushel, paying his agent 2% commission for buying. A policy at $1\frac{1}{2}$% was taken out to cover the cost of the grain and commission as protection during the transit from Des Moines to Chicago. Find the amount of the policy and the premium paid.

Life Insurance. Many men fail to make provision for their own support during that period of life when their earnings cease or, in case of death or disability, for the support of those who may be dependent upon these earnings. To make such provision is one of the main reasons why men, especially in early life when the cost of doing so is comparatively small, invest in life insurance.

Advantages of Life Insurance. Among the advantages recently cited by the Philadelphia Thrift Week Committee as reasons why the public should invest in life insurance are the following: It is a safe and sane investment; it provides for the support of dependents in case of emergency; it creates credit

by which unexpected bills can be paid; it guarantees an income for old age; it never decreases in value; it helps one to save.

Meaning of Life Insurance. If a person takes out life insurance, he agrees to pay a stated premium at regular intervals, in return for which the life insurance company receiving these premiums agrees to pay to a party specified in the policy a certain sum of money at the death of the person insured or at the end of a stated length of time. The party to whom this money is payable is called the *beneficiary.*

The beneficiary may be the insured, his estate, or any other person or any institution named in the policy.

Kinds of Policies. There are three general kinds of life insurance policies issued, and several modifications of each; namely, *ordinary life, limited payment life,* and *endowment.*

Ordinary life policies provide for the payment of the insurance at the death of the insured, and require premiums to be paid during the entire life.

Limited payment policies require the payment of premiums for a fixed number of years; if the insured outlives this period, he pays no further premiums, but the face of the policy is not paid by the company until his death.

Endowment policies provide for the payment of the insurance at the end of a stated period, should the insured live so long; or for the payment of the insurance at the death of the insured if it happens within the endowment period. As in the *limited life,* the premiums cease at the end of a stipulated period, usually in 10, 15, 20, 25, or 30 years.

Rate. The *rate* of life insurance is always stated as so many dollars and cents per year for each $1000 worth of insurance; that is, if the rate is $22.50, it means that the insured pays a premium of $22.50 a year for each $1000 for which he is insured.

Table. The following table shows the annual premiums charged by a leading company for $1000 insurance by the ordinary life, the limited payment, and the endowment plans.

Age	Ordinary Life	Limited Payment Life			Endowment		
		10-Payment	15-Payment	20-Payment	20-Year	25-Year	30-Year
21	18.40	46.30	34.19	28.25	48.08	37.66	31.00
22	18.80	47.00	34.71	28.69	48.17	37.76	31.11
23	19.23	47.73	35.26	29.15	48.25	37.87	31.24
24	19.67	48.47	35.82	29.63	48.35	37.98	31.37
25	20.14	49.24	36.40	30.12	48.46	38.10	31.51
26	20.63	50.04	37.00	30.63	48.58	38.24	31.69
27	21.15	50.87	37.63	31.16	48.69	38.37	31.85
28	21.69	51.72	38.27	31.71	48.83	38.53	32.04
29	22.26	52.61	38.94	32.28	48.96	38.70	32.25
30	22.85	53.50	39.64	32.87	49.12	38.89	32.49
35	26.35	58.58	43.51	36.22	50.11	40.11	34.03
40	30.94	64.59	48.22	40.38	51.70	42.14	36.56
45	37.08	71.81	54.06	45.73	54.41	45.52	42.72
50	45.45	80.66	61.54	52.87	58.96	51.11	

LIFE INSURANCE

In answering questions in this exercise refer to the above table.

Find the annual premium on a $1000 policy, when the age of the insured and the kind of policy are as follows:

Age of Insured	Kind of Policy	Kind of Policy
1. 21	10-Payment Life	20-Year Endowment
2. 40	15-Payment Life	25-Year Endowment
3. 28	20-Year Endowment	Ordinary Life
4. 28	25-Year Endowment	20-Payment Life
5. 30	Ordinary Life	20-Year Endowment
6. 27	Ordinary Life	25-Year Endowment
7. 25	30-Year Endowment	Ordinary Life
8. 29	20-Payment Life	Ordinary Life

9. Mr. Brown took out an ordinary life policy for $5000 when he was 21 years old. What must be his average savings per month to pay the annual premium on this policy? Answer correct to the nearest cent.

10. Find the annual premium on a $5000 20-year endowment policy taken out when the insured was 25 years old. If the insured lives to the end of the insurance period, what will be his age when his policy expires?

11. Find the annual premium that Frank Miller must pay on a 25-year endowment policy for $10,000 taken out May 1, 1921, if Mr. Miller was born June 11, 1899.

12. What is the difference in the amount of premiums paid on a $5000 20-payment life policy taken out when the insured was 25 years old, and a 10-payment life policy taken out when the insured was 35 years old for the same amount, if the insured lives until the policies mature?

13. How much less will the total premiums amount to on a 20-year endowment policy taken out when the insured was 25 than the total premiums on the same kind of policy taken out when the insured was 35, if the face of each policy is $5000?

14. In a recent year the amount of life insurance in force in the United States was $29,274,557,871 represented by 14,460,828 policies. This was an average of how many dollars to each policy, estimated correct to the nearest $1000?

REVIEW OF THE APPLICATIONS OF PERCENTAGE

1. In how many years will $1 placed at simple interest at 5% earn $1 interest?

2. What must be paid for $5000 U. S. Liberty Loan $3\frac{1}{2}\%$ bonds, market value 90.65, brokerage $\frac{1}{8}\%$? What is the semiannual income on these bonds?

3. Bring to school a newspaper containing the market report of bond sales, and read to the class the market price of different issues of Liberty Loan bonds sold the previous day. What must you now pay for a $3\frac{1}{2}\%$ Liberty Loan bond of $100?

4. Consult the table, page 173, to find the year at the end of which $1 placed at compound interest at 4% amounts nearest to $2.

5. What is the assessed valuation of my property if my tax is $68.40, the tax rate being 8 mills on each dollar of the valuation?

6. A dealer sold coal at $11.20 a ton, thereby making a profit of 40% of the cost, and a total profit of $6400. How many tons did he sell?

7. What inheritance tax must be paid in Michigan on $9500 which a man willed his children, if $2000 of this legacy is exempt from taxation, and the amount in excess of this is subject to an inheritance tax of 1%?

8. In a recent year the assessed value of the real estate in Akron, O., was $277,588,840. In that year the tax rate was $1.76 per $100. What was the total tax for that year on the real estate?

9. July 15, 1920, Armour and Company, through bankers, sold $60,000,000 of 7% ten-year gold notes, interest payable Jan. 15 and July 15 of each year until the notes mature. When will they mature? How much interest will Armour and Company be required to pay every 6 months? in 10 years?

10. The $60,000,000 gold notes (see problem 9) were sold to investors at 98.84. What did a note of $1000 of this issue cost an investor, the brokerage being included in the price given?

11. What did investors in the $60,000,000 gold notes (see problem 9) pay for the whole issue of these notes?

12. Consult the table, page 188, read the facts there set forth for your own state, and tell what they mean.

13. When Mr. Jordan was 25 years old, he invested in a 20-year endowment policy for $5000. His annual premiums require a saving of how much each month? (See table, page 218.)

14. When an article is bought at a discount of 20% and 20% from the list price and sold at the list price, what per cent is gained?

15. Last year I paid $38.40 tax on property assessed at $4800, which was 75% of its real value. What was the tax rate? This year the tax rate is the same as last year but my property is assessed at 90% of its real value, which is estimated to be the same as that of last year. What is the increase in my tax?

16. Find the proceeds of a promissory note dated June 1, at Columbus, O., payable in 60 da., face $250, if this note is discounted June 1 at 6%.

17. Mr. Ford bought a lot for $2000 and built a house on it costing $6500. The house was insured for 80% of its value at a premium of 30¢ per $100. The property was assessed at 80% of its cost and the tax rate was $1\frac{1}{4}$%. The house was destroyed by fire within a year and Mr. Ford sold the lot for $3000. Find his gain or loss including the premium on his policy and the taxes for one year.

18. After deducting a commission of $\frac{3}{4}$% and $25.80 for other expenses, an agent sent his principal $4500 as the net proceeds of a sale of goods. Find the commission charged.

19. A township is obliged to build three new bridges, the estimated cost of each being $1640. The estimated expenditure for building and repairing roads is $3830. Find the rate of taxation to meet these expenditures, if the assessed valuation of the property in this township is $1,750,000.

20. Mr. Britton bought a piece of property for $8000. The outlay on this property for one year was as follows: taxes, $1.80 per $100 on an assessed valuation of $7000; fire insurance, 40¢ per $100 on a policy of $5000; repairs, $70. The property also depreciates annually 2% of its cost. Mr. Britton rented this house at $60 per month. What was his annual per cent of net income?

21. The Federal Government imposes an inheritance tax on legacies in excess of $50,000. If the excess does not exceed $50,000, the rate on the excess is 1%. What federal tax is imposed on a legacy of $82,000?

22. The assessed valuation of the property in a town is $810,000; the rate of taxation is 1.6%. Ten per cent of the taxes cannot be collected, and it costs 2% for the collection of the remainder. What is the net amount of taxes received by the town?

23. My house valued at $7200 is rented for $60 a month. It is assessed for $\frac{5}{6}$ of its value, and the rate of taxation is 8 mills. The average cost of repairs per year is $75, and it is estimated that the property depreciates in value $1\frac{1}{2}\%$ annually. Would my income be increased or decreased and by how much, if I were to sell the house for $7200 and invest the money at 6%?

24. Mr. Cunningham owns property valued at $7000. He pays annually $105 taxes, $50 for repairs, and $25 for other expenses. What monthly rental must he get for his property in order to make 6% clear on his investment of $7000?

25. Brown, Alexander & Company imported from Paris 275 pieces of silk, each piece containing 100 yards, which they bought at 4 francs a yard. They paid 50% ad valorem duty and $75 for shipping charges. They sold the silk at $1.50 per yard. Find their gain, if the franc was equivalent to 19.3¢.

26. Write an equation which expresses the interest, i, on $\$d$ for n days at $r\%$.

27. The assessed valuation of a certain town in a recent year was $960,000. The school tax to be raised amounted to $8640. What must be the tax rate per $100? per $1? per $1000?

28. Carl Owen bought from the Moline Plow Company, on March 15, farm machinery worth $750. He gave in payment his note drawn for six months with interest at 6% per annum. The Moline Plow Company presented this note at their bank of deposit on April 5, when it was discounted at 6%. What amount was credited to the account of the Moline Plow Company?

29. On January 1, 1921, Mr. Adams borrowed $1000 from Mr. Mason at 6% per annum. On July 1, 1922, he paid $600. How much did Mr. Adams owe Mr. Mason on January 1, 1925, when he paid the debt, principal and interest?

30. Draw a check on the Union Market Bank of Chicago for the payment of one month's rent in advance on your house in favor of the owner (supplying some fictitious name), using the present date and $35 as the monthly rental.

31. Supposing you receive a check for $108 drawn by Samuel W. Brown on the First National Bank of San Francisco. You wish to transfer this check to Andrew Stevens. Write the check and properly indorse it.

32. I can borrow $1000 on a 90-day note at 6% from an individual or from a bank. How does the interest in the one case compare with the discount in the other? If I borrow from the bank what rate of interest do I actually pay? What is the difference between simple interest and bank discount?

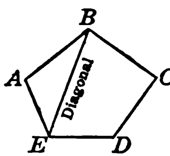
33. State how to find the bank discount on a non-interest-bearing note if the face of the note, the date of discount, the date of maturity, and the rate of discount are given.

CHAPTER IV

MEASURES

Surface. We speak of the surface of the earth, the surface of a lake, and so on. Anything that has length and breadth, without thickness, is called *surface.* Give two other illustrations of surface.

Plane Surface. We say that the carpenter has *planed* the board, meaning that he has made the surface of the board such that straight lines can be drawn on it in different directions. Such a surface is called *a plane surface* or *a plane.*



Polygon. If a portion of a plane is bounded by straight lines, the figure formed is called a *polygon,* as the figure at the right.

The straight lines which bound a polygon are called the *sides* of the polygon, the points in which the sides meet are called the *vertices,* the sum of the sides is called the *perimeter,* and a straight line joining two vertices but not forming a side is called a *diagonal.*

Polygons are named according to the number of their sides. For example, a polygon of three sides is called a *triangle,* a polygon of four sides is called a *quadrilateral,* and a polygon of six sides is called a *hexagon.*

POLYGONS

1. How many sides has the polygon shown on this page? How many vertices?

2. How many diagonals can be drawn in the polygon shown on page 224?

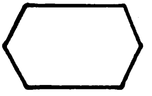
3. How many diagonals can be drawn in a square? in a triangle?

4. Find out what name is given to a polygon of 5 sides; 8 sides; 10 sides.

5. Measure carefully each side of the polygon on page 224 and find its perimeter.

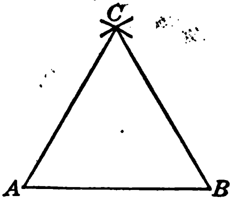
6. Name some special kinds of quadrilaterals which you have studied.

7. How many sides has the figure shown at the right? What is the name of this polygon? Measure carefully each side and find the perimeter.



TRIANGLES

1. A triangle all of whose sides are equal is called an *equilateral triangle*. Measure the sides of triangle ABC and tell whether or not it is equilateral. What is its perimeter?



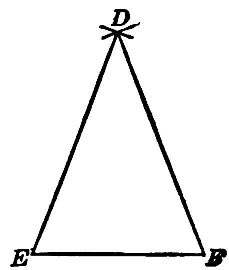
2. Triangle ABC was constructed by using each end of AB as a center and a line equal to AB as a radius, and with the compasses drawing arcs cutting each other at C, and then joining C with A and B. Construct an equilateral triangle with each side 1 in. long; with each side 2 in. long; with each side 3 in. long.

3. **The angles of an equilateral triangle are equal.** With your protractor test the angles of triangle ABC.

4. **The sum of the angles of any triangle is 180°.** Test to see that this is true for triangle DEF (page 226).

5. A triangle two of whose sides are equal is called an *isosceles triangle.* Test triangle DEF and tell whether or not it is isosceles. How long is each side of DEF?

6. Triangle DEF was constructed by using each end of EF as a center and a line equal to ED as a radius, and with the compasses drawing arcs cutting each other at D, and then joining D with E and F. EF is called the *base* of DEF. Construct an isosceles triangle with its base 1 in. and each equal side 2 in.

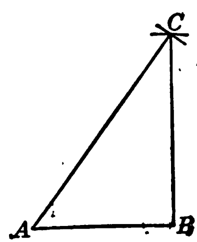


7. **In an isosceles triangle, the angles opposite the equal sides are equal.** Test to see that this is true in triangle DEF.

8. Two angles of a triangle are 60° and 70°; what is the third angle?

9. The angle opposite one of the equal sides of an isosceles triangle is 50°; how many degrees are there in each of the other angles of the triangle?

10. Draw a line 3 in. long corresponding to AB in the figure at the right. Using B and A as centers and drawing arcs, find a point, C, 4 in. from B and 5 in. from A. Draw AC and BC. Angle ABC is a *right angle* (test) and ABC is a *right triangle.* The side opposite the right angle in a right triangle is called the *hypotenuse.* Which side of triangle ABC is the hypotenuse?



11. How many degrees in each angle of an equilateral triangle?

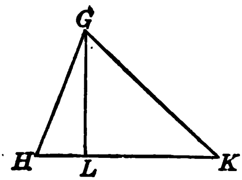
12. If one angle of a right triangle is 20°, how many degrees are there in each of the other angles?

13. One angle of a triangle is 100° and another is 30°; is the third angle acute, right, or obtuse?

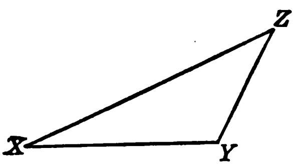
14. Two angles of a triangle are 20° and 50°; is the third angle acute, right, or obtuse?

15. Two angles of a triangle are 30° and 60°; is the third angle acute, right, or obtuse?

16. The *base* of a triangle is the side upon which it is supposed to stand, and the perpendicular drawn to this side from the opposite vertex is the *altitude* of the triangle. Which line is the base of triangle *GHK*? Which is the altitude?



17. Draw a large triangle of the shape of *XYZ*. Using *XY* as the base, draw as nearly as you can the line representing the altitude. Draw the altitude using *YZ* as the base; using *XZ* as the base.



The area of a triangle is equal to one half the product of its base and altitude.

If K represents the area of a triangle, b the number of units in the base of the triangle, and a the number of units in its altitude, then,

$$K = \frac{a \times b}{2}$$

An expression such as $K = \frac{a \times b}{2}$ is called a *formula*. This formula may be used to solve any problem involving only the area of a triangle, if the value of any two of the three letters used in the formula are known and the value of the third is required.

Using the formula for finding the area of a triangle, find:

18. K when $a = 4$, and $b = 6$.

19. K when $a = 15.8$, and $b = 1.25$.

20. a when $b = 6$, and $K = 30$.

21. b when $a = 10$, and $K = 50$.

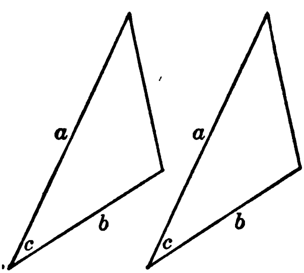
22. Using a line 2 in. long as the base, construct a triangle with one of the other sides $1\frac{1}{2}$ in. long and the third side $2\frac{1}{2}$ in.

SUGGESTION. Use each end of the 2 in. line as a center, and draw arcs using for one a radius of $1\frac{1}{2}$ in. and for the other a radius of $2\frac{1}{2}$ in.

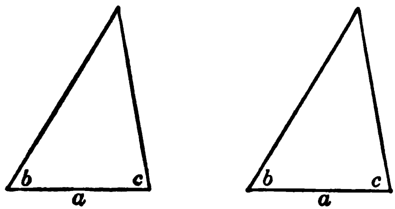
23. Draw a second triangle with sides equal to those in the triangle of problem 22. Cut out the two triangles and place one upon the other in such a way as to learn whether or not the two triangles are equal. If two triangles have the three sides of the one equal, respectively, to the three sides of the other, how do the triangles compare in shape and size?

24. Draw a triangle whose sides are 1 in., 2 in., and $1\frac{1}{2}$ in. Draw three other triangles assigning definite lengths for each of their sides.

25. Draw two triangles as shown in the figure at the right, after measuring and comparing the sides marked a, those marked b, and the angles marked c. Cut the triangles out, place one upon the other in such a way as to learn whether or not the two triangles are equal. If two triangles have two sides and the included angle of the one equal, respectively, to two sides and the included angle of the other, how do the triangles compare in shape and size?



26. Measure and compare the sides marked a in the triangles below, also the angles marked b and those marked c. Draw two triangles of the same size as these, and place the triangles one upon the other in such a way as to learn whether or not they are equal. If two triangles have two angles and the included side of the one equal, respectively, to two angles and the included side of the other, how do the triangles compare in shape and size?



27. What is the area of a right triangle whose base is 2 in. and altitude $1\frac{1}{2}$ in.? Construct this triangle and measure the hypotenuse.

28. Construct a right triangle with base 12 in. and altitude 5 in. Find the area of the triangle and measure its hypotenuse.

29. Find the area of the gable end of a house which is 25 ft. wide, the height of the gable being 10 ft.

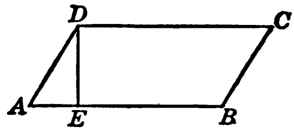
30. Harold and Fred make a kite of two cross sticks 16 in. and 20 in. long, fastening them together at right angles in the center of the shorter piece and 6 in. from the end of the longer piece. Draw on a scale of $\frac{1}{4}'' = 1''$ the figure represented by these cross pieces, and join the ends of the lines. How many square inches of paper will be required to cover the kite?

QUADRILATERALS

You have already learned that two straight lines are said to be parallel if they are everywhere the same distance apart; or we may say: **Two straight lines are parallel if they lie in the same plane and cannot meet however far they may be extended.**

Point out four pairs of parallel lines.

1. A quadrilateral whose opposite sides are parallel is called a *parallelogram*. The side on which a parallelogram is supposed to rest is called the base, and the perpendicular from the side opposite the base to the base is called the altitude. In the parallelogram $ABCD$, which side is the base? Which line is the altitude?

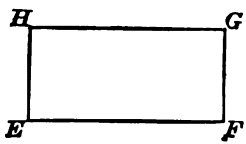


2. Measure the sides of $ABCD$ and compare AB and CD; BC and AD. How do the *opposite* sides of a parallelogram compare in length? Draw a parallelogram whose sides are 3 in. and 4 in.

3. In a parallelogram, as $ABCD$, two sides like AB and AD are called *adjacent* sides. If the adjacent sides of a parallelogram are 10 ft. and 5 ft., what is the perimeter of the parallelogram?

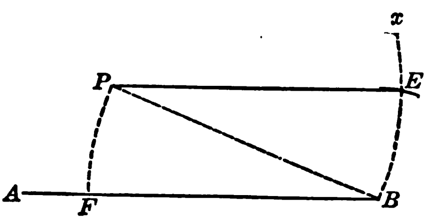
4. A *rectangle* is a parallelogram whose angles are all right angles. Test rectangle $EFGH$ to see that this is true. What is the sum of the angles of $EFGH$?

5. In rectangle $EFGH$, measure the distance from E to G; from F to H. How do the diagonals of a rectangle compare in length? Draw a rectangle 4 in. by 3 in.



6. What name is given to a rectangle which has all of its sides equal?

7. Practice until you can draw through any point, as P (figure at the right) outside a given straight line, as AB, a straight line, as PE, parallel to AB.



This may be done as follows: Given AB. With B as a center and BP as a radius, draw an arc cutting AB at F. With P as a center and the same radius as before, draw the indefinite arc Bx. Now place one point of the compasses at F and the other at P. Then without changing the distance between the points of the compasses, place one point at B and draw an arc cutting Bx at E. Draw PE. Then PE is parallel to AB.

8. Construct a parallelogram having adjacent sides 2 in. and 1 in. and the angles between them 60°.

9. Construct a rectangle having the base 2 in. and altitude 1 in.

10. Construct a square having a side 2 in. long; a side 3 in. long. What is the perimeter of each square?

11. If K represents the area of any parallelogram, b the number of units in the base, and a the number of units in the altitude,

$$K = a \times b$$

Express in words what is stated in this formula.

Using the formula for finding the area of any parallelogram, find:

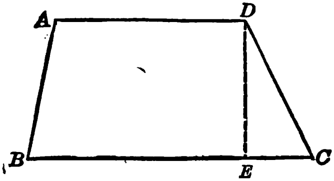
12. K when $a = 8$, and $b = 10$.

13. K when $a = 6.25$, and $b = 12.5$.

14. a when $K = 100$, and $b = 20$.

15. b when $K = 100$, and $a = 12.5$.

16. A quadrilateral two and only two of whose sides are parallel is called a *trapezoid*, as $ABCD$. The parallel sides are called the *bases* of the trapezoid, and the perpendicular joining the bases is called the *altitude*. Name the bases of $ABCD$; the altitude.



17. If K represents the area of any trapezoid, b the number of units in the upper base, B the number of units in the lower base, and a the number of units in the altitude,

$$K = \frac{1}{2} a (B + b)$$

Express in words what is stated in this formula.

Using the formula for finding the area of a trapezoid, find:

18. K when $a = 6$, $B = 10$, and $b = 8$.

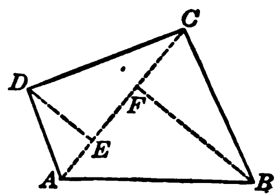
19. K when $a = 12$, $B = 12.5$, and $b = 7.25$.

20. a when $K = 28$, $B = 8$, and $b = 6$.

21. $B + b$ when $K = 150$, and $a = 10$.

22. A quadrilateral no two of whose sides are parallel is called a *trapezium;* as $ABCD$. Measure each side of $ABCD$ and find its perimeter.

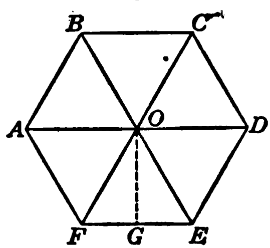
23. Is the area of trapezium $ABCD$ equal to the sum of the areas of the triangles ABC and ADC? If diagonal AC is 32 ft. long and the perpendiculars DE and BF upon the diagonal are 14 ft. and 24 ft., respectively, what is the area of the trapezium?



REGULAR POLYGONS

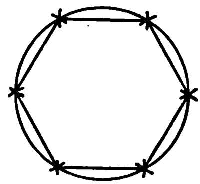
1. A polygon whose sides are equal and whose angles are also equal is called a *regular polygon.* Name a kind of triangle which is a regular polygon. Name another regular polygon.

2. How many sides has the regular polygon shown at the right? It is called a regular *hexagon.*



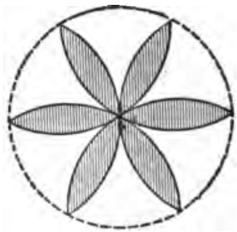
3. In the regular polygon, shown on page 232, O is called the *center*, OA is called a *radius*, and the perpendicular OG drawn from O to FE is called the *apothem*. Name all of the radii of this regular polygon.

4. The regular hexagon is much used in architecture and designs. It may be constructed by drawing a circle and dividing it as shown in the figure at the right. The straight lines (chords of the circle) are each equal in length to the radius of the circle.

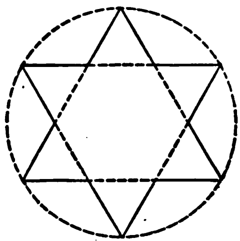


5. Construct a regular hexagon having each side 1 in. long; 2 in. long.

6. Locate on a circle the vertices of a regular hexagon. Then using each vertex as a center and a line equal to a side of the regular hexagon as a radius, describe arcs and shade (or color) as shown in the opposite figure. Erase the dotted line. Have you ever seen this design used?



7. Locate on a circle the vertices of a regular hexagon. Draw straight lines joining vertices as shown in the opposite figure, then shade (or color) and erase the dotted lines. The design is called a six-pointed star.



8. Study the figure for problem 7 to find an easy way of constructing an equilateral triangle. Show the class how to construct an equilateral triangle in this way.

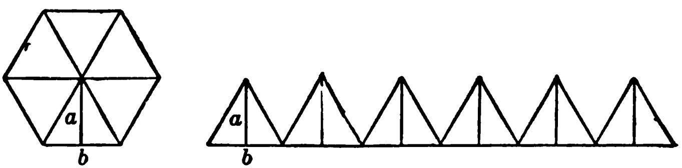
9. Using what you have learned in the above problems, try to construct some original design.

10. Construct a regular hexagon having each side 1 in. long and erase the circle used in constructing it. With a radius of $\frac{1}{2}$ in. and each vertex as a center, draw arcs outside the hexagon and extend them from midpoint to midpoint of the two adjacent sides. The design thus constructed is sometimes called a *six-foil.*

11. Study the patterns in oilcloths, linoleums, rugs, and draperies, also the ornaments on the walls, floors, or ceilings of public buildings, to see if any of the geometrical figures you have been studying are used in their designs. Sketch the designs you find and bring them to class for discussion.

12. Study the following to learn how the area of a regular polygon may be found and what the formula is for finding it.

How may a regular polygon be divided into equal triangles? See drawing below. How does a side of the regular polygon compare with the base of each of the triangles? How does the apothem of the regular polygon compare with the altitude of each of the triangles? Give reasons for your answers. How does the sum of the bases of the triangles compare with the perimeter of the polygon? How does the sum of the areas of the triangles



compare with the area of the polygon? Knowing how to find the area of one of the triangles, state how you think you would find the area of the polygon. Give reasons for your answers.

The area of a regular polygon is equal to half the product of its apothem and the perimeter.

If a represents the number of units in the apothem, P the number of units in the perimeter, and K the area of a regular polygon, then,

$$K = \frac{a \times P}{2}$$

13. Find the area of a regular hexagon one of whose sides is 8 ft. and whose apothem is 6.93 ft.

SUGGESTION. In a regular polygon, $K = \dfrac{a \times P}{2}$

Here $a = 6.93$, and $P = 6 \times 8$, or 48,

Therefore, $$K = \frac{6.93 \times 48}{2}$$

In finding the value of K in problem 13, observe that 6.93 and *not 6.93 ft.* is used; also 48 and *not 48 ft.* The omission of the unit of measure in this formula and in the other formulae of measurements involving products will provide, at least partially, against the use of such loose expressions as *3 ft.* × *2 ft.* = *6 sq. ft.* The important thing is to make sure that the linear units employed in the calculation are expressed in terms of the same unit; the area will then express the number of square units of the corresponding denomination.

14. Find the area of a regular hexagon one of whose sides is 10 ft. and whose apothem is 8.66 ft.

15. Find the area of a regular polygon of 8 sides (*octagon*) one of whose sides is 10 in. and whose apothem is 12.07 in.

16. Find the area of a regular polygon of 5 sides (*pentagon*) one of whose sides is 4 yd. and whose apothem is 2.75 yd.

17. Find the area of a regular hexagon one of whose sides is 20 ft. and whose apothem is 17.32 ft.

The Circle. If the circumference of any circle is divided by its diameter, the quotient will be found in every case to be 3.1416, nearly. The Greek letter π (pronounced *pi*) stands for this quotient. Therefore, if the circumference of a circle is represented by C, the diameter by D, and the radius by R,

$$(1)\ \frac{C}{D} = \pi \qquad (2)\ C = \pi D \qquad (3)\ C = 2\pi R$$

How was (2) obtained from (1)? (3) obtained from (2)?

Sometimes $3\frac{1}{7}$ instead of 3.1416 is used to express the value of π.

THE CIRCLE

1. How do you find the circumference of a circle when you know its radius?

2. Find the circumference of a circle whose radius is $\frac{1}{2}$ in.; 6 in.; 1 ft. 3 in.; 2 yd.; 20 ft.; $\frac{1}{2}$ mi.

3. How do you find the circumference of a circle when you know its diameter?

4. Find the circumferences of the circles whose diameters are equal, respectively, to the radii of the circles in problem 2.

5. If the radius of a circle is 1 in., how does its circumference compare with the circumference of a circle whose radius is 4 in.?

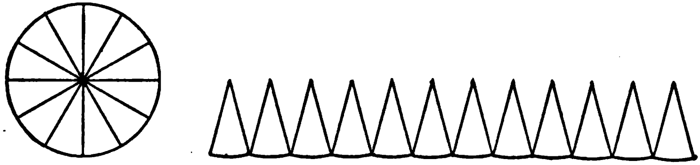
6. Can you tell from problem 5 that the circumference of a circle 6 ft. in diameter is six times the circumference of a circle whose diameter is 1 ft.?

7. A wire may be bent so as to form a square $5\frac{1}{2}$ in. on a side. If this wire were bent into the form of a circle, what would be its radius?

8. A locomotive went from Altoona, Pa., to Harrisburg, Pa., a distance of 131.1 mi., in 112 min. The diameter of the driving wheels is 80 in. How many revolutions per minute did the driving wheels make if there was no slipping?

9. If flywheels on machinery are revolved beyond a certain degree of rapidity, they are apt to fly to pieces. How many revolutions per minute may a flywheel 8 ft. in diameter make and yet the surface speed of the rim not exceed a mile a minute?

Area of a Circle. By the ***area*** of a circle is meant the area of the surface enclosed by the circle. This surface may be divided, as here shown, into a series of figures (*sectors*) each of which resembles a triangle, with the curved side (*arc*) corresponding to the base of the triangle and the radius of the circle corresponding to the altitude of the triangle.



Just as the area of a triangle is $\frac{1}{2}a \times b$ (see page 227), so it is proved in geometry that the area of a sector is $\frac{1}{2}R \times b$, if R denotes the number of units in the radius and b the number of linear units in the arc.

1. How does the sum of the arcs of all the sectors compare with the circumference (C) of the circle?

2. Using R and C, what may you write for the sum of the areas of the sectors?

3. How does the sum of the areas of the sectors compare with the area of the circle?

4. Using R and C, what may you write for the area of the circle?

The area of a circle is equal to the product of half the radius and the circumference.

If C represents the number of units in the circumference, R the number of units in the radius, and S the area of a circle,

$$S = \tfrac{1}{2} R \times C, \qquad (1)$$

$$\text{and } C = 2\,\pi R. \qquad (2)$$

Substituting $2\,\pi R$ for C in (1)

$$S = \tfrac{1}{2} R \times 2\,\pi R; \qquad (3)$$

$$\text{or, } S = \pi R^2 \qquad (4)$$

Equation (4) is a short way of saying that the area of a circle equals π times the radius squared, or, as we say, the area is expressed in terms of the radius; *i.e.*, if we know the radius of a circle, we can find its area by multiplying the square of its radius by π.

5. If the radius is expressed in inches, the area is expressed in what unit of measure? How is the area expressed if the radius is expressed in feet? in yards? in miles?

6. If you know R, how can you find C from equation (2)?

7. If you know C, how can you find R from equation (2)?

8. If you know S and C in equation (1), how can you find R?

9. If you know S and R in equation (1), how can you find C?

10. If you know R, how could you find S from equation (4)?

11. Since $2R$ equals D, express C in equation (2) in terms of π and D.

12. According to equation (1) what two facts concerning a circle must you know before you can find its area? According to equation (4) what one fact must you know? Which equation is the more useful? Why?

13. How can you find the area of a circle if you know its circumference and its radius? If you know the radius can you find the circumference? If you know the radius, do you know all

that is necessary to find the area of a circle? If you know the circumference, can you find the radius? How?

14. What formula would you use to find the area of a circle whose radius is 2 in.?

ILLUSTRATIVE PROBLEMS

1. Find the area of a circle whose radius is 4 ft.

EXPLANATION: 4 ft. = the radius.

$$S = \pi R^2.$$

$$S = 3.1416 \times 4^2, \text{ or } 50.2656.$$

That is, the circle contains 50.2656 sq. ft.

2. Find the area of a circle whose circumference is 31.416 ft.

EXPLANATION: 31.416 ft. = the circumference.

$$C = 2\pi R. \quad (1)$$

$$R = 31.416 \div 6.2832, \text{ or } 5. \quad (2)$$

$$S = \pi R^2.$$

$$\text{Therefore, } S = 3.1416 \times 5^2, \text{ or } 78.54.$$

That is, the circle contains 78.54 sq. ft.

CIRCLES

Find the areas of circles whose radii are as follows:

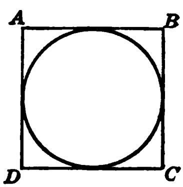
1. 6 in. 2. 4 yd. 3. 12 rd. 4. 2 mi. 5. 2 ft. 6 in

Find the areas of circles whose diameters are as follows:

6. 10 yd. 7. 5 mi. 8. 6.4 ft. 9. 2.75 rd. 10. 5.9 mi.

Find the areas of circles whose circumferences are as follows:

11. 18.8496 ft. 12. 31.416 rd.
13. 3.1416 in. 14. 12.5664 in.
15. 628.32 yd. 16. 157.08 ft.

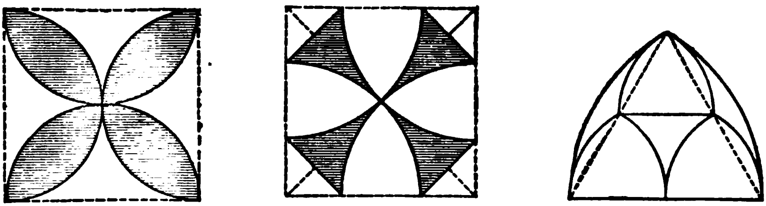


17. Find the perimeter and area of a square circumscribed about a circle whose diameter is 8 ft.

18. Find the difference between the circumference of a circle whose diameter is 12 ft. and the perimeter of its circumscribed square.

19. Find the difference between the area of a circle and the area of a circumscribed square whose length is 10 ft.

20. Draw each of the designs shown below. Shade (or color) as indicated. Erase the dotted lines.



21. How could you lay off on the ground a circle with a radius of 50 ft.? What would be the diameter of this circle?

22. If you wish to lay off a circular running track one eighth of a mile around, how many feet, correct to the nearest tenth of a foot, must you use as a radius?

23. James measured the distance around a tree and found it to be $6\frac{1}{4}$ ft. How thick is the tree, correct to the nearest inch, where he made the measurement?

24. The basin of one of the fountains in the Boston Public Gardens is 20 ft. in diameter; it is surrounded by a circular walk 5 ft. wide. How much did the walk cost at 30¢ per square foot? Draw a diagram before solving this problem.

25. If a lawn roller is 3 ft. long and 4 ft. in circumference, how many revolutions will it make while Joseph rolls lengthwise a lawn 45 ft. by 112 ft., making no allowance for turning at the ends or for overlapping?

Square Root. You have learned how to find the area of a square if the number of units in one side is given, but not how to find the number of units in one side if the area of a square is given. If the number of units in the side of a square is 3, the area of the square is 9, or 3^2. Therefore, 9 is called the *square* of 3, and 3 is called the *square root* of 9.

The number of units in the side of a square is the square root of the area.

Meaning of Square Root. The square root of any number is a number whose square is the given number.

Thus, the square root of 64 is 8, since $8^2 = 64$. Again the square root of $\frac{9}{16}$ is $\frac{3}{4}$, since $(\frac{3}{4})^2$, that is, $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.

Symbol of Square Root. The symbol $\surd$, called the *radical sign*, written before a number, indicates that the square root of the number is to be taken.

Thus, $\sqrt{9}$ (read *the square root of* 9) equals 3.

Perfect Square. A number which is the square of a whole number, a mixed number, or a fraction is called a *perfect square.*

Thus, 25, 2.25, and $\frac{16}{25}$ are perfect squares since $25 = 5^2$, $2.25 = (1.5)^2$, and $\frac{16}{25} = (\frac{4}{5})^2$; but 3 is not a perfect square, since the square of no whole number, mixed number, or fraction is equal to 3. There is a number, however, 1.732, whose square, which is 2.999824, is nearly equal to 3. Therefore, we say $\sqrt{3} = 1.732$, nearly.

SQUARES AND SQUARE ROOTS

1. Find the squares of these numbers:

2.5	$\frac{1}{2}$	$2\frac{2}{3}$	.7	.07	.87	45.01
.005	.022	3.001				

2. Name the square root of each of the following:

4   49   1   81   144 $\frac{16}{81}$ $\frac{4}{9}$ $\frac{9}{25}$

3. Name the number of units in the side of a square whose area is:

25   16   100   9   36   64   121

4. Name the square root of each of the following:

$\frac{1}{4}$ $\frac{1}{9}$ $\frac{4}{25}$ $\frac{9}{64}$ $\frac{16}{121}$ .01   .09   .81   1.44

5. Find the square root of 1225.

In problems 2 and 3 we could tell at a glance the two equal factors, and hence the square root, of each of the perfect squares given there, but if we are asked to give the square root of 1225, we cannot determine at once its two equal factors. It is possible, however, to find these factors, and hence the square root of 1225, by factoring. As an aid to factoring, we may use the form at the right.

$$\begin{aligned} 1225 &= 5 \times 5 \times 7 \times 7 \\ &= 5 \times 7 \times 5 \times 7 \\ &= 35 \times 35 \end{aligned}$$

Therefore, $\sqrt{1225} = \sqrt{35 \times 35} = 35$

$$\begin{array}{r|l} 5 & 1225 \\ \hline 5 & 245 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

By factoring find the square root of each of the following numbers:

	a	*b*	*c*	*d*	*e*	*f*	*g*
6.	324	441	576	784	1296	1764	1849
7.	2025	2916	3969	4356	1156	676	1089

8. What is the square of 10? of 11? Estimate to the nearest whole number the square root of 108; of 119.

9. Make a table of the squares of numbers from 12 to 18. Using this table, estimate to the nearest whole number the square root of 149; of 210; of 230; of 250; of 262; of 284; of 299; of 344.

10. Find the square root of 2116.

The square root of 2116 could be found by factoring, but there is also a general method for finding the square root.

METHOD: First point off the number into periods of two figures each, beginning at units.

```
   21'16|46
   16
86 |516
   |516
```

The largest perfect square which is contained in 21 is 16, the square root of which is 4, or the first figure of the root. Subtract the square of 4 from 21 and annex 16, the next period, to the remainder, making the dividend 516. Double 4, the root found, for a trial divisor. Divide 516, exclusive of the right-hand figure, by 8 for the next figure of the root, which is found to be 6. Annex 6 to 8, making the complete divisor 86. Multiply 86 by 6, the last figure of the root found; subtract the product 516 from 516; nothing remains. The square root of 2116 is 46.

Find the square root of:

11. 1024	12. 1225	13. 7396	14. 2025
15. 3721	16. 5041	17. 8281	18. 3249
19. 9025	20. 3364	21. 7744	22. 9801

23. Find the square root, correct to hundredths, of 142.3.

```
      1'42'.30'00'00 | 11.928
      1
   21|42
     |21
  229|2130
     |2061
   2382|6900
       |4764
  23848|213600
```

SUGGESTION. Point off the number into periods of two figures each, rightward and leftward from the decimal point. Notice that in finding each new trial divisor we multiply by 2 the entire quotient thus far found. Read 11.928 correct to the nearest hundredth.

Find the square root of:

24. 53.29	25. 18.49	26. 94.09	27. 51.84

Find the square root, to the nearest hundredth, of:

28. 32 29. 321 30. 3.42 31. .023

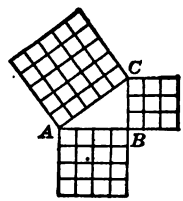
NOTE. If a fraction in its lowest terms is a perfect square, its terms are perfect squares. When the terms of a fraction in its lowest terms are not perfect squares, it is best to reduce it to a decimal before extracting the square root.

Extract the square root of:

32. $\frac{121}{144}$ 33. $\frac{256}{729}$ 34. $7\frac{1}{9}$ 35. $\frac{1089}{1156}$ 36. $\frac{2}{8}$

Observe that in each problem of this exercise you have found the number of units in the side of a square whose area is the given number.

APPLICATIONS OF SQUARE ROOT



1. In the right triangle *ABC*, which side is the hypotenuse? How many square units are there in the square constructed on *AC*? on *AB*? on *BC*?

2. Compare the number of square units in the square whose side is the hypotenuse with the number of square units in the other two squares combined.

The relation between the three squares shown in the figure above may be stated thus:

In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

This truth, known as the Pythagorean Theorem, was first proved by Pythagoras, a Greek mathematician, who lived about 2500 years ago.

3. In a right triangle, if *h* represents the number of units in the hypotenuse, *a* the number of units in the altitude, and *b* the number of units in the base, check each of the following, using the figure given above:

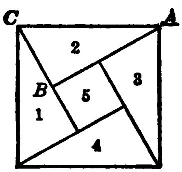
(1) $h^2 = a^2 + b^2$ (2) $a^2 = h^2 - b^2$ (3) $b^2 = h^2 - a^2$

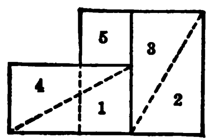
(4) $h = \sqrt{a^2 + b^2}$ (5) $a = \sqrt{h^2 - b^2}$ (6) $b = \sqrt{h^2 - a^2}$

Using formulae (4), (5), and (6) of problem 3,

4. Find h if $a = 6$ and $b = 8$
5. Find a if $h = 25$ and $b = 20$
6. Find h if $a = 9$ and $b = 12$
7. Find b if $h = 29$ and $a = 21$
8. Find h if $a = 5$ and $b = 12$
9. Find a if $h = 45$ and $b = 36$

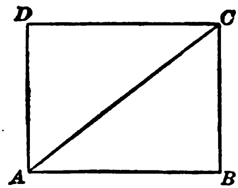
10. The truth of the Pythagorean theorem was shown more than 1000 years ago in the following manner by a famous Hindoo mathematician: He showed that the square constructed on the hypotenuse of right triangle ABC, can be divided into four right triangles 1, 2, 3 and 4, each equal to ABC, and a small square 5, whose side is equal to the difference of the sides forming the right angle of triangle ABC. By changing the position of 1 and 2, the figure shown at the left may be changed to the form shown in the figure on the right. The area of this figure is equal to the square on AB plus the square on BC.





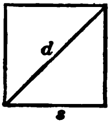
Draw on cardboard a figure like that shown on the left, divide it as indicated, and rearrange as shown at the right. When you have done so, what have you proved? Give reasons for your answer.

11. A guy wire is to be fastened 30 ft. from the ground to a telegraph pole, standing upright and on a level surface, then stretched and fastened at the ground to a stake 18 ft. from the foot of the pole. Allowing 2 ft. at each end for fastening, how long a guy wire will be needed? Draw a diagram showing the pole and the wire, and mark the distances.



12. Two adjacent sides and the diagonal of a rectangle form what kind of figure? In the rectangle $ABCD$, $AB = 4$ and $BC = 3$. Find AC.

13. What is the length of the longest line which can be drawn on a rectangular sheet of paper 12 in. long and 9 in. wide?



14. In the square at the right, a side (s) equals 2. What is the length of the diagonal (d), correct to the second decimal place?

15. What is the length of the longest line that can be drawn on a floor 24 ft. long and 18 ft. wide?

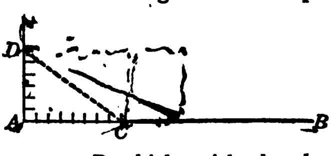
16. How long is the side of a square which contains the same area as a rectangle 50 ft. long and 16 ft. wide?

17. How many more feet of fencing are needed to enclose a rectangular lot 80 ft. long and 20 ft. wide than are needed to enclose a square lot of equal area.

18. A baseball diamond, or infield, of regulation size for men is 90 ft. square. How long is a straight throw from first base to third?

19. A ladder 26 ft. long is stood up against the side of a building. How many feet must it be drawn out at the bottom so that the top may be lowered 2 ft.?

The right triangle is frequently employed in staking out lines, as the lines for a tennis court, or placing foundation walls for buildings. For example, the line AB having been determined, DA may readily be located so as to be perpendicular to AB at A. This may be done by fixing point C on AB, 8 feet from A, fastening a tape measure (or



any cord) at A, then taking hold of the tape at D which, with the tape extended, is 6 ft. from A, drawing it into such position that a 10-foot pole or

10 feet of tape will just reach from C to D. AD is the required perpendicular. With the aid of another person, practice, when it is convenient to do so, until you can stake out lines at right angles to each other.

20. Draw on the blackboard a line AB as in the figure on page 246, and locate C 20 in. from A. Let a classmate hold one end of a 15 in. string at A while you hold one end of a 25 in. string at C. With the strings extended bring the loose ends together at the fixed point D. Draw AD. Why is A a right angle? Erect a perpendicular to AB at C; to AD at D.

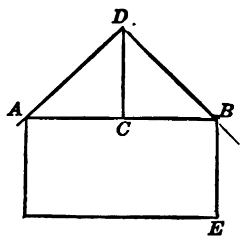
21. Using the formula $K = \pi R^2$, find R when $K = 314.16$; find D; find C.

22. Using the formula $K = \pi R^2$, find R when $K = 78.54$; find D; find C.

23. A square courtyard, as large as possible, is made within a circle whose diameter is 66 ft., and the remainder of the circle is sodded. How many square feet are sodded? Answer correct to the nearest .1 ft.

SUGGESTION. The diameter of the circle is the diagonal of the square. Before attempting to solve the problem, draw a diagram.

24. In the figure at the right AB represents the width of a building and CD the *rise*, or height, of the gable roof. The *pitch* of the roof is found by dividing CD by AB. For example, if CD is 16 ft. and AB is 32 ft., the pitch of the roof is $\frac{16}{32}$, or $\frac{1}{2}$. What is the pitch of the roof if AB is 32 ft. and CD is 20 ft.?



25. In the figure for problem 24, DA represents a rafter. How long must it be cut for a house 32 ft. wide if the pitch of the roof is $\frac{1}{2}$ and the rafter is to extend 1 ft. beyond the eaves?

26. Ask a carpenter to show you how he finds the pitch of a roof by using a rule and steel square; also, how he finds the length of the rafters required.

27. Find the diameter of a circle whose area is 314.16 sq. in.

28. What is the radius of a circle equal in area to a rectangle containing 45239.04 sq. ft.?

29. A baseball diamond is 90 ft. square. Edward stands 10 ft. from second base directly between it and third base. How far does he have to throw to reach the home plate?

Cubes and Cube Root. The cube of 2 (written 2^3) is $2 \times 2 \times 2 = 8$. The cube of 3 (written 3^3) is $3 \times 3 \times 3 = 27$. 3 is the cube root of 27. What is the cube root of 8? The cube root of 8 is written $\sqrt[3]{8}$.

Read the first two lines below carefully. Then supply the missing numbers in the lines following.

The cube of 4 is 64 and the $\sqrt[3]{64}$ is 4.

The cube of 6 is 216 and the $\sqrt[3]{216}$ is 6.

The cube of 5 is ... and the $\sqrt[3]{\ldots}$ is ...

The cube of 8 is ... and the $\sqrt[3]{\ldots}$ is ...

The cube of 7 is ... and the $\sqrt[3]{\ldots}$ is ...

The cube of 9 is ... and the $\sqrt[3]{\ldots}$ is ...

Your knowledge of cubes and cube root can be applied to volumes; for instance, if the edge of a cube is 3 units, its volume is 27 cubic units. If you know that the volume of a cube is 27 cubic units, you can find its edge by finding the cube root of 27, which is 3.

What is the edge of a cube whose volume is 1000 cu. ft.? 729 cu. in.?

What is the edge of a cube which contains 27 cu. yd.?

Cube Root of a Perfect Cube. Just as we were able to find the square root of a perfect square by factoring, so we can find the cube root of a perfect cube by factoring. For example:

$$\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7 \quad (1)$$

$$\sqrt[3]{1000} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5} \quad (2)$$

Arranging the factors of (2) in three groups $\Big\} = \sqrt[3]{(2 \times 5)\ (2 \times 5)\ (2 \times 5)}$

$$= \sqrt[3]{10 \times 10 \times 10} = 10$$

FINDING CUBE ROOTS

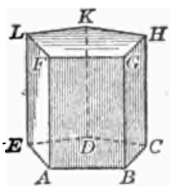
Find the cube root of these perfect cubes by factoring:

1. 64	2. 125	3. 216	4. 343	5. 729
6. 1728	7. 4096	8. 2744	9. 3375	10. 5832

11. What is the edge of a cube whose volume is 4096 cu. ft.? 1728 cu. ft.? 1331 cu. yd.?

There is a general method of finding the cube root of any number, just as there is a general method of finding the square root, but it is not one of the subjects discussed in an elementary arithmetic.

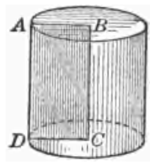
Right Prism. The figure at the right represents a *right prism*. *ABCDE* and *FGHKL*, called *bases*, are parallel, which means that they cannot meet however far they may be extended; they are also equal polygons. The other boundaries, called *lateral faces*, are rectangles. *AF*, *BG*, and so on, are called *lateral edges*. In a right prism any lateral edge may be taken as the *altitude* of the prism. Name the lateral faces of this prism.



There are prisms which are not right prisms; these are called *oblique prisms*. Only right prisms will be treated in this book; they will be called merely prisms. The cube and the rectangular solid, which are classes of right prisms, have already been treated.

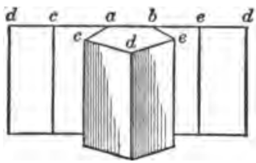
Meaning of Lateral Area of a Prism. The sum of the areas of the lateral faces of a prism is called the lateral area.

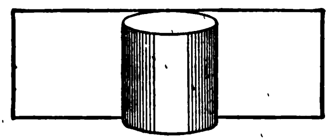
Right Circular Cylinder. The figure at the right may be thought of as having been formed by the revolution of rectangle *ABCD* about *BC* as an *axis*. The figure is a *right circular cylinder*. The surface formed by the side *AD* is the *lateral surface* and its area is the *lateral area*. The surfaces bounded by the circles formed by *A* and *D* are the *bases*. The axis *BC* is the *altitude*.



There are cylinders which are not right circular cylinders, but they will not be considered in this book. Right circular cylinders will be spoken of merely as cylinders.

Areas. The lateral surface of a prism or of a cylinder may be thought of as unrolled like a rectangular sheet of paper. How





does the perimeter of the base of the prism or the circumference of the base of the cylinder compare with the base of the rectangle? How does the altitude of the prism or the cylinder compare with the altitude of the rectangle? As you know how to find the area of a rectangle, how would you say the lateral area of a prism might be found? the lateral area of a cylinder?

1. **The lateral area of a prism is equal to the product of its altitude and the perimeter of the base.**

2. **The lateral area of a cylinder is equal to the product of its altitude and the circumference of the base.**

NOTE. See note, page 121.

Formulae. If H is the number of units in the altitude of a prism (see page 250) and P the number of units in its perimeter, show that its lateral area equals $P \times H$. If S represents its lateral area, then:

$$S = P \times H. \quad (1)$$

If, in a cylinder, S represents its lateral area, H the number of units in its altitude, and P the number of units in its perimeter (circumference), show that:

$$S = P \times H. \quad (2)$$

If P represents the circumference of the base of the cylinder and R its radius, show that $P = 2\,\pi R$. If $2\,\pi R$ is substituted for P in equation (2), show that:

$$S = 2\,\pi RH. \quad (3)$$

In equation (1), what facts must be known concerning a prism before you can find its lateral area? What facts must be known concerning a cylinder before you can find its lateral area using equation (2)? using equation (3)?

AREAS OF PRISMS AND CYLINDERS

1. Find the lateral area of a prism each side of whose square base is 6 ft. and whose altitude is 3 ft.

SUGGESTION. In a prism, $S = P \times H$.
$\therefore S = 4 \times 6 \times 3.$

2. Find the lateral area of a cylinder whose altitude is 6 in. and whose radius is 4 in.

SUGGESTION. In a cylinder, $S = 2\,\pi RH$.
$\therefore S = 2 \times 3.1416 \times 4 \times 6.$

3. Find the lateral area of a prism whose base is a square 4 ft. long, and whose altitude is 6 ft.

4. Find the lateral area of a prism whose base is a regular hexagon 2 ft. on a side, if the altitude of the prism is 8 ft.

5. Find the lateral area of a cylinder whose radius is 4 in. and whose altitude is 1 ft.

6. Find the number of square feet in the entire surface of a cube whose edge is 3 in.

Volumes. The rectangular solid (Figure 1), the prism (Figure 2), and the cylinder (Figure 3) have *equal altitudes* and the

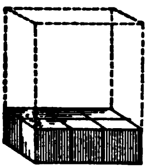


FIG. 1.

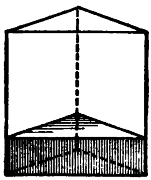


FIG. 2.

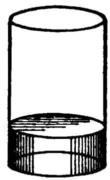


FIG. 3.

same number of square units in their bases. How many square units in the base of Figure 1? Figure 2? Figure 3? In Figure 1 a layer 1 unit high contains how many cubic units? How many cubic units in each layer 1 unit high in Figures 2 and 3? How does the number of cubic units in 1 layer 1 unit high in Figure 1 compare with the number of cubic units 1 layer high in Figures 2 and 3?

As Figures 1, 2, and 3 have the same altitudes, how does the number of layers in the three figures compare? Then how do the volumes of the three figures compare? How would you say the number of cubic units in each of these figures, that is the volume of each, might be found?

The volume of a prism or of a cylinder is equal to the product of its base and altitude.

NOTE. By *the product of its base and altitude*, as used in the foregoing principle, is meant the *number* of square units in the base and the *number* of linear units in the altitude, the base and altitude being expressed in terms of corresponding units. Throughout this book when a product involving base and altitude is spoken of, it will be with this meaning.

Formulae. If, in a prism, V represents the volume, H the number of units in the altitude, and B the number of square units in the base, show that:

$$V = B \times H. \quad (1)$$

What facts must you know concerning a prism before you can find its volume, using equation (1)?

If, in a cylinder, V represents the volume, H the number of units in the altitude, B the number of square units in its base, show that:

$$V = B \times H. \quad (2)$$

If B is the number of square units in the base of a cylinder and R its radius, show that:

$$B = \pi R^2 \quad (3)$$

If πR^2 is substituted for B in equation (2), show that:

$$V = \pi R^2 H. \quad (4)$$

What facts must you know concerning a cylinder before you can find its volume, using equation (2)? using equation (4)?

VOLUMES OF PRISMS AND CYLINDERS

1. Find the volume of a prism whose base contains 64.96 sq. ft. and whose altitude is 8 ft.

SUGGESTION. In a prism, $V = B \times H$.

$$\therefore V = 64.96 \times 8.$$

2. Find the volume of a cylinder whose altitude is 5 in. and whose radius is 6 in.

SUGGESTION. In a cylinder, $V = \pi R^2 H$. 37

$$\therefore V = 3.1416 \times 6^2 \times 5.$$

3. Find the volume of a prism whose base is a square 8 ft. on a side, and whose altitude is 18 in.

4. What is the volume of a cylinder whose radius is 6 in., and whose altitude is 1 ft. 6 in.?

5. Find, to the nearest gallon, the capacity of a cylindrical tank, the radius of whose base is 3 ft. and whose altitude is 6 ft.

6. What is one side of the base of a square prism which contains 150 cu. ft. and whose altitude is 6 ft.?

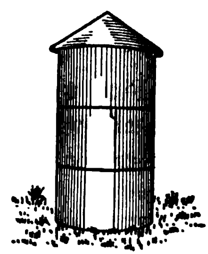
7. Find the weight of a granite pillar of uniform size, whose base is a circle with a radius of 12 in. and whose height is 14 ft., if 1 cu. ft. of granite weighs $165\frac{5}{8}$ lb.

7 8. What must be the depth of a bin 10 ft. long and 5 ft. wide to hold 12 tons of wheat, counting 1 bu. (60 lb.) as equal to $\frac{5}{4}$ of a cubic foot?

9. The tank connected with Mr. Henderson's windmill is 8 ft. high and 6 ft. in diameter. Find the capacity of the tank, correct to the nearest gallon.

10. Find the altitude of a cylinder that contains 125.664 cu. ft., if the radius of its base is 2 ft.

A *silo*, shown at the right, is a structure used to preserve shredded corn fodder and some other farm crops as feed principally for dairy cows. A silo is built round so that it can be more easily reinforced to withstand the bursting pressure, and because in this form it has no corners for air pockets, making it possible to pack the green feed tightly. The size of the silo depends upon the number of cows to be fed and the length of time they are fed silage. To keep silage from molding, at least 2 in. in depth should be fed each day. Some cows will eat 25 lb. of silage, others as much as 40 lb.; the average ration, however, is about 30 lb. per day. An acre of corn yields from 10 to 20 tons of silage.



End.

11. The following table (Circular 87, Extension Service of University of Wisconsin) gives the relation of size and capacity of silo to the amount of silage to be used daily; check each number under *Tons:*

Number of cows	Silage for 180 days at 30 lb. per day	Size of Silo		Silage for 240 days at 30 lb. per day	Size of Silo	
		Inside diameter	Depth of silage		Inside diameter	Depth of silage
	Tons	Feet	Feet	Tons	Feet	Feet
10	27	10	20	36	10	25
15	40.5	10	28	54	10	33
25	67.5	12	30	90	12	26
30	81	12	34	108	14	34
40	108	14	34	144	14	41
50	135	14	40	180	16	40

12. Find the number of cubic feet of silage in the silo whose inside diameter is 12 ft. if the depth of silage is 30 ft. (See above table.)

13. From the above table, find the average weight of a cubic foot of silage in the silo whose inside diameter is 14 ft. if the depth of silage is 40 ft.

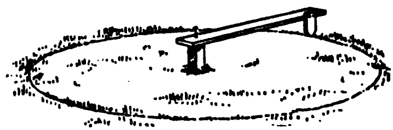
14. If a silo is 18 ft. in diameter and is filled with silage to a depth of 36 ft., how many cubic feet of silage does it contain?

15. If a silo is filled to a depth of 36 ft., the average weight of the silage is estimated to be 42.8 lb. per cubic foot. Using this estimate, find the number of tons in the quantity of silage as found in problem 14.

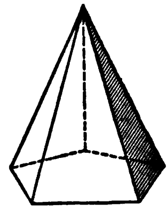
Will a cubic foot of silage at the bottom of a silo weigh more or less than a cubic foot near the top? Why? It is estimated that 10 ft. from the top corn silage weighs 33.1 lb. per cubic foot; 20 ft. from the top, 46.2 lb.; 30 ft. from the top, 56.4 lb.; 36 ft. from the top, 61 lb.

16. How many days, correct to the nearest day, will the silage in problem 15 last a herd of 60 cows allowing each 30 lb. a day?

17. A device like the one in the accompanying illustration may be used to mark out on the surface of the ground the location of a silo. If the circle there represented is 18 ft. in diameter and a foundation of this size is to be dug to a depth of 6 ft., how many cubic feet of earth must be removed?

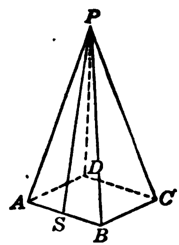


Regular Pyramids. The figure at the right is called a *regular pyramid;* it is a solid whose base is a regular polygon and whose *lateral faces* are equal isosceles triangles which meet in a point, called the *vertex* of the pyramid.



Name the lateral faces of the pyramid shown below on this page; the base; the vertex.

The sum of the areas of the lateral faces is the *lateral area,* the perpendicular from the vertex to the base is the *altitude,* and the line drawn from the vertex to the middle point of a side of the base is the *slant height.* In the pyramid, page 258, name the altitude; the slant height.



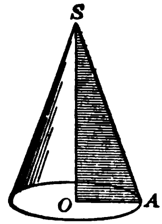
In this book regular pyramids only are treated; they will be called merely pyramids.

Lateral Area of a Pyramid. In the pyramid $P-ABCD$, how would you find the area of one of the triangular faces, as PAB?

$$\begin{aligned}
\text{Area } PAB &= \tfrac{1}{2}\, PS \times AB. \quad \text{Why?}\\
\text{Area } PBC &= \tfrac{1}{2}\, PS \times BC. \quad \text{"}\\
\text{Area } PCD &= \tfrac{1}{2}\, PS \times CD. \quad \text{"}\\
\text{Area } PDA &= \tfrac{1}{2}\, PS \times DA. \quad \text{"}\\
\text{Lateral area} &= \tfrac{1}{2}\, PS \times (AB + BC + CD + DA).
\end{aligned}$$

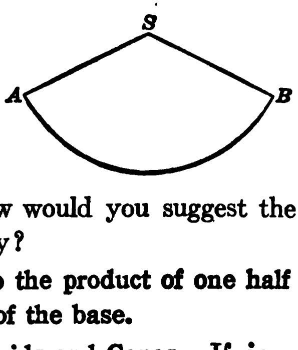
The lateral area of a pyramid is equal to the product of one half the slant height and the perimeter of the base.

Cone of Revolution. The figure at the right may be thought of as having been formed by the revolution of the right triangle SOA about SO as an *axis*. The figure is a *cone of revolution*. The surface formed by the side SA in making this revolution is the *lateral surface*, and its area is the *lateral area* of the cone. The circle formed by the side OA is the *base*, and the axis, SO, is the *altitude*. The side SA is called the slant height of the cone.



In this book cones of revolution only are treated; they will be called merely cones.

Lateral Area of a Cone. If we should slit the surface of a cone and flatten it out, we would have the figure shown below, which is part of a circle. AB of this figure corresponds to what line of the cone? SA of the figure corresponds to what line of the cone? Give reasons for your answers in each case. When we find the area of a sector we multiply its arc by one half the radius. How would you suggest the area of the figure SAB be found? Why?



The lateral area of a cone is equal to the product of one half the slant height and the circumference of the base.

Formulae for Lateral Areas of Pyramids and Cones. If, in a pyramid, S denotes the lateral area, L the number of units in the slant height, and P the number of units in the perimeter of the base, show that:

$$S = \frac{P \times L}{2}. \quad (1)$$

If, in a cone, S denotes the lateral area, L the number of units in the slant height, and P the number of units in the circumference of the base, then show that:

$$S = \frac{P \times L}{2}. \quad (2)$$

If P is the circumference of which R is the radius, then $P = 2\,\pi R$. If $2\,\pi R$ is substituted for P in equation (2), show that:

$$S = \pi RL. \quad (3)$$

AREAS OF PYRAMIDS AND CONES

1. Find the lateral area of a pyramid whose base is a regular hexagon 8 ft. on a side, if the slant height of the pyramid is 10 ft.

SUGGESTION. $S = \dfrac{P \times L}{2}.$

$\therefore S = \dfrac{6 \times 8 \times 10}{2}.$

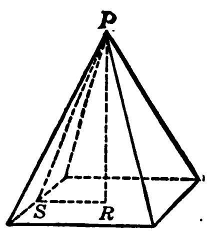
2. Find the lateral area of a cone the radius of whose base is 6 in. and whose slant height is 8 in.

SUGGESTION. $S = \pi RL.$
$S = 3.1416 \times 6 \times 8.$

3. Find the lateral area of a pyramid whose base is a regular pentagon 10 in. on a side and whose slant height is 12 in.

4. Find the lateral area of a cone whose radius is 1 ft. and whose slant height is 18 in.

5. A side of a square pyramid is 6 ft. and the altitude (PR) is 4 ft. Find the length of the slant height PS. Find the lateral area.

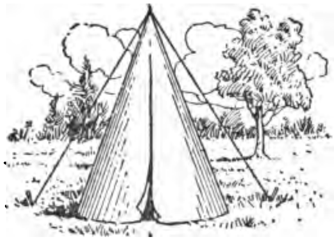


SUGGESTION. PRS is a right triangle.

6. Find the lateral area of a pyramid whose slant height is 5 ft. and whose altitude is 3 ft., if its base is a square.

7. The slant height of a cone, the radius of whose base is 6 in., is 10 in. Find the altitude. Find the lateral area.

8. How many square yards of canvas are required to make a conical tent 18 ft. in diameter at the base and 12 ft. high?

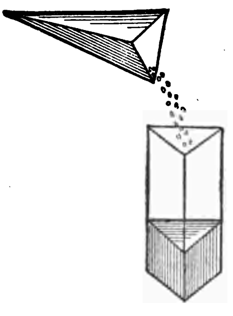


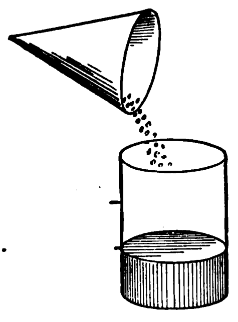
9. If canvas costs $1.75 a square yard, what will be the expense of the canvas for a tent in the shape of a pyramid, whose base is a square 8 ft. on a side and whose altitude is 10 ft.?

10. How many square yards of canvas are required for a tent of 8 sides each 6 ft., if the slant height of the tent is 20 ft.?

11. The pyramid of Cheops has a square base 746 ft. on a side; its slant height is 608 ft. Find its lateral area.

Volumes of Pyramids and Cones. Taking a hollow pyramid and a hollow prism of equal base and equal altitude, show that





the pyramid holds one third as much as the prism. In the same way show that a cone holds one third as much as a cylinder of equal base and equal altitude. Hence it follows that:

The volume of a pyramid or of a cone is equal to one third of the product of its base and altitude.

Formulae. If, in a pyramid, V represents the volume, H the number of units in the altitude, and B the number of square units in the base, show that:

$$V = \frac{B \times H}{3}. \quad (1)$$

If, in a cone, V represents the volume, H the number of units in the altitude, and R the number of units in the radius of the base, show that:

$$V = \frac{\pi R^2 H}{3}. \quad (2)$$

VOLUMES AND AREAS OF PYRAMIDS AND CONES

1. Find the volume of a pyramid the area of whose base is 93.53 sq. yd. and whose altitude is 6 yd.

2. Find the volume of a cone the radius of whose base is 3 in. and whose altitude is 6 in.

3. Find the volume of a pyramid whose base is a square 6 in. on a side and whose altitude is 9 in.

4. How many cubic inches will a conical vessel hold that is 9 in. high and the radius of whose base is 3 in.?

5. Find the volume of a pyramid whose base is a square 6 ft. on a side, if the slant height of the pyramid is 5 ft.

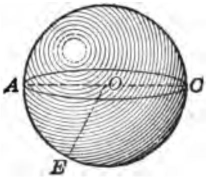
6. Find the lateral area and volume of a cone the radius of whose base is 8 in. and whose slant height is 10 in.

7. Find the volume of a cone the radius of whose base is 12 in. and whose slant height is 15 in.

8. Find the lateral area and volume of a pyramid whose base is a square 24 ft. on a side, the altitude of the pyramid being 16 ft.

9. Find the lateral area and volume of a cone whose altitude is 15 ft. and the diameter of whose base is 40 ft.

Sphere. A solid bounded by a curved surface every point of which is equally distant from a point within, its *center*, is called a *sphere*.

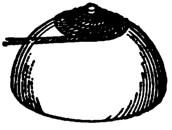


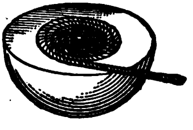
A straight line drawn from surface to surface through the center is called a *diameter* of the sphere. A straight line drawn from the center to the surface is called a *radius*, and the circumference of a circle whose radius is a radius of the sphere is called a *circumference* of the sphere.

Name the diameter and also the radius in the picture of a sphere shown above.

If a plane cuts a sphere in such a way as to pass through the center, it divides the sphere into two equal parts called *hemispheres*. A hemisphere is bounded by a curved surface and the surface enclosed by a circle. The circle has the same radius as the sphere.

Area of a Sphere. By winding the curved surface of a hemisphere closely with a string and then the plane surface, as in the picture below, it is found that twice as much string is required to cover the curved surface as to cover the plane surface. The curved surface, therefore, has twice as much area as the plane surface.



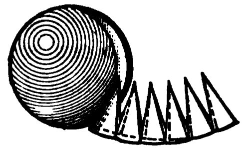


The area of the plane surface, that is, the area of the circle, is πR^2. Therefore the area of the curved surface is $2\pi R^2$ and the area of the sphere is $4\pi R^2$.

Formula. Denoting the area of the sphere by S and its radius by R,

$$S = 4\pi R^2$$

Volume of a Sphere. A sphere may be divided into a number of solids which resemble pyramids; they are called *spherical pyramids*. Their common vertex is the center of the sphere. It is shown in geometry that the volume of each is the area of its curved surface multiplied by $\frac{1}{3}$ of the radius of the sphere, that is by $\frac{1}{3}R$. The sum of the areas of these curved surfaces is the area of the sphere, or $4\pi R^2$. This area multiplied by $\frac{1}{3}R$ gives $\frac{4}{3}\pi R^3$, the volume of all of the pyramids, or the volume of the sphere.



Formula. If V denotes the volume of the sphere and R the radius,

$$V = \frac{4}{3}\pi R^3.$$

FINDING AREAS AND VOLUMES OF SPHERES

1. Find the area and volume of a sphere whose radius is 2 in.

Find the areas of spheres with the following radii:

2. 4 ft. 3. 3.2 in. 4. 6 yd. 5. 4.5 ft.

Find the volumes of spheres with the following radii:

6. 1 in. 7. 1.5 in. 8. 5 ft. 9. .5 ft.

10. Find the area of a sphere whose circumference is 12.5664 ft. Find its volume.

11. What is the radius of a sphere whose surface contains 113.0976 sq. ft.? What is its volume?

12. Considering the earth to be a sphere whose radius is 4000 miles, find the area of the earth's surface; its volume.

13. The diameter of a sphere, the diameter of a cylinder, and the altitude of the cylinder are each 3 in. Find the volume of each, the area of the surface of the sphere, and the lateral surface of the cylinder. Compare the volumes and also the areas.

BUILDING AN INEXPENSIVE MODERN HOUSE

The house illustrated above has been so planned[1] as to provide all the room and conveniences necessary for the average home buyer, at the lowest possible cost. By the compact arrangement of halls and chambers every available inch of space has been utilized.

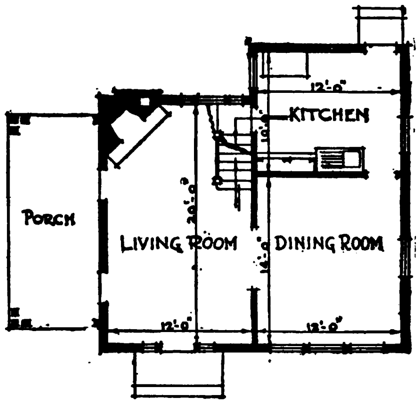
Although the house is only 25 by 25 feet, a spacious living room and dining room are made possible. There are three good sized chambers on the second floor, all of which are well lighted, which insures good ventilation. There is also ample closet room, a feature which is lacking in many inexpensive houses. A wide porch extends the entire length of the house.

The exterior is extremely attractive, presenting a cozy home, inexpensive to erect and at the same time containing features found in houses of greater cost. Cream-colored stucco walls, bottle-green shutters, and a maroon colored roof give the necessary color to enliven the entire design. The first and the second floor plans are shown on the next page.

[1] Courtesy of Andrew Charles Borzner, 717 Walnut St., Philadelphia.

FLOOR PLANS

1. Read from the first floor plan the dimensions of the kitchen; the dining room; the living room.

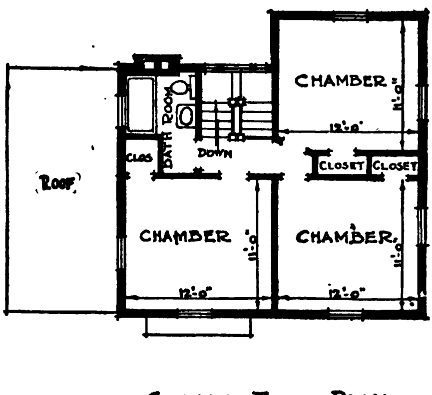


FIRST FLOOR PLAN

2. The plans show how many closets on the second floor? Do they all have the same floor space?

3. The scale used in these plans of the first floor and second floor is $\frac{1}{16}$ of an inch to a foot. How many feet long is the porch?

4. The plan of the living room should measure how many inches to represent the 20 ft. dimension? the 12 ft. dimension? (See problem 3 for scale.)



SECOND FLOOR PLAN

5. Read the dimensions of each chamber. How many inches on the plan represents each of these dimensions?

6. On the plans as first drawn the scale used was 1 in. to 8 ft. What length of line was then drawn to represent the width of the porch, which is $7\frac{1}{2}$ ft. wide?

PROBLEMS ON THE COST OF A MODERN HOME

1. Edward's father built a house using the plans shown on pages 263 and 264. The excavations were as follows: under the kitchen and dining room (first floor plan, page 264), 26′6″ × 14′ × 4′6″; under the living room, 13′ × 22′ × 4′6″; for trenches for porch, 36′ × 1′4″ × 2′6″; for cement entrance, 4′ × 6′ × 2′6″. Compute the cost of the excavation at $0.75 per cubic yard.

2. There were 58 perches of stone masonry at $8 per perch; what was the cost of the stone masonry?

3. The cement work consisted of 521 sq. ft. of cellar floor at $0.20 per square foot; 144 sq. ft. of porch floor at $0.30 per square foot; entrance platform and steps, $40; rear steps, $20; entrance walk, 20′ × 4′ at $0.40 per square foot. Find the cost of the cement work.

4. Find the outlay for brick work as follows: 62 ft. of 9″ × 13″ flue lining, at $0.55 per linear foot; 5000 rough bricks in place at $30 per M; 100 face bricks and 100 fire bricks for fireplace at $5 per C; damper for fireplace, $7.50; labor for constructing fireplace, $34.50.

5. In building the house, 1944 8″ hollow tile were used; they cost 36¢ each, in place. What was the cost of the hollow tile used in this house?

6. The bill of rough lumber purchased at $55 per M, was as follows; find the cost of this lumber:

35 pieces 2″ × 10″ × 12′	35 pieces 2″ × 8″ × 12′
35 pieces 2″ × 6″ × 12′	33 pieces 2″ × 6″ × 20′
12 pieces 2″ × 6″ × 10′	9 pieces 2″ × 10″ × 12′
5 pieces 2″ × 8″ × 14′	38 pieces 2″ × 4″ × 18′

7. Besides the rough lumber there were purchased the following: 2000 linear feet 1″ × 2″ shingle laths at $14 per 1000 linear feet;

1400′ of North Carolina flooring at \$65 per M; 300′ North Carolina "roofers" at \$40 per M; 5000 shingles at \$19 per M. Find the cost of this bill of lumber.

8. Find the carpenter's bill, which was for 148 da. work, 8 hr. per day, at \$0.80 per hour.

9. Find the bill for plastering: 231 sq. yd. of exterior stucco at \$0.75 per square yard; 208 sq. yd. on tile walls (interior) at \$0.74 per square yard; 352 sq. yd. lath and plaster at \$0.97 per square yard.

10. Other items of expense in building this house were as follows: millwork, \$600; hardware, \$80; painting and glazing, \$375; electrical work and fixtures, \$172; plumbing and fixtures, \$485; pipeless furnace, etc., \$330. Find the total of these other items of expense.

11. Using the results of problems 1–10, find what this house cost Edward's father.

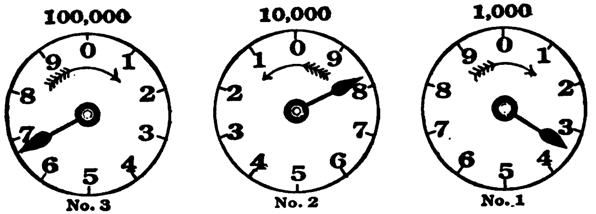
12. Edward's father built this house on a lot which he bought graded and paved at \$0.25 per square foot. The lot is 40′ × 120′. How much did the house and lot cost him?

Measuring Gas. Gas is one of the most convenient kinds of fuel for the home, especially for cooking purposes. The amount of gas consumed is measured in cubic feet by a *gas meter;* but in sending out monthly bills to their customers, gas companies base their charges on 1000 cubic feet.

How to Read a Gas Meter. The dial (marked 1), page 267, indicates one hundred cubic feet from one figure to the next. The dial (marked 2) indicates one thousand cubic feet from one figure to the next. The dial (marked 3) indicates ten thousand cubic feet from one figure to the next.

If the hand on dial No. 1 is between the figures 3 and 4, the lesser of the two numbers is read, the index reading 3 hundred cubic feet. If the hand on dial No. 2 is between the figures 8 and 9, this dial reads 8 thousand cubic feet.

If the hand on dial No. 3 is between the figures 6 and 7, the index reading on this dial will be 60 thousand cubic feet. The complete index on the 3 dials would read 68,300 cubic feet. From the present reading subtract the read-



CUBIC FEET

ing of the previous month, multiply the difference in cubic feet by the rate per cubic foot, and you have the amount of your bill in dollars and cents.

Read dials from right to left, setting down figures as read.

PROBLEMS

1. At $1.30 per 1000 cu. ft., find the cost of the gas consumed during February in a home in which the meter, as shown by the bill sent, registered as follows: Index of meter March 1, 36,000 cu. ft.; index of meter February 1, 30,000 cu. ft.

2. On the bill for March, the index of the meter (see problem 1) Apr. 1 was given as 41,800 cu. ft. Between what two numbers did the hand on each dial stand when the meter was read? How many cubic feet were consumed in this home during March? What was the gas bill at $1.30 per 1000 cu. ft.?

Tell where the hand on each dial would stand for the following readings:

3. 68,300 cu. ft. 4. 71,200 cu. ft. 5. 76,100 cu. ft.

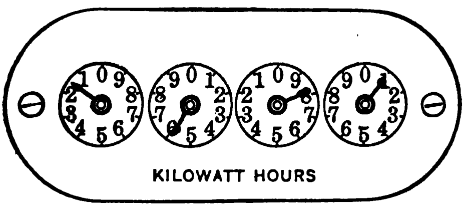
6. The readings as given in problems 3 to 5, were those of Mrs. Fielder's gas meter, May 1, June 1, and July 1. At $1.30 per M, what was her gas bill for May? for June?

7. If you use gas at home, read your own gas meter at the beginning of two successive months, and then check your monthly bill.

8. Mrs. Hill received her gas bill stating that the index of the gas meter in her home indicated 17,600 cu. ft. on June 1, and 22,100 cu. ft. July 1, that the rate is $1.30 per M, with a discount of 2% if the bill is paid before the 10th of the month. She paid the bill July 8th; how much did she pay?

Measuring Electrical Energy. Electricity may be used as a source of light, power, and heat. The rate at which electrical energy is used is expressed in *watts*. The unit of electrical energy is the *watt hour*, which is the electrical energy capable of working at the rate of one watt for one hour. A *kilowatt hour* is 1000 watt hours and is the unit that most electric meters register and upon which rates for electricity are usually based.

How to Read an Electric Meter. The electric meter is provided with four small dials, and each dial has a pointer like a hand of a clock. The meter registers kilowatt hours. These pointers are so connected with each other that every complete revolution of one causes the next pointer to the left to move one division. The dials, therefore, register from left to right in thousands, hundreds, tens, and units.



In reading the counter shown on this page, the first pointer is between 1 and 2, but the next pointer to the right is only between 5 and 6, so the first pointer could not yet have reached 2. The first dial, therefore, reads 1 thousand. The hundreds' pointer is almost at 6, but not quite, because the next pointer has not reached 10. The second dial, therefore, reads 5 hundred.

The third dial reads 8 tens, or 80. If there is any question as to whether the pointer has passed the 8, reference to the next dial to the right shows that its pointer has passed the 0, therefore the tens' pointer must have passed the 8. The units' dial clearly reads 1. The reading is therefore 1581 kilowatt hours.

PROBLEMS

1. If you use electricity at home, read your meter at the beginning of two successive months and check your bill.

2. At $0.13 per kilowatt hour, find the amount of the bill sent a patron for the month of March if the meter in his home read 810 kilowatt hours April 1, and 777 kilowatt hours March 1.

3. An ordinary 16-candle-power carbon filament lamp, which uses 55 watts, burning for 20 hr. would make the meter register 20×55 watt hours, or 1100 watt hours; this is the same as how many kilowatt hours? At $0.13 per kilowatt hour, find the cost of using this lamp for 20 hr.

4. A Westinghouse mazda lamp of 20-candle-power, which uses 25 watts, would in 20 hr. make the meter register 20×25 watt hours, or 500 watt hours. How many kilowatt hours is this? At $0.13 per kilowatt hour, find the cost of using this lamp for 20 hr.

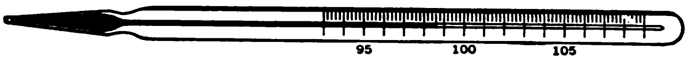
5. The mazda lamp (see problem 4) will give how many per cent more light than the carbon filament (see problem 3)?

6. A Westinghouse 8-in. fan running 60 hr. will make the meter register 1.2 kilowatt hours. At 13¢ per kilowatt hour, what is the cost of running the fan for 60 hr.?

7. An electric 6-pound iron used for 20 hr. makes the meter register 8.8 kilowatt hours. The use of this iron for 20 hr. would cost how much based on 12¢ per kilowatt hour?

8. A washing machine motor of average size uses 200 watts per hour. What is the cost of running such a motor for 5 hr., based on a charge of 12¢ per kilowatt hour?

Measuring Temperature. Here is a picture of a thermometer which is used to find the temperature of the body:

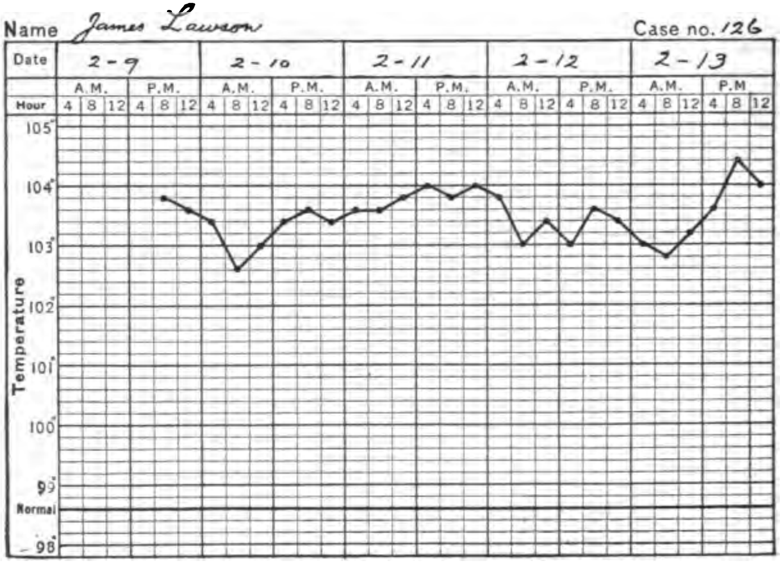


PROBLEMS ON TEMPERATURE

1. When a person is well, the temperature of his body is about 98.6 degrees, which is called *normal.* How many degrees above normal is the temperature of a patient when his temperature is 104.2°? 102.4°? 99.2°? 101.6°?

2. Examine this temperature chart of a boy admitted to a hospital ill with pneumonia. The chart shows that his temperature was taken how many times and at what hours the day he was admitted? How many degrees was it above normal each time?

GRAPHIC CHART

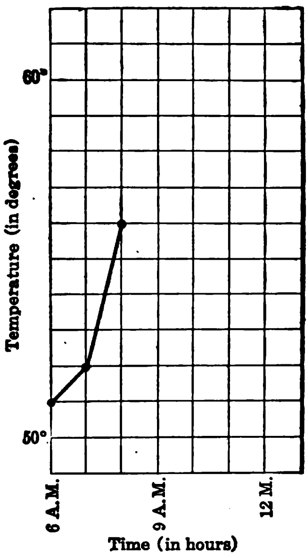


3. Read the patient's temperature for each time shown for the second day; for the third day; for each of the other days.

4. The chart shown on page 270 is made out for only the first five days of the patient's illness. What and when was the highest temperature shown during this period? the lowest? The highest was how many degrees above normal? the lowest?

5. From a newspaper record of the temperature in Edward's home city for a certain day in March, he copied the following; read what he copied.

6 A.M., 51°
7 A.M., 52°
8 A.M., 56°
9 A.M., 58°
10 A.M., 63°
11 A.M., 66°
Noon, 69°
1 P.M., 70°
2 P.M., 72°
3 P.M., 73°
4 P.M., 72°
5 P.M., 71°
6 P.M., 70°



6. Edward ruled paper and made a graph, a part of which is shown on this page. Rule paper and complete the graph.

7. Observe the temperature of your schoolroom each hour for one day from 9 A.M. to 3 P.M., and make a chart like the one you made for problem 6.

The Metric System. The metric system of weights and measures, first adopted in France in 1799, is in general use in nearly all the countries of Europe and South America and in Mexico. It is almost universally used for scientific measurements. Its use is permitted in the United States, and is even required in certain work of the departments of the Government, such as the medical work of the Navy and War Departments. It is the legal system of Porto Rico and the Philippines.

Units. The fundamental unit of the metric system is the *meter* — the unit of length. From the meter, the units of surface (*square meter*), volume (*cubic meter*), capacity (*liter*), and weight (*gram*) were derived. All other units are the decimal subdivisions or multiples of these. The relation of the liter and the gram to the meter is as follows:

A liter is a cube 0.1 *of a meter on an edge.*

A gram is the weight of a cube of pure water 0.01 *of a meter on an edge at the greatest density of water,* 39.2° *Fahrenheit.*

Prefixes. The names of the decimal subdivisions and the multiples of the meter, the liter, and the gram, are made up by combining the words *meter*, *liter*, and *gram* with the six prefixes here shown:

	Prefixes	Meaning		Units
Latin	milli-	.001		
	centi-	.01		
	deci-	.1		*meter* for length
			prefixed to	*gram* for weight
	deka-	10		*liter* for capacity
Greek	hecto	100		
	kilo-	1000		

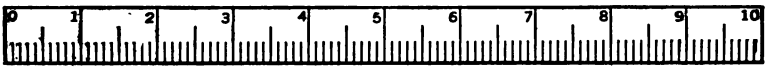
Measures of Length. The primary unit of length is the *meter*; it is equal to 39.37 inches, nearly. A meter is how many yards (express correct to the nearest tenth of a yard)?

TABLE

10 *millimeters* $(^{mm})$	= 1 *centimeter* $(^{cm})$	= 0.3937 in.
10 centimeters	= 1 decimeter $(^{dm})$	
10 decimeters	= 1 *meter* $(^{m})$	= 39.37 in.
10 meters	= 1 dekameter $(^{Dm})$	
10 dekameters	= 1 hectometer $(^{Hm})$	
10 hectometers	= 1 *kilometer* $(^{Km})$	= 200 rd.
10 kilometers	= 1 myriameter $(^{Mm})$	

NOTE. Important units are printed in italic type.

This rule is 1 decimeter long. How many centimeters long is it? Count the millimeters in each centimeter.



As our system of money is decimal, a sum of money made up of, say, \$5000, \$42, 7 dimes, 3 cents, and 8 mills may be expressed \$5042.738 and read *5042.738 dollars* or *five thousand forty-two dollars, seventy-three cents, eight mills.* In the same way $5^{Km}\ 4^{Dm}\ 2^{m}\ 7^{dm}\ 3^{cm}\ 8^{mm}$ may be expressed 5042.738^{m} and read *five thousand forty-two and seven hundred thirty-eight thousandths meters* or *5 kilometers, 42 meters, 738 millimeters.*

In measures of length in the metric system 10 units of any denomination make one of the next higher denomination; therefore, reductions from one denomination to the next lower or higher may be made by moving the decimal point one place to the right or one place to the left.

Thus, $0.638^{m} = 6.38^{dm} = 63.8^{cm} = 638^{mm}$,

and $1825^{mm} = 182.5^{cm} = 18.25^{dm} = 1.825^{m}$.

MEASURES OF LENGTH

1. Read as meters:

5^{Km} 7^{Km} 48^{cm} 56^{cm} 175^{mm} 46^{mm}

2. Read as centimeters:

7^{m} 2^{m} 1.5^{m} 1.25^{m} 200^{mm} 75^{mm}

3. Read as millimeters:

0.48^{m} 0.007^{m} 0.428^{m} 3^{cm} 1.6^{cm} 0.48^{cm}

4. Read as meters:

$2^{Km}\,5^{m}$ $7^{Km}\,256^{m}$ $2^{m}\,6^{cm}$ $4^{m}\,24^{mm}$ $1^{m}\,75^{cm}\,2^{mm}$

5. Read as centimeters:

$1^{m}\,6^{cm}$ $4^{cm}\,2^{mm}$ $2^{cm}\,46^{mm}$ $7^{cm}\,275^{mm}$

6. Write in kilometers:

$2^{Km}\,7^{m}$ $2^{Km}\,7^{m}\,6^{cm}$ 7568^{m} 356.8^{m}

7. Write in meters:

$2^{m}\,6^{cm}\,56^{mm}$ 275^{cm} $85^{cm}\,7^{mm}$ 1.426^{Km}

8. Write in centimeters; in millimeters:

$2^{cm}\,7^{mm}$ $1^{m}\,76^{dm}$ $1^{m}\,27^{cm}$ $1^{m}\,14^{mm}$

9. Express a decimeter in inches, then check your result by measuring as nearly as you can the rule shown on page 273.

10. If a centimeter be taken as 0.4 in., how much will this differ from the value given in the table, page 273? Measure a centimeter to see that it is but little longer than $\frac{3}{8}$ in.; that we may say: $1^{cm} = 0.4$ in., nearly.

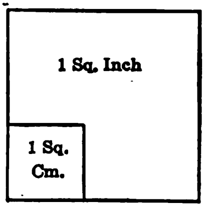
11. Make a rule a decimeter long, using cardboard or stiff paper, then placing your rule so that its edge lies along the upper edge of the metric rule, page 273, mark your rule off in centimeters and number them. Using your decimeter rule, measure on the blackboard a line 1 meter long and compare it with a line 1 yard long. Compare your answer with the one you gave to the question at the foot of page 272.

12. Draw a circle with radius 3^{cm}.

13. The table page 273 states that 1 kilometer = 200 rd. This is the approximate length of a kilometer. From this statement find what part of a mile is equal to 1 kilometer.

14. Compare a decimeter with 4 in.

Measures of Surface. In square measure the units are squares whose sides are the units of length, as in our system. The principal unit is the *square meter*, which is equal to 10.763 sq. ft., nearly. For land measure a square 10 meters long, called an *are* (meaning *area*), is the unit.



TABLE

100 *square millimeters* ($^{\text{sqmm}}$)	= 1 *square centimeter* ($^{\text{sqcm}}$)	= 0.155 sq. in.
100 square centimeters	= 1 square decimeter ($^{\text{sqdm}}$)	
100 square decimeters	= { 1 *square meter* ($^{\text{sqm}}$) / 1 centare ($^{\text{ca}}$) }	= 10.763 sq. ft.
100 centares	= 1 *are* ($^{\text{a}}$)	= 119.6 sq. yd.
100 ares	= 1 hectare ($^{\text{Ha}}$)	= 2.5 *A.*

NOTE. Instead of *sqm.*, *sqcm.*, etc. we may write m^2, cm^2, etc.

In square measure in the metric system 100 units of any denomination make one of the next higher denomination; therefore, reductions from one denomination to the next lower or higher may be made by moving the decimal point two places to the right or two places to the left.

Thus, $1.4^{\text{sqdm}} = 140^{\text{sqcm}} = 14000^{\text{sqmm}}$,

and $75^{\text{sqcm}} = 0.75^{\text{sqdm}} = 0.0075^{\text{sqm}}$.

MEASURES OF SURFACE

1. Read 0.15^{sqm} as square centimeters; as square millimeters.
2. Read 25^{sqcm} as square millimeters; as square meters.
3. Express 8^{sqcm} 25^{sqmm} as square millimeters.
4. How many ares are there in a piece of land in the form of a rectangle 200^{m} long and 80^{m} wide? how many hectares? how many acres (see table)?

5. Taking 2.5 acres as the equivalent of 1 hectare, find what part of a hectare is equivalent to 1 acre.

6. Draw a square 1 decimeter long and divide it into square centimeters. How many square centimeters does it contain?

7. The table states that $1^{sqcm} = 0.155$ sq. in. Using this statement and observing carefully the drawing at the top of page 275, estimate the number of square centimeters in a square inch. Check your estimate by dividing 1 by 0.155, finding the quotient correct to the nearest 0.1. What does the quotient express?

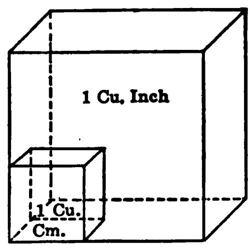
8. A rectangular field of wheat is 50.5^{Dm} long and 24^{Dm} wide. How many hectares does it contain?

Measures of Volume. In cubic measure the units are cubes whose edges are the linear units. The *cubic meter* (sometimes called the *stere*, meaning "*solid*") is the unit of volume.

TABLE

1000 *cubic millimeters* (^{cmm})	=	1 *cubic centimeter* (^{ccm})
1000 cubic centimeters	=	1 *cubic decimeter* (^{cdm})
1000 cubic decimeters	=	1 *cubic meter* (^{cbm}), or stere (^{s}) = 35.315 cu. ft.

NOTE. Instead of *ccm.*, *cbm.*, etc. we may write *cm*^{3}, *m*^{3}, etc.



In cubic measure in the metric system 1000 units of any denomination make one of the next higher denomination; therefore, reductions from one denomination to the next lower or higher may be made by moving the decimal point three places to the right or three places to the left.

Thus, $.45^{cdm} = 450^{ccm}$, and $245^{cmm} = .245^{ccm}$

MEASURES OF VOLUME

1. Read and write 4^{cdm} as cubic centimeters; as a decimal part of a cubic meter.

2. Read, then write 125^{cmm} as a decimal part of a cubic centimeter.

3. How many cubic centimeters are there in a cube whose edge is 0.5^{dm} long? how many cubic millimeters?

4. How many cubic centimeters are there in a bar of silver in the form of a rectangular solid $1.4^{cm} \times 3.5^{cm} \times 8.4^{cm}$? how many cubic millimeters? what decimal part of a cubic decimeter?

5. How many cubic centimeters will a flask in the form of a cylinder hold if it is 10^{cm} high and the radius of its base is 2^{cm}?

Measures of Capacity. The primary unit of capacity is the *liter*, which is exactly one cubic decimeter; it is equivalent to 1.0567 liquid quarts or 0.908 dry quarts, nearly.

TABLE

10 milliliters (^{ml})	= 1 centiliter (^{cl})	
10 centiliters	= 1 deciliter (^{dl})	
10 deciliters	= 1 *liter* (^{l})	$= \begin{cases} 0.908 \text{ qt. (dry)} \\ 1.0567 \text{ qt. (liq.)} \end{cases}$
10 liters	= 1 dekaliter (^{Dl})	
10 dekaliters	= 1 hectoliter (^{Hl})	$= \begin{cases} 2.8375 \text{ bu.} \\ 26.417 \text{ gal.} \end{cases}$
10 hectoliters	= 1 kiloliter (^{Kl})	

In measures of capacity 10 units of any denomination make one of the next higher denomination; therefore, reduction from one denomination to the next lower or higher may be made by moving the decimal point one place to the right or one place to the left.

Thus, $2.4^{dl} = 24^{cl} = 240^{ml}$,

and $4.8^{cl} = .48^{dl} = .048^{l}$.

MEASURES OF CAPACITY

1. Read as liters:

4^{Kl} 5^{Dl} 6^{dl} 4^{ml} $2^{l}\ 5^{cl}$ $5^{cl}\ 8^{ml}$

2. Read 200^{ml} as centiliters; as deciliters; as liters; as a decimal part of a kiloliter.

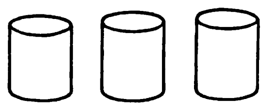
3. Which holds the more, a liter or a quart dry measure? a liter or a quart liquid measure? (See table.)

4. Compare a dekaliter with a peck. Which is the greater? Express the difference in quarts.

5. How many liters does a cubical vessel hold which is 5^{dm} long? 1^{m} long?

6. How many cubic centimeters does a flask hold that has a capacity of 8^{cl}?

Measures of Weight. The primary unit of weight is the *gram*, which is the weight of one cubic centimeter of pure water at a temperature of 39.2 degrees Fahrenheit. It is equivalent to 15.432 grains.



Relative Size of Avoirdupois Ounce, 30-Gram, and Troy Ounce Weights

TABLE

10 *milligrams* (^{mg})	= 1 *centigram* (^{cg})	
10 centigrams	= 1 decigram (^{dg})	
10 decigrams	= 1 *gram* (^{g})	= 15.432 gr.
10 grams	= 1 dekagram (^{Dg})	
10 dekagrams	= 1 hectogram (^{Hg})	
10 hectograms	= 1 *kilogram* (^{Kg})	= 2.2046 lb. (avoir.)
10 kilograms	= 1 myriagram (^{Mg})	
10 myriagrams	= 1 quintal (^{q})	
10 quintals	= 1 *Metric Ton* (^{MT})	= 2204.6 lb. (avoir.)

In measures of weight in the metric system 10 units of any denomination make one of the next higher denomination; there-

fore reduction from one denomination to the next lower or higher may be made by moving the decimal point one place to the right or one place to the left.

Thus, $.4^{g} = 4^{dg} = 40^{cg} = 400^{mg}$,

and $456^{mg} = 45.6^{cg} = 4.56^{dg} = .456^{g}$.

MEASURES OF WEIGHT

1. Read and write as grams:

   2^{Kg} 1.5^{Kg} 0.25^{Kg} 250^{mg} 25^{mg} 5^{mg}

2. Read and write as kilograms:

   45^{g} 750^{g} 1250^{g} 75.5^{g} 140.5^{g}

3. Read and write as milligrams:

   2^{g} 25^{g} 2.5^{g} 1.75^{g} 0.02^{g} 0.075^{g}

4. The unit of weight for gems and precious stones is the *carat*, which is equal to 200^{mg}. A carat of radium is estimated to be worth $20,000. What is a gram of radium worth at this estimate?

5. Our silver coins weigh 1^{g} for each 4¢ of their value. What is the weight of a half dollar?

6. A cubic centimeter of water weighs a gram. Show that a cubic decimeter, or a liter, of water weighs a kilogram, and that a cubic meter of water weighs a metric ton.

7. Find the weight of a liter of milk, if it is 1.03 times as heavy as water.

8. Find the weight of 2^{ccm} of silver, which is 10.47 times as heavy as water.

9. Find in milligrams the weight of a centiliter of alcohol, which is 0.792 times as heavy as water.

10. The weight of the olive oil in a certain flask is 36^{g} 600^{mg}. How many cubic centimeters of this oil are there, if it is 0.915 times as heavy as water?

REVIEW OF MEASURES

1. How many acres does a rectangular field contain that is 40 rd. long and 120 rd. wide?

2. How many acres does a field contain if it has the form of a right triangle whose base is 16 rd. and hypotenuse 20 rd.?

3. The length of a rectangle which contains 2 A. is 20 rd. What is its width? its perimeter?

4. A square lawn 120 ft. on a side is surrounded by a walk 8 ft. 3 in. wide. Find the cost of paving the walk at $0.50 per square yard.

5. Find the volume and the lateral area of a prism whose base is a right triangle, the perpendicular sides of which are 20 ft. and 21 ft., if the altitude of the prism is 11 ft.

6. Find the lateral area and the volume of a prism whose base is a right triangle, the perpendicular sides of which are 3 ft. and 4 ft., if the altitude of the prism is 10 ft.

7. Mr. Crosby has a rectangular lot whose frontage is 60 ft. and depth 120 ft.; on it he builds a house 48 ft. long and 42 ft. wide. The remainder of the lot is made into a yard. Find the area of the yard in square feet.

8. A rectangular mirror 36 in. long and 20 in. wide is surrounded by a frame 4 in. wide. Find the area of the frame.

9. A rug 24 ft. long and 18 ft. wide is placed in the center of a floor 28 ft. long and 22 ft. wide. How much will it cost to finish the floor remaining uncovered at $12\frac{1}{2}$¢ per square foot?

10. A rectangular plot of ground 176 yd. long and 22 yd. wide at an agricultural college experiment station produced 1266 lb. of wheat. What was the yield in bushels per acre?

11. The entire surface of a cube contains 54 sq. in. What is the length of one edge? What is the volume of the cube?

12. The bases of a trapezoid are 12 ft. and 10 ft. and the altitude is $8\frac{1}{2}$ ft. Find the area.

13. Find the total area of a cylindrical hot water tank if the altitude is 8 ft. and the radius of its base 1 ft. (Use $\pi = \frac{22}{7}$.)

14. How many square inches are there in the surface of a frame 1 in. wide around a clock dial 8 in. in diameter? 18.2744

15. Trees are planted 40 ft. apart on both sides of a certain street for the distance of a mile. Find the number of trees.

16. For one inch of rainfall what weight of water falls on one square yard of ground, 1 cu. ft. of water weighing 62.5 lb.?

17. How many cubic yards of earth must be removed in digging a well 45 ft. deep and 5 ft. in diameter?

18. Find the weight of a granite block 4 ft. long, 3 ft. wide, and 18 in. high, granite weighing 2.65 times as much as an equal bulk of water, which weighs 62.5 lb. per cubic foot.

19. A cubic foot of iron was formed into a bar 3 in. square. Find its length.

20. Find the cost, at 15 cents per square foot, of lining the sides and bottom of a cubical tank which is 6 ft. long.

21. The length of the minute hand of the clock on City Hall, Philadelphia, is 10 ft. 8 in. How many feet does the point of this hand move in an hour?

22. A line 50 ft. long extends from the top of a center pole 40 ft. high to the edge of a circus ring. Find the area of the ring.

23. Two buildings 94 ft. and 78 ft. high are directly opposite each other, with a street 30 ft. wide between them. What is the length of the shortest line connecting the tops of the buildings?

24. How many revolutions per minute are made by a locomotive wheel 5.1 ft. in diameter, when the locomotive is running 45 miles per hour?

25. Regarding 1 cu. ft. as equal to $\frac{16}{25}$ of a heaped bushel, how many bushels of apples, heaped measure, will a box hold if it is 5 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. deep?

26. How many gallons will a cubical tank hold that is 4 ft. long?

27. Estimating that a bushel is equal to $\frac{5}{4}$ of a cubic foot, how many bushels of wheat will a box hold that is 6 ft. long, 4 ft. wide, and 5 ft. deep?

28. Find, correct to the nearest square yard, the amount of rubber cloth in the covers of 100 balls, spherical in form, the diameter of each to be 1 ft.

29. A rectangular field 40 rd. long and 37.5 rd. wide was bought at \$75 per acre. The purchaser had it inclosed by a fence which cost \$1.20 per rod. Find the total cost.

30. A farmer has a field in the form of a right triangle containing 2 A. 142 sq. rd. If one of the perpendicular sides is ~~242 yd.~~, find the other.

31. 1 cu. ft. of water equals how many quarts? Answer correct to the nearest 0.01 qt.

32. Find the cost of 86 planks, each 16 ft. by 6 in. by 3 in., at \$48 per M board feet.

33. Find in feet per minute the flow of water through a pipe 8 in. in diameter, if the discharge is 50 gal. per minute. Answer correct to the nearest 0.01 ft. (Use $\pi = \frac{22}{7}$.)

34. *ABCD* is a field whose diagonal *AC* is 18 rd., and the perpendiculars *DE* and *BF* drawn to the diagonal are 12 rd. and 6 rd. Find the number of acres in the field.

35. A cylindrical tank car is 25 ft. long and 5 ft. in diameter. How many gallons of oil will it contain, and what will the oil be worth at 16¢ per gallon?

CHAPTER V

RATIO AND PROPORTION

Ratio. The ratio of 6 to 3 is $6 \div 3$, or 2. The ratio of 3 to 6 is $3 \div 6$, or $\frac{1}{2}$. The ratio of $\frac{1}{2}$ to $\frac{1}{4}$ is $\frac{1}{2} \div \frac{1}{4}$, or 2. The ratio of a to b is $a \div b$, or $\frac{a}{b}$.

Meaning of Ratio. The ratio of one number to another number is the quotient of the first divided by the second.

The ratio of one amount to a similar amount (expressed in terms of the same unit) is the number of units in the first divided by the number of units in the second. For example, the ratio of 6 ft. to 3 ft. $= 6 \div 3$, or 2.

Writing Ratios. The ratio of 3 to 5 may be written as a fraction, thus, $\frac{3}{5}$; or with the sign of ratio, which is the colon, thus, 3 : 5. The numbers 3 and 5 in this ratio are called the *terms* of the ratio.

Since a ratio may be expressed as a fraction, the principles used in fractions are applicable also to ratios.

FINDING RATIOS

1. State at sight the ratio of each number in the first line to the number under it in the second line:

2	5	4	8	12	10	5	$2\frac{1}{2}$	$\frac{3}{4}$	$12\frac{1}{2}$
4	15	6	10	9	15	$2\frac{1}{2}$	$7\frac{1}{2}$	$\frac{1}{2}$	$37\frac{1}{2}$

2. State at sight the ratio of each number in the second line in problem 1 to the number above it in the first line.

3. State at sight the ratio of:

2 to 3	2 to 5	3 to 7	5 to 5	8 to 7	6 to 8	$\frac{3}{4}$ to $\frac{1}{4}$
2 to a	b to 4	x to g	a to c	b to c	r to s	6% to 8%

4. Write six ratios each equal to 2; to 3; to 4.

5. State at sight the ratio of:

4 yd. to 2 yd.	6 pk. to 2 pk.	8 lb. to 2 lb.	8 lb. to 4 lb.
2 ft. to 4 ft.	2¢ to 6¢	2¢ to 8¢	$6 to $8
10 in. to $2\frac{1}{2}$ in.	$12\frac{1}{2}$ rd. to $2\frac{1}{2}$ rd.	1 yd. to 4 ft.	$1\frac{1}{2}$ ft. to 8 in.

6. The ratio of 10 to 5 tells how many 5's there are in 10. The ratio of 5 to 10 tells what part 5 is of 10. State what each ratio in problem 1 tells.

7. State at sight the ratio of:

0.1 to 0.01 0.01 to 0.1 0.5 to 5 5 to 0.05.

8. In the ratios 4 : 3 and 3 : 4, what is the first term of the first ratio? the second term of the second ratio? the second term of the first ratio? the first term of the second ratio?

Each of the ratios, 4 : 3 and 3 : 4, is called the *inverse* of the other. Show by four illustrations that the product of a ratio and its inverse is 1.

Name at sight the number represented by x in the following ratios:

	a	*b*	*c*	*d*	*e*
9.	$\frac{x}{2} = 6$	$\frac{12}{x} = 2$	$\frac{8}{2} = x$	$\frac{x}{b} = 1$	$\frac{4}{x} = 2$
10.	$\frac{3}{x} = \frac{1}{2}$	$\frac{x}{4} = \frac{3}{4}$	$\frac{x}{2} = \frac{1}{2}$	$\frac{5}{x} = \frac{1}{3}$	$\frac{x}{a} = b$
11.	$\frac{x}{5} = \frac{4}{5}$	$\frac{5}{x} = \frac{1}{4}$	$\frac{x}{a} = .4$	$\frac{x}{5} = .4$	$\frac{6}{x} = \frac{2}{3}$

12. The circumference whose diameter is 2 in. is 6.2832 in. What is the ratio of the circumference to the diameter?

13. The circumference whose radius is 5, is 31.416. What is the ratio of the circumference to the radius? What is the ratio of the circumference of any circle to its diameter?

14. It is estimated that the average yearly amount spent in the United States for food is \$$466\frac{2}{3}$ per family, and that of the food bought. the waste averages \$35 per family. What is the ratio of the cost of the food wasted to the amount spent?

15. A meter is 39.37 in. Find correct to the nearest 0.01, the ratio of a meter to a yard.

16. Estimating the population of the United States at 105,000,000 (in round numbers) and the number of families at 21,000,000, what is the ratio of the population to the number of families? The answer gives, at this estimate, how many persons as the average of a family?

17. Mrs. Wagner and her two children, Samuel and Emily, belong to a Christmas Savings Club. Mrs. Wagner belongs to the dollar class, Samuel to the 50-cent class, and Emily to the 25-cent class. When Emily has saved \$7.50, how much has each of the other two saved if all have kept up their dues?

18. What is the ratio of the areas of two squares whose sides are 2 in. and 4 in., respectively?

19. What is the ratio of one side of a square to its perimeter?

20. What is the ratio of one side of an equilateral triangle to its perimeter?

21. What is the ratio of the width of your school room to its length?

22. The radii of two circles are 4 in. and 8 in. What is the ratio of their diameters? of their circumferences?

23. Find the areas of two circles whose radii are 4 in. and 8 in., respectively; find the ratio of their areas.

24. The areas of two circles are πR^2 and πr^2. What is the ratio of their areas?

25. What is the ratio of the area of a rectangle 8 in. $\times$ 6 in. to the area of a rectangle 8 in. $\times$ 2 in.?

26. What is the ratio of the area of a rectangle 9 in. $\times$ 4 in. to the area of a rectangle 3 in. $\times$ 4 in.?

27. What is the ratio of two rectangles which have equal bases, each represented by b, and whose altitudes are a and h?

28. What is the ratio of the area of a rectangle 6 in. $\times$ 3 in. to the area of a rectangle 4 in. $\times$ 2 in.?

29. Find the ratio of the area of a rectangle b units long and a units wide, to the area of a rectangle l units long and w units wide.

30. Find the ratio of the area of a triangle whose base is 6 in. and altitude 8 in. to the area of a triangle whose base is 6 in. and altitude 4 in.

31. If in problem 30 the base of each triangle is b in., the altitude of the first a in. and that of the second h in., what is the ratio of their areas?

32. Find the ratio of the area of a triangle whose base is 10 in. and altitude 6 in. to the area of a triangle whose base is 2 in. and altitude 6 in.

33. If in problem 32 the altitude of each triangle is a in., the base of the first b in., and that of the second l in., what is the ratio of their areas?

34. What is the ratio of the area of the triangle whose base is b in. and altitude a in. to the area of the triangle whose base is l in. and altitude w in.?

35. In problem 34, what is the ratio of the area of the first triangle to the area of the second, when $b = 8$, $a = 6$, $l = 9$, and $w = 12$?

36. What is the ratio of the volume of a pyramid to that of a prism if they have equal bases and equal altitudes?

37. If $\frac{a}{b} = r$, does $\frac{na}{nb} = r$? Give reason for your answer.

38. If $\frac{a}{b} = r$, does $\frac{a \div n}{b \div n} = r$? Give reason for your answer.

DIVIDING A NUMBER INTO PARTS WHICH HAVE A GIVEN RATIO

1. \$45 was given as first and second prizes to the two contestants having the best compositions in a contest of the eighth grade pupils of a certain school. The money was divided in the ratio of 5 to 4. How much did each receive?

SUGGESTION. Divide \$45 into 5 + 4, or 9, equal parts. The winner of the first prize should receive how many of these equal parts? The winner of the second prize should receive how many of them?

2. Books which cost \$360 are to be given to the two schools of a town in the ratio of the number of pupils in the two schools. If there are 400 pupils in one of the schools and 500 in the other, how many dollars' worth of books should be given to the smaller school? to the larger?

3. A good winter mixture for making hens lay is 100 lb. of wheat to 100 lb. of cracked corn and 50 lb. of oats. How many pounds of each are there in a ton of the mixture?

SUGGESTION. Divide 2000 lb. into 100 + 100 + 50, or 250 equal parts; or into 2 + 2 + 1, or 5 equal parts.

4. Frank and James together earned \$30. Frank earned \$2 as often as James earned \$1; how much did each earn?

5. Solder for lead consists of 1 part tin to $1\frac{1}{2}$ parts lead; how much of each of these substances is necessary to make 5 lb. of solder?

6. In making a gargle a woman mixed peroxide and warm water in the ratio of 1 to 4; how many tablespoonfuls of this gargle did she make if she used 4 tablespoonfuls of peroxide?

7. William charged three neighbors \$6 for clearing their pavements of snow during January. The first neighbor had 40 ft. of pavement, the second 60 ft., and the third 50 ft. How much did each pay?

8. Three potato raisers in Canton, Maine, shipped a carload of potatoes to Boston, the freight rate being \$0.44 per hundredweight. One man owned 150 bu., another 250 bu., and the third 300 bu. How much freight should each pay?

9. After ducklings are 10 days old, a good feed for them consists of the following parts by weight:

3 parts bean	$1\frac{1}{2}$ parts beef scrap
1 part corn meal	$2\frac{1}{2}$ parts green food
1 part flour	

How many pounds of each ingredient are there in 360 lb. of this mixture?

10. Divide 720 in the ratio of 4 to 5.

11. Divide 720 in the ratio of 2 to 3 to 4.

12. The sum of two numbers is 960, and the ratio of the two numbers is 9 to 7. What are the numbers?

Proportion. The ratio $\frac{8}{4}$ equals 2; the ratio $\frac{14}{7}$ equals 2. As these ratios are equal, $\frac{8}{4} = \frac{14}{7}$. This equation is called a *proportion.* The only new thing about a proportion is its name. You have previously learned that:

$\frac{\textit{A Dividend}}{\textit{A Divisor}} = \frac{\textit{A Dividend}}{\textit{A Divisor}}$ *provided the quotients in both divisions are the same.*

As a proportion is an equation in which the equal ratios are the members, any operation performed upon the members of an equation which does not destroy the equality of the members, may also be performed upon the ratios of a proportion without affecting the proportion.

In problems of proportion we usually have three *terms* of the proportion given and are required to find the fourth. For example, we may have the proportion

$$x : 8 = 6 : 12,$$

in which x represents the number which is required. Writing the proportion in the fractional form, we have

$$\frac{x}{8} = \frac{6}{12}$$

Now multiplying each member of the equation by 8, we have

$$x = 8 \times \frac{6}{12}, \text{ or } 4.$$

If the unknown number in a proportion is not the numerator of the first ratio, we may write the ratios in such a way as to make the unknown number the numerator of the first ratio. In some proportions this is done by interchanging the ratios, in others by interchanging the numerator and denominator of each ratio, and in still others by making both these changes. For example, $\frac{4}{8} = \frac{x}{12}$ may be written $\frac{x}{12} = \frac{4}{8}$; $\frac{4}{x} = \frac{6}{12}$ may be written $\frac{x}{4} = \frac{12}{6}$; $\frac{4}{8} = \frac{6}{x}$ may be written $\frac{x}{6} = \frac{8}{4}$.

The ratio of two similar amounts may equal the ratio of two other similar amounts, and the two ratios form a proportion. For example, 6 yd. : 3 yd. = \$8 : \$4 since each ratio is equal to 2. But x yd. : 3 yd. = \$8 : \$4 should be written $x : 3 = 8 : 4$, as a first step in finding the value of x.

Check. To check the result in proportion, see that the product of the first and fourth terms is equal to the product of the second and third terms.

FINDING A MISSING TERM IN A PROPORTION

In the following expressions, supply the number which will make each expression a proportion:

1. $\frac{15}{5} = \frac{27}{}$ 2. $\frac{}{3} = \frac{12}{2}$ 3. $\frac{8}{} = \frac{14}{7}$

4. $\frac{9}{12} = \frac{}{20}$ 5. $\frac{6}{8} = \frac{}{24}$ 6. $\frac{64}{32} = \frac{}{11}$

7. $\frac{75}{} = \frac{45}{3}$ 8. $\frac{121}{11} = \frac{33}{}$ 9. $\frac{}{5} = \frac{2}{4}$

In the following, x stands for the missing term of each proportion; find the value of x:

10. $\frac{x}{8} = \frac{3}{4}$ 11. $\frac{6}{x} = \frac{12}{4}$ 12. $\frac{9}{4} = \frac{x}{72}$

13. $\frac{4}{5} = \frac{20}{x}$ 14. $\frac{42}{x} = \frac{14}{2}$ 15. $\frac{x}{6} = \frac{2}{10}$

16. $\frac{2\frac{1}{2}}{5} = \frac{12\frac{1}{2}}{x}$ 17. $\frac{2\frac{1}{2}}{12\frac{1}{2}} = \frac{5}{x}$ 18. $\frac{3}{8} = \frac{x}{37\frac{1}{2}}$

19. $x : 24 = 2 : 6$ 20. $9 : x = 36 : 24$ 21. $4 : 20 = x : 25$

22. $6 : 25 = x : 100$ 23. $7 : 1 = x : 12\frac{1}{2}$ 24. $\frac{1}{4} : \frac{1}{2} = x : \frac{1}{8}$

25. $.5 : x = .25 : 1$ 26. $1.25 : 5 = 10 : x$

27. $\$x : \$10 = 6$ yd. : 15 yd.

Types of Proportion Problems. Problems which may readily be solved by proportion are of one or the other of *two types*. When the type to which a problem in proportion belongs is recognized, its solution is comparatively easy. The explanation of the two types follows:

First Type. If a train running 40 mi. per hour runs 200 mi. in a certain time, it would run 250 mi. in the same time if its speed were 50 mi. per hour.

The statement may be abridged thus:

Rate	Distance
40 mi.	200 mi.
50 mi.	250 mi.

Observe that:

(1) When the rate is 40 mi., the corresponding distance is 200 mi.

(2) When the rate is 50 mi., the corresponding distance is 250 mi.

(3) $40 : 50 = 200 : 250$.

(4) The ratio of the two values of rate is equal to the ratio of the corresponding values of distance.

(5) If any term of the proportion of (3) were missing, it could be replaced by x and found. Test.

Two quantities are said to be directly proportional when the ratio of any two values of the one equals the ratio of the corresponding values of the other.

Second Type. A train running 40 mi. per hour runs a certain distance in 5 hr.; if its speed were 50 mi. per hour, it would run the same distance in 4 hr.

The statement may be abridged thus:

Rate	Time
40 mi.	5 hr.
50 mi.	4 hr.

Observe that:

(1) When the rate is 40 mi., the corresponding time is 5 hr.

(2) When the rate is 50 mi., the corresponding time is 4 hr.

(3) $40 : 50 =$ the inverse of $5 : 4$; that is, $40 : 50 = 4 : 5$.

(4) That the ratio of the two values of rate is equal to the inverse of the ratio of the corresponding values of time.

(5) If any term of the proportion of (3) were missing, it could be represented by x and found. Test.

Two quantities are inversely proportional when the ratio of any two values of the one equals the inverse of the ratio of the corresponding values of the other.

Test to Determine Type. A test to determine to which type a problem of proportion belongs is as follows:

(1) If when the second quantity is increased the first is also increased, the first is directly proportional to the second.

(2) If when the second quantity is increased the first is decreased, the first is inversely proportional to the second.

Illustrative Problems. 1. Last year a corn-club boy raised 130 bu. of corn on 2 acres of land. If the father's 10-acre field of corn produces this year at the same rate at which the boy's 2 acres did, what will be the yield from it?

EXPLANATION.

YIELD	ACREAGE
x bu.	10 A.
130 bu.	2 A.

The yield is directly proportional to the acreage, for if the acreage is increased the yield is increased, provided the yield per acre does not vary. Therefore,

$$x : 130 = 10 : 2$$

$$x = \frac{130 \times 10}{2} = 650$$

That is, 650 bu. = the required yield.

2. At the rate of 25 bu. per acre, how many acres of wheat will yield as much as 40 A. yield at the rate of 20 bu. per acre?

EXPLANATION.

ACREAGE	YIELD PER ACRE
x A.	25 bu.
40 A.	20 bu.

The acreage is inversely proportional to the yield per acre, for if the yield per acre is increased, the acreage is decreased, provided the quantity produced does not vary. Therefore,

$$x : 40 = 20 : 25$$

$$x = \frac{40 \times 20}{25} = 32$$

That is, 32 = the number of acres required.

PROBLEMS IN PROPORTION

1. In the following tell whether the first quantity named is directly or inversely proportional to the second:

(1) The cost of land and the quantity bought.
(2) The quantity of land bought for a certain price and the cost per acre.
(3) The principal on interest and the interest earned.
(4) The time required to go a certain distance and the speed.
(5) The time required to do work and the force.
(6) The quantity of work done in a day and the force working.
(7) The force required to do a piece of work and the time.

2. Steers should be fed 2.84 lb. of hay, or its equivalent, per day, for each 100 lb. of their live weight. How much hay, or its equivalent, should be fed per day to a steer which weighs 1050 lb.?

3. A contractor used 16 bbl. of cement for 10 cu. ft. of concrete work. He estimates that 40 cu. ft. more of concrete work of the same kind will have to be built to finish the job. How many more barrels of cement will be required?

4. When hay is selling at $28 per ton, what should be charged for 1250 lb. of hay?

5. When $\frac{3}{8}$ of an inch on a certain scale represents 3 ft., what does $1\frac{1}{2}$ in. on the same scale represent?

6. It is estimated that 100 lb. of good hay possesses the same feeding value as $62\frac{1}{2}$ lb. of corn. How many bushels of corn possess the same feeding value as 1 ton of hay?

7. If a certain sum when expended for sugar at 25¢ per pound was sufficient to purchase 16 lb., how many pounds will the same sum purchase when sugar is 8¢ a pound?

8. If the cost of burning ten 20-candle-power mazda lamps 30 hr. each is $0.90, what is the cost of burning these lamps 2½ hr. each?

9. A farmer's crop of 240 bu. of oats weighs 30 lb. to the bushel; if he sells it to a miller who calculates 32 lb. to the bushel, for how many bushels should the farmer be paid?

10. If a train running 40 mi. an hour runs a certain distance in 5 hr., in what time will it run the same distance at the rate of 50 mi. per hour?

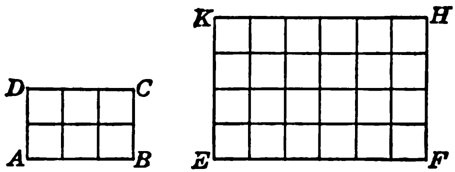
11. In a plan of a farm 1.25 in. is made to represent 10 rd. What is the length of a line on the plan if it represents a side of the farm 18 rd. long? 32 rd. long?

12. How many hours will it require a man to cut a 24½-acre field of grass at the rate of 7 acres in 8 hr.?

13. A spray used for codling moths consists of a dilution of 3 lb. of arsenate of lead to 50 gal. of water. How much arsenate of lead must be used to make sufficient spray for an orchard of 50 trees if it requires 4 gal. of spray for each tree? (This is about the quantity of spray required for trees ten years old.)

Similar Figures. Figures which are of the same shape are called *similar figures.*

Thus, these two rectangles are similar figures, or are said to be similar.



Corresponding Lines. In similar figures lines that are similarly placed are said to be *corresponding.*

Thus, in the rectangles $ABCD$ and $EFHK$, AB and EF are corresponding lines, also AD and EK. Name the other pairs of corresponding lines. The radii of two circles are corresponding lines; also the radii of two spheres.

Rectangle $ABCD$ is 3 units long and 2 units wide; rectangle $EFHK$ is how many units long? how many units wide?

In the similar rectangles $ABCD$ and $EFHK$, what is the ratio of AB to EF? of AD to EK?

The proportion formed by these equal ratios is

$$\frac{AB}{EF} = \frac{AD}{EK}, \text{ which means}$$

$$\frac{\textit{Length of smaller rectangle}}{\textit{Length of larger rectangle}} = \frac{\textit{Width of smaller rectangle}}{\textit{Width of larger rectangle}}$$

This relationship exists between the ratio of any two corresponding lines and the ratio of any other two corresponding lines of any two similar figures.

In similar figures corresponding lines are proportional.

Relation of Areas of Two Similar Figures. What is the area of rectangle $ABCD$ (see page 294)? of $EFHK$? What is the ratio of area $ABCD$ to area $EFHK$? What is the ratio of the *squares* of the corresponding sides AB and EF of these two similar rectangles? How do the two ratios compare? What proportion can be formed with these two ratios? Since:

$$\frac{6}{24} = \frac{9}{36} \qquad (1), \text{ we may write}$$

$$\frac{\textit{Area } ABCD}{\textit{Area } EFHK} = \frac{\overline{AB}^2}{\overline{EF}^2} \qquad (2)$$

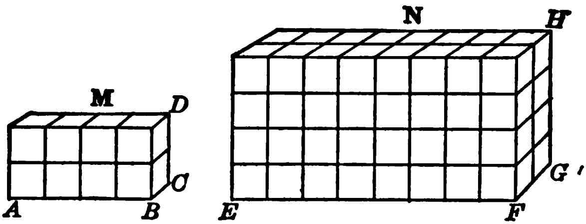
Remark. By $\overline{AB}^2$ is meant the square of the *number* of units in AB. By $\overline{AB}^2 : \overline{EF}^2$ is meant the ratio of the square of the *number* of units in AB to the square of the *number* of units in EF.

The relation expressed by equation (2) is true for any two similar figures; that is,

The areas of two similar figures are to each other as the squares of any two of their corresponding lines.

Would the members of equation (2) still be equal if the second ratio were $\frac{\overline{AD}^2}{\overline{EK}^2}$ instead of $\frac{\overline{AB}^2}{\overline{EF}^2}$? Prove your answer.

Relation of Volumes of Similar Solids. What is the volume of rectangular solid *M*? of rectangular solid *N*? What is the



ratio of the volume of *M* to the volume of *N*? What is the ratio of the cubes of the corresponding edges *AB* and *EF*? How do the two ratios compare? What proportion can be formed with these ratios? Since:

$$\frac{8}{64} = \frac{64}{512} \quad (1), \text{ we may write}$$

$$\frac{Volume\ M}{Volume\ N} = \frac{\overline{AB}^3}{\overline{EF}^3} \quad (2)$$

What is meant by $\overline{AB}^3$? by $\frac{\overline{AB}^3}{\overline{EF}^3}$? See remark, page 295.

The relation expressed by equation (2) is true for any two similar solids; that is,

The volumes of two similar solids are to each other as the cubes of any two of their corresponding lines.

Would the members of equation (2) still be equal if the second ratio were $\frac{\overline{CD}^3}{\overline{GH}^3}$? if it were $\frac{\overline{BC}^3}{\overline{FG}^3}$? Prove your answer.

SIMILAR FIGURES

1. The altitudes of two similar triangles are 6 ft. and 9 ft., and the base of the first is 10 ft. What is the base of the second?

SUGGESTION. Let x be the number of feet in the required base.

2. The bases of two similar triangles are 5 ft. and 6 ft. If the first contains 40 sq. ft., how many square feet are there in the second?

3. The area of an equilateral triangle whose side is 6, is 15.6, correct to the nearest 0.1. What is the area of the equilateral triangle whose side is 12?

4. If the ratios of the diameters of two circles are 1 to 2, compare their circumferences; their areas.

5. If the edge of one cube is twice the edge of another, the volume of the larger is how many times that of the smaller?

6. A farmer has two fields of the same shape two of whose corresponding sides are 40 rd. and 24 rd. The first field contains 12.6 acres; how many acres does the second field contain?

7. A conical piece of iron was found to weigh 8 lb. Its height was 6 in. What is the weight of a piece of iron of the same shape whose height is 12 in.?

8. At the same time that a pole, 7 ft. long, set erect, casts a shadow of 6 ft., a tree casts a shadow of 36 ft. What is the height of the tree?

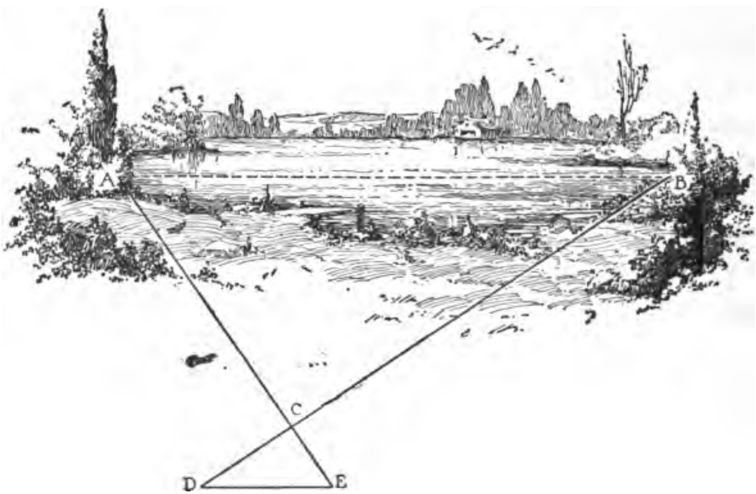
9. The diameters of the earth and the moon are as 1 to $\frac{8}{11}$. The earth is how many times as large as the moon?

10. If 240 gallons of water will flow through a certain orifice 3 in. in diameter in a certain time, how much water will flow through an orifice 6 in. in diameter in the same time?

SUGGESTION. The amounts of water or other liquid that flow through two similar orifices are to each other as the squares of the corresponding lines of the orifices.

11. The diameters of the earth and the sun are as 1 to 112. The sun is how many times as large as the earth?

12. Some boys wishing to measure the distance from A to B on opposite sides of the lake shown in the picture, set stakes at



A, B, and C, and measured lines AC and BC. Then they set a stake at D in line with B and C and one at E in line with A and C, having first fixed the points D and E so that $CE : CD = AC : BC$. AC was found to be 40 ft. and BC 60 ft.; so they made

CE 10 ft. and *CD* 15 ft. They then had two similar triangles, *CED* and *CAB*. They measured *DE*, which they found to be 18 ft. They then used the following proportion to find *AB*, or the distance across the lake. What did they find this distance to be?

$$AB : DE = AC : CE$$

or, $$AB : 18 = 40 : 10$$

If convenient, measure the distance across an imaginary lake on your school grounds, using the method which these boys used.

13. A motor boat which is to be built 10 ft. long should be how wide and how deep if it is to be built on the same model as one 20 ft. long, 4 ft. wide, and 20 in. deep?

14. How high must a silo be built which is to be 12 ft. in diameter and which is to have the same shape as a silo 30 ft. high and 14 ft. in diameter?

Food Values. During recent years our Government, with a view to conserving food materials and to raising the standard of physical efficiency, has, through various agencies, endeavored to enlighten the people on the relative values of the different food materials and the proportion in which they should enter into our diet to secure the ends sought.

Literature is available on the subject, some of the best of which is in the form of bulletins issued by the Department of Agriculture, Washington, D. C. (See Bulletin 28.)

Three Important Constituents of Food. Three compounds constitute important materials of our bodies. They are also important measurable constituents of our foods. These compounds are as follows:[1]

[1] Certain recent advances in connection with the problem of nutrition, such as consideration of the variation in the quality of the proteins from various sources and the functions of what are known as *health preservers*, *vitamines*, are beyond the scope of this treatment of nutrition.

1. **Protein,** which builds tissue, repairs waste, and may yield energy or help to store fats; contained in *eggs, meat, fish, nuts, milk,* etc.

2. **Fats,** which yield energy or are stored as fats; as, *fats* and *oils.*

3. **Carbohydrates,** which yield energy, or are stored as fats; as, *sugar* and *starch.*

Calorie. Protein, taken in excess of that required for growth and repair of waste, the fats, and the carbohydrates are burned (oxidized) in the body; the energy resulting therefrom is measured in terms of heat units, called *calories.*

Meaning of Calorie. The calorie (C) is approximately the amount of heat (fuel value) necessary to raise one pound of water 4 degrees Fahrenheit.

Calorie Requirement. The calories needed per day for normal individuals depend upon age, normal weight, and physical activity. In general, for each pound of normal weight the requirement per day is:

Infants.......	40 to 50 C	Adults (light work)...	15 to 20 C
Growing children	20 to 40 C	Adults (heavy work)..	20 to 30 C

Normal Weight. Normal weight may be found, *approximately,* by the following formulae, h representing the number of inches in a person's height, and w the number of pounds in his weight:

$$w = 5\tfrac{1}{2} \times (h - 60) + 110, \text{ if you are over 5 feet in height.}$$
$$w = 110 - 5\tfrac{1}{2} \times (60 - h), \text{ if you are under 5 feet in height.}$$

For example, the normal weight for an individual 5 feet 10 inches in height may be obtained, approximately, from the first formula thus:

$$\begin{aligned} w &= 5\tfrac{1}{2} \times (70 - 60) + 110. \\ &= 5\tfrac{1}{2} \times 10 + 110, \text{ or } 165. \end{aligned}$$

Normal weight, 165 lb.

What is the normal weight for a person whose height is 5 ft. 6 in.? 6 ft.? 4 ft. 8 in.? 5 ft. 2 in.?

Balanced Diet. To provide the elements necessary for health a diet should be a *balanced diet* as follows:

Estimated in calories { 10 % to 18 % protein.
25 % to 40 % fats.
50 % to 65 % carbohydrates.

Fuel Values. One ounce of protein or of carbohydrates has a fuel value of 116 calories. One ounce of fat has a fuel value of 264 calories. These values may be used to find the calories of energy (fuel value) contained in each ounce of the proteins, fats, and carbohydrates of foods.

Thus, the calories contained in a day's standard allowance of a man engaged as shown in the following table, are found by using the fuel values given above. (Check the fuel values in this table.)

DIETARY STANDARDS

	Nutrients				Fuel Value
	Total	Protein	Fat	Carbohydrates	
	ounces	ounces	ounces	ounces	calories
Man with light exercise..	21	3.5	3.5	14	2954
Man with moderate muscular work..........	25	4.5	4.5	16	3566
Man at active muscular work................	28	5.25	5.25	17.5	4015

NOTE. A woman whose occupation requires considerable bodily activity may require about 2400 calories per day. A balanced diet for her may be made up thus:

	Calories	*Ounces (approximately)*
10 % protein	240........	(240 ÷ 116), or 2.0
25 % fat.............	600........	(600 ÷ 264), or 2.3
65 % carbohydrates...	1560........	(1560 ÷ 116), or 13.4

TABLE

Food Materials (Edible portion)	Water	Nutrients (Edible portion minus water)					Fuel Value of 1 ounce, to nearest calorie
		Total	Protein	Fat	Carbohydrates	Mineral Matters, or Ash	
	per cent	per cent	per cent	per cent	per cent	per cent	calories
Apples	82.0		.5	.5	16.6	.4	21
Beef	62.2		18.4	18.5		.9	70
Bread (white)	35.4		9.5	1.2	52.8	1.1	75
Butter	10.0		.5	88.0		1.5	233
Cheese (American)	31.6		28.8	36.2		3.4	133
Eggs	73.5		14.9	10.6		1.0	45
Ham (smoked)	40.3		16.2	38.8		4.7	121
Mackerel (salt)	42.2		22.0	22.6		13.2	85
Milk (whole)	87.0		3.3	4.0	5.0	.7	20
Potatoes (white)	78.9		2.1	.1	18.0	.9	24
Raisins	14.0		2.5	4.7	74.7	4.1	102
Rice	12.4		7.4	.4	79.4	.4	102
Sugar (granulated)					100.0		116

PROBLEMS

1. What is the meaning of *calorie? nutrients?*

2. What per cent of the edible portion of apples are nutrients? See above table. Check your result.

SUGGESTION. Water + nutrients = edible portion.

3. Fill in the column of totals in the table. Check your results.

4. Find from the last column of the above table the fuel value of each of the following helpings: beef, 4 oz.; ham, 3 oz.; butter, $\frac{1}{2}$ oz.; milk, (1 glass), 8 oz; granulated sugar, 1 oz.

SUGGESTION. See Fuel Values, page 301.

5. What is the weight in ounces and the fuel value of the protein in 1 lb. raisins? of the fat? of the carbohydrates? What is the fuel value of 1 lb. raisins?

Using food materials in the above table, make and solve five problems like problem 5.

GENERAL REVIEW

Your answer to each of the first nine questions should be accompanied by an illustrative example. The answer to each illustration should be checked, as should all other answers in this review.

1. How is the value of a fraction affected if both terms are multiplied by the same number? divided by the same number? if the numerator is multiplied by a number? if it is divided by a number? if the same operations are performed upon the denominator?

2. How do you reduce a fraction to lowest terms? to a fraction having a required denominator? How is the value of a fraction affected by adding a number to the numerator? by subtracting a number from the numerator? by performing the same operations upon the denominator?

3. How do you reduce unlike fractions to similar fractions? an improper fraction to a mixed number? a mixed number to an improper fraction? How do you find the sum of two fractions? the difference? the product? the quotient? How do you find what fractional part one number is of another?

4. How do you reduce a decimal to a common fraction? a common fraction to a decimal?

5. How do you add decimals? How do you find the difference of two decimals? the product? the quotient? How do you determine the number of decimal places in a product? in a quotient?

6. Write the formulae for finding the areas of all surfaces mentioned in this book; the volumes of all solids. Express these formulae in words.

7. How do you find a given per cent of a number? a number when a certain per cent of it is given? what per cent one number is of another?

8. What is the general method of finding interest? the 6% method? the 60-day method?

9. How do you find the amount due on a note if several partial payments have been made? How do you find compound interest? bank discount on a non-interest-bearing note? on an interest-bearing note?

10. Show how the tables of square and cubic measure are developed from the table of linear measure. Write from memory the table of arc measure; angle measure; avoirdupois weight; time; United States money; counting.

11. What is your record now in each test in this book?

12. If the cost of hauling 3000 lb. of milk 5 mi. is estimated at $5, what is the estimated cost of hauling 1 lb. of milk 1 mi.?

13. If physical inefficiency is responsible for an average loss of 9 days each year by the country's 30,000,000 wage earners, how many dollars are lost, estimating wages at $2.50 per day?

14. One quart of milk is equal in food value to $\frac{4}{5}$ lb. of beefsteak. At current prices the cost of milk as a food is what per cent of the cost of beefsteak?

15. It is estimated that in a recent year farmers of this country burned 68,244,000 cords of wood valued at $345,866,000. What was the average value of the wood per cord?

16. Express at sight as common fractions in lowest terms: $.12\frac{1}{2}$; $.33\frac{1}{3}$; $.37\frac{1}{2}$; $.66\frac{2}{3}$; $.87\frac{1}{2}$; $.62\frac{1}{2}$; $.6\frac{1}{4}$; $.16\frac{2}{3}$; $.18\frac{3}{4}$.

17. An American airman recently established the world's record for parachute leaping when he dropped 24,600 ft. How many miles was this, correct to the nearest .1 of a mile?

18. If for every 8 lb. of cotton lint, 15 lb. of cotton seed are produced, how many pounds of cotton lint are produced to $4\frac{1}{2}$ tons of cotton seed?

19. Estimating that a glass of milk ($\frac{1}{2}$ pt.) has the same food value as 2 eggs, what is the cost at current prices of the milk that has the food value of 1 doz. eggs?

20. If one side of a rectangle is 16 ft. and another side is 12 ft., what is the perimeter? What is the length of the diagonal?

21. How many square yards of plastering are required for the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 9 ft. high, making no deductions for openings?

22. In a recent year a stick of timber in the state of Washington was hand hewn, which measured 54 in. square and 70 ft. in length. How many feet of inch lumber did the stick contain?

23. What part of an acre does a lot in the form of a rectangle contain, if it is 165 ft. long and 33 ft. wide?

24. Construct a parallelogram with base 4 in. and adjacent sides each 2 in., making the angle between the base and one of the adjacent sides 30°. What is its perimeter? What is the area?

25. Find the interest on $600 for 6 da. at 6%.

26. Simplify: $\dfrac{.875 \times \frac{3}{8} \times .16\frac{2}{3} \times .625 \times 2.5}{.375 \times \frac{7}{8} \times .33\frac{1}{3} \times .125 \times 5}$. (Cancel.)

27. What is the volume of a cube whose edge is 10 in.? What is the area of its entire surface? What is its diagonal?

28. How many $\frac{1}{2}$-in. cubes can be placed in a box $4\frac{1}{2}$ in. long, $2\frac{1}{2}$ in. wide, and 2 in. high?

29. Estimating 1 bu. as $\frac{5}{4}$ of a cubic foot, how many bushels of grain will a bin hold that is $10' \times 8' \times 4'$?

30. Given the volume and two of the dimensions of a rectangular solid, how do you find the third dimension? Illustrate.

31. A dealer who gained 25% of the cost when he sold oranges at 60¢ a dozen, gained what per cent of the cost when he sold them at 2 for 15¢?

32. What is the semiannual income on \$500 of U. S. Liberty Loan $3\frac{1}{2}\%$ bonds?

33. The distance between two places on a map is $1\frac{3}{8}$ in. If the actual distance between these places is 13.75 mi., the map is drawn on a scale of how many miles to the inch?

34. A bushel of oats contains 10 oz. of nitrogen, 3 oz. of phosphoric acid, and 2 oz. of potash. How many pounds of each of these substances were taken from the soil by a crop of 6 A. of oats yielding 40 bu. per acre?

35. In the formula $S = \pi R^2$, find S in terms of π when $R = 5$. Find R if S is 36π.

36. Mr. Hunt had his law library insured for \$1200 at a rate of 1% for five years. What was the premium for five years? What was the rate of premium for one year?

37. A town has property assessed at \$4,275,800 and 624 polls assessed at \$1.50 each. What must be the tax rate to raise a tax of \$22,315?

38. The publisher's price for a certain book is \$1.20, but he sells to retailers at a discount of $16\frac{2}{3}\%$. If the retailer sells the book at the publisher's price, what per cent does he gain on the cost? If the retailer sells the book at an advance of 10% on its cost, what does he get for it?

39. 60π expresses the lateral area of a cylinder whose altitude is 10. What is the radius?

40. A manufacturer sells goods to a retailer at a profit of 20%, and the retailer sells them to the consumer at a profit of 25%. The cost of the goods to the consumer is what advance on the manufacturing cost?

41. Find the profit on 20 shares of stock (par value \$50), bought at $\$58\frac{1}{4}$ per share and sold at $\$60\frac{1}{4}$, if two semiannual

dividends of $2\frac{1}{2}\%$ each were received between the transactions, usual brokerage on the sale and the purchase.

42. Find the interest on \$1100 from Nov. 22 to Feb. 28 of the following year, if the rate is 5%.

43. The altitude of a prism is 12 ft. Its base is a right triangle with the hypotenuse 15 ft. long, and one side 12 ft. long. Find the lateral area and the volume of the prism.

44. Given the diameter and height of a cylindrical tank, how can you find the number of gallons of water it will hold?

45. Find the annual premium that a man, born November 20, 1895, must pay on a 20-year endowment policy for \$10,000 taken out March 1, 1922.

46. When 6% stock (par value \$50) is bought at $\$51\frac{1}{8}$ per share, brokerage 15¢, what rate of income, correct to the nearest .01%, does it pay the investor?

47. Mr. Haines purchased 1000 bu. of wheat for me at \$1.40 a bushel. I sent him a check for \$1488, which paid the cost of the wheat, freight charges amounting to \$60, and his commission. What was his rate of commission?

48. How many shares of stock at $88\frac{1}{2}$ can be purchased with the proceeds of 50 bonds (face value \$100) at $110\frac{1}{4}$, brokerage on the bonds $\frac{1}{8}\%$, and on the stock 15¢ per share, and what will be the surplus?

49. Mrs. Lloyd insured her dwelling for \$1800 and her garage for \$300, each for 5 years. The annual rate on the dwelling was 8¢ per \$100 and on the garage $22\frac{1}{2}$¢ per \$100. What premium did she pay?

50. Find the missing term in each of the following:

a. $x : \frac{1}{2} = \frac{1}{4} : \frac{1}{8}$ *b.* $.5 : 1.5 = x : 3$

c. $9 : x = 12\frac{1}{2}\% : 87\frac{1}{2}\%$ *d.* $R : 4 = 6 : x$

51. Find $\sqrt{77 \times 21 \times 33}$; $\sqrt{2809}$; $\sqrt{54.76}$.

52. A tomato-club girl cultivates $\frac{1}{10}$ of an acre. If 12 sq. ft. are allowed for each hill, how many tomato plants will she have?

53. The average yield of tomatoes per $\frac{1}{10}$ of an acre is about 1000 lb. Estimating 50 lb. of tomatoes per bushel, how many bushels will be produced on $\frac{1}{10}$ of an acre?

54. If one bushel of tomatoes will put up 18 No. 3 cans, how many cans can be put up from the $\frac{1}{10}$ of an acre? If the tomatoes sell at 20¢ a can, and it cost 9¢ to grow a can, what will be the net profit on $\frac{1}{10}$ of an acre? on 1 acre?

55. A dealer paid $1800 for an automobile, which he sold through one of his agents at a profit of 20% of the cost. What was the selling price and what was the profit if the agent received a commission of 5% of the selling price?

56. A store allows its employees a discount of 25% from the price of all goods purchased. A $10 pair of shoes is marked down 20%; what is its cost to an employee?

57. Write a promissory note for $300 dated at your place of residence March 15 of the current year, payable in 60 da. Make William White the payee and yourself the maker. Properly indorse the note for discount and find the proceeds, if the note is discounted at a bank in your state April 1.

58. Find the cost of gilding a hemispherical dome 30 ft. in diameter at 50¢ a square foot.

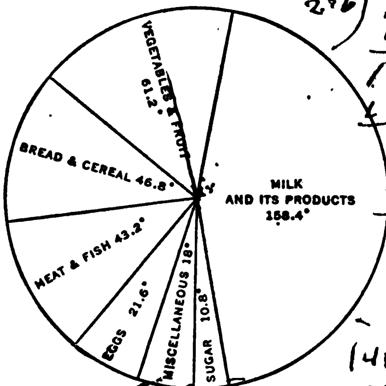
59. A woman deposited $300 in a savings bank at the beginning of a certain year and withdrew principal and interest at the end of 18 mo. How much was she entitled to if the savings bank paid 4% interest, compounded semiannually?

60. Find the rate of commission charged on the sale of a consignment of poultry, if the net proceeds were $460 after the commission, $25, also storage charges amounting to $15, had been deducted.

61. It is usual to allow 10 lb. of shelled corn and 10 lb. of hay or the equivalent to produce 1 lb. of beef. With corn at 80¢ a bushel (56 lb.) and hay at \$25 a ton, how much will it cost per 100 lb. to fatten a steer?

62. A dealer bought a bunch of 72 bananas for \$1.50. He retailed one half of them at the rate of 3 for 10¢, 2 doz. at the rate of 5¢ each, and the remainder at 3¢ each. What per cent of the cost did he gain? what per cent of the selling price?

63. The accompanying graph shows how you should divide the money spent for food for the best result. When \$10 is spent for food, how much should be spent for milk and its products? vegetables and fruit? bread and cereal? meat and fish? eggs? miscellaneous? sugar? Check your answers.



64. What per cent of the money spent for food should be spent for the articles or article as shown in the graph?

65. A piece of land in the form of a trapezoid contains 4 A. If the length of one of the parallel sides is 118 rd. and of the other 10 rd., how far are they apart?

66. In the formula $s = (a + l)\frac{n}{2}$, find s when $a = 1$, $l = 100$, and $n = 100$. The correct answer is the sum of the numbers 1, 2, 3, and so on, to 100.

67. A playground is in the form of a rectangle. If you were asked to find its area, what measurements would you make and how would you find the area in square yards?

TABLES OF DENOMINATE AMOUNTS

United States Money

10 mills	=	1 cent (¢, ct., or c.)
10 cents	=	1 dime (d)
10 dimes	=	1 dollar ($)

Although the gold dollar is no longer coined, it is represented by 25.8 grains of standard gold .9 pure. The standard silver dollar consists of 412.5 grains of standard silver. All gold and silver coins are .9 pure metal and .1 copper.

Counting

12 units	=	1 dozen (doz.)
12 dozen	=	1 gross (gro.)
12 gross	=	1 great gross
20 units	=	1 score

7

Liquid Measure

4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)
31½ gallons	=	1 barrel (bbl.)
2 barrels	=	1 hogshead (hhd.)
1 gallon	=	231 cu. in.

7

Dry Measure

2 pints (pt.)	=	1 quart (qt.)
2 quarts	=	1 small measure
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)
1 bushel	=	2150.42 cu. in.

Time

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (da.)
7 days	=	1 week (wk.)
52 weeks	=	1 year (yr.) (nearly)
365 days	=	1 common year
366 days	=	1 leap year
100 years	=	1 century

The exact length of the year is 365 days 5 hours 48 minutes 49.7 seconds. But, as it is not convenient to have other than a whole number of days in a year, it has been decreed that:

Every year whose number is not divisible by 4 shall consist of 365 days, called a **common year**.

Every year whose number is divisible by 4 shall consist of 366 days, called a **leap year**, except when it is a centennial year.

A **centennial year** whose number is divisible by 400 shall be a leap year, and when not so divisible, a common year.

Angles and Arcs

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference
1 right angle, 1 quadrant	=	90°

Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards	= 1 rod (rd.)
320 rods	= 1 statute mile (mi.)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

Surveyor's Linear Measure

7.92 inches	= 1 link (li.)
100 links	= 1 chain (ch.)
80 chains	= 1 statute mile (mi.)

Other Linear Units

(1) A **fathom** = 6 feet, used at sea to measure cables, cordage, and the depth of the sea. (2) A **hand** = 4 inches, used in measuring a horse's height. (3) A **geographic mile, or knot** = 6086 ft. = the length of one minute of the equatorial circumference on the earth. (4) A **league** (England and U. S.) = 3 geographic miles, used to measure distances at sea.

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

1 A. = 4840 sq. yd. = 43,560 sq. ft. = 6,272,640 sq. in.

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 short ton (T.)
2240 pounds	= 1 long ton
1 lb. avoir.	= 7000 troy grains

The long hundredweight, 112 pounds, and the long ton, 2240 pounds, are used in custom houses and in certain states in some wholesale and retail transactions.

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

The unit of weight for gems and precious stones is the **carat**, which equals 200 milligrams, or about 3.08 troy grains. The term *carat* is also used to denote the purity, or fineness, of gold, 24 carats denoting pure gold.

The troy pound contains 5760 grains, which is also the unit of apothecaries' weight.

Apothecaries' Weight

Medicines are sold in prescriptions by apothecaries' weight.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

Foreign Money

£1 (British money)	= $4.8665
1 franc (French money)	= .193
1 lira (Italian money)	= .193
1 mark (German money)	= .238

These are pre-war equivalents. At the present time the value of foreign money varies considerably from these equivalents.

Approximate Equivalents

1 bushel contains $1\frac{1}{4}$ cubic feet.
1 barrel ($31\frac{1}{2}$ gal.) contains $4\frac{1}{5}$ cubic feet.
1 cubic foot contains $7\frac{1}{2}$ gallons.
1 cubic foot of water weighs $62\frac{1}{2}$ pounds.
1 gallon of water weighs $8\frac{1}{3}$ pounds.
1 ton of hay occupies 500 cubic feet.

Legal Weights per Bushel in Texas for the Principal Commodities

Commodities	Lb.	Commodities	Lb.
Alfalfa seed	60	Milo maize	50
Apples, dried	28	Oats	32
Apples, green	50	Onions	57
Barley	48	Peaches	50
Beans, green or string	24	Peanuts, green, Georgia or Virginia	32
Beans, wax	24	Peanuts, Spanish	24
Beans, white	60	Peanuts, roasted	20
Beans, castor	46	Pears	58
Bran	20	Peas, dried	60
Buckwheat	52	Peas, green in pod	32
Clover seed	60	Potatoes, Irish	60
Coal, anthracite	80	Potatoes, sweet	50
Broom corn seed	48	Rye	56
Corn meal, unbolted	48	Salt, coarse	55
Corn, in the ear, after Dec. 1	70	Salt, fine	50
Corn, in the ear, new crop, before Dec. 1	72	Shorts	20
Corn, shelled	56	Sorghum seed	50
Cotton seed	32	Spinach	12
Cranberries	33	Timothy seed	45
Cucumbers	48	Tomatoes	56
Millet	50	Turnips	55
		Wheat	60

Miscellaneous Legal Weights in Texas

Lime, unslacked	per barrel, 180 pounds net
Lime, hydrated	per sack, 100 pounds net
Lime, hydrated	per bag, 40 pounds net
Lime, agricultural	per sack, 100 pounds net
Lime, agricultural	per bag, 50 pounds net
Bank sand	per cubic yard, 2500 pounds
Torpedo sand	per cubic yard, 3000 pounds
Gravel	per cubic yard, 3000 pounds

Texas Land Measure

Unit of measure: The vara = $33\frac{1}{3}$ inches; 36 varas = 100 feet.
1900.8 varas = one mile = 5280 feet.
5645.4 square varas = one acre = 43,560 square feet.

One	league	= 5000.0 varas square	= 4428.4 acres	= 13,889 feet square
Third	"	= 2886.7 " "	= 1476.1 "	= 8,019 " "
Quarter	"	= 2500.0 " "	= 1107.1 "	= 6,944 " "
One	labor	= 1000.0 " "	= 177.1 "	= 2,778 " "

1900.8 varas square	= 640.0 acres	= 5280 feet square
1344.0 " "	= 320.0 "	= 3733 " "
950.4 " "	= 160.0 "	= 2640 " "

To reduce varas to feet — multiply by 100, then divide by 36.
To reduce feet to varas — multiply by 36, then divide by 100.
To reduce sq. vrs. to acres — multiply by 177, then divide by 1,000,000.

METRIC TABLES

Measures of Length

NOTE. Important units are printed in italic type.

10 *millimeters* (mm)	= 1 *centimeter* (cm)	= 0.3937 in.
10 centimeters	= 1 decimeter (dm)	
10 decimeters	= 1 *meter* (m)	= 39.37 in.
10 meters	= 1 dekameter (Dm)	
10 dekameters	= 1 hectometer (Hm)	
10 hectometers	= 1 *kilometer* (Km)	= 200 rd.
10 kilometers	= 1 myriameter (Mm)	

12) 314
26 2

Measures of Surface

100 *square millimeters* ($^{\text{sqmm}}$)	= 1 *square centimeter* ($^{\text{sqcm}}$)	= 0.155 sq. in.
100 square centimeters	= 1 square decimeter ($^{\text{sqdm}}$)	
100 square decimeters	= 1 *square meter* ($^{\text{sqm}}$) / 1 centare ($^{\text{ca}}$)	= 10.763 sq. ft.
100 centares	= 1 *are* ($^{\text{a}}$)	= 119.6 sq. yd.
100 ares	= 1 hectare ($^{\text{Ha}}$)	= 2.5 *A.*

NOTE. Instead of *sqm.*, *sqcm.*, etc., we may write m^2, cm^2, etc.

Measures of Volume

1000 *cubic millimeters* ($^{\text{cmm}}$)	= 1 *cubic centimeter* ($^{\text{ccm}}$)	
1000 cubic centimeters	= 1 *cubic decimeter* ($^{\text{cdm}}$)	
1000 cubic decimeters	= 1 *cubic meter* ($^{\text{cbm}}$)	= 35.315 cu. ft.

Measures of Capacity

10 milliliters ($^{\text{ml}}$)	= 1 centiliter ($^{\text{cl}}$)	
10 centiliters	= 1 deciliter ($^{\text{dl}}$)	
10 deciliters	= 1 *liter* ($^{\text{l}}$)	= 0.908 qt. (dry) / 1.0567 qt. (liq.)
10 liters	= 1 dekaliter ($^{\text{Dl}}$)	
10 dekaliters	= 1 hectoliter ($^{\text{Hl}}$)	= 2.8375 bu. / 26.417 gal.
10 hectoliters	= 1 kiloliter ($^{\text{Kl}}$)	

Measures of Weight

10 *milligrams* ($^{\text{mg}}$)	= 1 *centigram* ($^{\text{cg}}$)	
10 centigrams	= 1 decigram ($^{\text{dg}}$)	
10 decigrams	= 1 *gram* ($^{\text{g}}$)	= 15.432 gr.
10 grams	= 1 dekagram ($^{\text{Dg}}$)	
10 dekagrams	= 1 hectogram ($^{\text{Hg}}$)	
10 hectograms	= 1 *kilogram* ($^{\text{Kg}}$)	= 2.2046 lb. (avoir.)
10 kilograms	= 1 myriagram ($^{\text{Mg}}$)	
10 myriagrams	= 1 quintal ($^{\text{q}}$)	
10 quintals	= 1 *Metric Ton* ($^{\text{MT}}$)	= 2204.6 lb. (avoir.)

INDEX

ANDERSON ARITHMETIC, BOOK THREE ANSWERS

Page 3. — 22. 9; 4; 44; 1926; 1865; 40; 60; 1109; 1809; 1924; 400; 105; 595; 1722; 1820.

Page 4. — 23. XIX: XXXVIII; CXIX: MDCCLXXVI; MDCCCLXIII; XLIV; XLIX; CDIX; MCDXCII; MCMXXVII.

Page 5. — 4. 68; 86; 69; 85; 69; 77; 82; 82; 75; 95.

Page 6. — 6. 5909. **7.** 5145. **8.** 5652. **9.** 6971. **10.** 5596. **11.** 5288. **12.** 5121. **13.** 6231. **14.** 5548. **15.** 5293. **16.** 6181. **17.** 6010.

Page 7. — 2. 64,988,905. **3.** 2,895,274. **4.** 140,543,261. **5.** 37,013,841. **6.** 99,007,866. **7.** 79,487,529. **8.** 85,592,661. **9.** 81,523,414. **10.** 161,205,-963. **11.** 373,486,250. **12.** 73,520,483. **13.** 763,779,210.

Page 8. — 4. 409,152. **5.** 93,636. **6.** 2,704,240. **7.** 545,205. **8.** 3,471,-880. **9.** 723,424. **10.** 355,355. **11.** 7,538,304. **12.** 1,512,940. **13.** 758,992. **14.** 185,688. **15.** 244,140. **16.** 1,930,632. **17.** 368,632. **18.** 1,087,060. **19.** 684,387.

Page 9. — 21. 1,707,404; 1,533,810; 568,342; 2,325,684; 16,779,168; 19,-045,402; 16,144,242. **22.** 3,280,542; 2,947,005; 1,091,991; 4,468,482; 32,238,-864; 36,593,121; 31,018,941. **23.** 5,928,526; 5,325,765; 1,973,423; 8,075,346; 58,261,392; 66,130,313; 56,056,773. **24.** 28,451,468; 25,558,770; 9,470,614; 38,754,228; 279,601,056; 317,364,634; 269,020,914. **25.** 6,097,974; 5,477,985; 2,029,827; 8,306,154; 59,926,608; 68,020,437; 57,658,977. **26.** 4,505,450; 4,-047,375; 1,499,725; 6,136,950; 44,276,400; 50,256,475; 42,600,975. **27.** 6,-467,744; 5,810,160; 2,152,912; 8,809,824; 63,560,448; 72,145,072; 61,155,312. **28.** 65,186,502; 58,558,905; 21,698,571; 88,791,642; 640,607,184; 727,129,101; 616,366,521.

Page 10. — 2. *a.* 360; *b.* 420; *c.* 480; *d.* 1230; *e.* 1620. **3.** *a.* 375; *b.* 435; *c.* 490; *d.* 635; *e.* 2845.

Page 11—4. *a.* 1700; *b.* 4100; *c.* 4800; *d.* 8750; *e.* 19,800. **5.** *a.* 4450; *b.* 4550; *c.* 3850; *d.* 6050; *e.* 39,300. **6.** *a.* 925; *b.* 1400; *c.* 1350; *d.* 7300; *e.* 9650. **7.** *a.* 1375; *b.* 1875; *c.* 2175; *d.* 12,?75; *e.* 19,725. **3.** 2×2×2×7; 3×3×7; 7×11; 2×2×2×2×3; 5×7; 2×2×2×11; 2×5×11; 2×2×2×3×5; 2×3×7; 2×2×7; 3×3×5; 2×3×3×3; 2×2×3×3; 2×2×2×3×3; 2×2×3×11; 3×3×11. **4.** 1061.

Page 12. — 4. 481. **5.** 913. **6.** 506. **7.** 205, rem. 44. **8.** 850. **9.** 930. **10.** 481, rem. 65. **11.** 1305, rem. 14. **12.** 921. **13.** 702. **14.** 745, rem. 23.

15. 590. **16.** 760. **17.** 531. **18.** 428, rem. 14. **19.** 411, rem. 43. **21.** *a.* 342; *b.* 233, rem. 375; *c.* 290, rem. 462; *d.* 159, rem. 285. **22.** *a.* 509, rem. 188; *b.* 348; *c.* 433, rem. 195; *d.* 237, rem. 291. **23.** *a.* 1146, rem. 270; *b.* 783, rem. 159; *c.* 975, rem. 231; *d.* 534, rem. 144. **24.** *a.* 568, rem. 130; *b.* 388, rem. 126; *c.* 483, rem. 231; *d.* 264, rem. 666. **25.** *a.* 1193, rem. 1085; *b.* 275, rem. 88; *c.* 212, rem. 2503; *d.* 165, rem. 7183. **26.** *a.* 3216, rem. 522; *b.* 740, rem. 4998; *c.* 572, rem. 798; *d.* 446, rem. 6146. **27.** *a.* 5417, rem. 424; *b.* 1247, rem. 4983; *c.* 963, rem. 4191; *d.* 752, rem. 3062. **28.** *a.* 6661, rem. 401; *b.* 1534, rem. 2601; *c.* 1184, rem. 6087; *d.* 925, rem. 907.

Page 14. — **2.** All; the tomato plants; the fruit trees; all. **6.** 31; 31; 11; 7; 15; 9; 8; 9; 9; 9.

Page 15. — **3.** 62. **4.** 64. **5.** 66. **6.** 200. **7.** 930. **8.** 40. **9.** 11. **10.** 3. **11.** 35. **12.** 54.

Page 16. — **1.** $102.50. **2.** $1450. **3.** 400 ft. **4.** $6.78. **5.** $10,000. **7.** 109,800,000 gal. **8.** 10 da. **9.** 1092. **10.** $920.80.

Page 17. — **11.** 3 to 0; 2 to 1; 4 to 6; 3 to 5; 7 to 5; 3 to 1; 0 to 3; 5 to 2; 6 to 7; 2 to 2. **12.** $1.60.

Page 19. — **5.** $\frac{6}{12}$, $\frac{21}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{10}{12}$, $\frac{9}{12}$, $\frac{18}{12}$, $\frac{15}{12}$; $\frac{12}{24}$, $\frac{42}{24}$, $\frac{4}{24}$, $\frac{6}{24}$, $\frac{20}{24}$, $\frac{18}{24}$, $\frac{36}{24}$, $\frac{30}{24}$. **6.** $\frac{24}{48}$; $\frac{8}{48}$; $\frac{40}{48}$; $\frac{30}{48}$; $\frac{32}{48}$; $\frac{4}{48}$; $\frac{28}{48}$; $\frac{21}{48}$; $\frac{36}{48}$; $\frac{6}{48}$; $\frac{15}{48}$; $\frac{14}{48}$.

Page 20. — **7.** $\frac{30}{60}$; $\frac{20}{60}$; $\frac{45}{60}$; $\frac{50}{60}$; $\frac{40}{60}$; $\frac{5}{60}$; $\frac{21}{60}$; $\frac{54}{60}$; $\frac{18}{60}$; $\frac{8}{60}$; $\frac{48}{60}$; $\frac{33}{60}$. **13.** *a.* $\frac{25}{30}$, $\frac{15}{30}$, $\frac{12}{30}$; *b.* $\frac{14}{24}$, $\frac{9}{24}$, $\frac{20}{24}$; *c.* $\frac{14}{16}$, $\frac{5}{16}$, $\frac{12}{16}$; *d.* $\frac{20}{50}$, $\frac{2}{50}$, $\frac{3}{50}$. **14.** *a.* $\frac{8}{36}$, $\frac{27}{36}$, $\frac{30}{36}$; *b.* $\frac{63}{72}$, $\frac{30}{72}$, $\frac{40}{72}$; *c.* $\frac{18}{60}$, $\frac{50}{60}$, $\frac{35}{60}$; *d.* $\frac{20}{48}$, $\frac{42}{48}$, $\frac{9}{48}$. **15.** *a.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, $\frac{5}{12}$; *b.* $\frac{30}{120}$, $\frac{72}{120}$, $\frac{100}{120}$, $\frac{105}{120}$; *c.* $\frac{24}{48}$, $\frac{32}{48}$, $\frac{36}{48}$, $\frac{15}{48}$; *d.* $\frac{10}{20}$, $\frac{8}{20}$, $\frac{15}{20}$, $\frac{14}{20}$.

Page 21. — **3.** *a.* $1\frac{1}{4}$; *b.* $1\frac{1}{2}$; *c.* $1\frac{1}{3}$; *d.* $1\frac{1}{2}$; *e.* $1\frac{1}{18}$; *f.* $1\frac{5}{16}$; *g.* $1\frac{5}{8}$; *h.* 2; *i.* $1\frac{9}{10}$; *j.* $1\frac{19}{48}$. **4.** *a.* 2; *b.* $2\frac{3}{10}$; *c.* $2\frac{7}{12}$; *d.* $2\frac{1}{3}$; *e.* $2\frac{3}{8}$; *f.* $1\frac{47}{48}$; *g.* $2\frac{13}{30}$; *h.* $1\frac{14}{15}$; *i.* $1\frac{13}{18}$; *j.* $1\frac{99}{100}$. **5.** $12\frac{1}{2}$; $10\frac{1}{4}$; 8; $12\frac{2}{5}$; $10\frac{13}{16}$; $8\frac{3}{10}$; $6\frac{9}{10}$; $13\frac{1}{6}$. **6.** $6\frac{7}{9}$; $16\frac{3}{10}$; $11\frac{39}{40}$; $12\frac{11}{40}$; $9\frac{31}{36}$; $14\frac{11}{24}$ $9\frac{13}{15}$; $18\frac{11}{15}$. **7.** $11\frac{1}{3}$; $17\frac{11}{12}$; $23\frac{7}{8}$; $15\frac{17}{18}$; $21\frac{5}{6}$; $9\frac{5}{8}$; 10; $11\frac{19}{48}$. **8.** $11\frac{3}{8}$; $15\frac{11}{24}$; $13\frac{11}{20}$; $21\frac{19}{24}$; $14\frac{23}{36}$; $16\frac{47}{48}$; $11\frac{2}{7}$; $15\frac{53}{100}$.

Page 22. — **9.** $13\frac{1}{4}$; $21\frac{1}{2}$; $20\frac{5}{8}$; $16\frac{3}{10}$; $14\frac{25}{48}$; $25\frac{5}{18}$; $24\frac{1}{9}$; $18\frac{21}{50}$. **10.** **Test 1.** $\frac{5}{8}$; $\frac{11}{12}$; $\frac{23}{36}$; $1\frac{21}{40}$; $\frac{13}{15}$; $1\frac{1}{15}$; $\frac{3}{4}$; $1\frac{11}{36}$; $1\frac{1}{10}$; $1\frac{1}{2}$. **Test 2.** $1\frac{1}{3}$; $\frac{1}{2}$; $1\frac{11}{60}$; $1\frac{13}{20}$; $\frac{7}{20}$; $\frac{13}{15}$; $\frac{41}{48}$; $1\frac{3}{8}$; $\frac{23}{24}$; $1\frac{1}{10}$. **Test 3.** $\frac{13}{16}$; $\frac{3}{4}$; $\frac{19}{24}$; $1\frac{2}{5}$; $1\frac{5}{16}$; $1\frac{11}{24}$; $1\frac{1}{6}$; $\frac{5}{6}$; $\frac{11}{12}$; $1\frac{23}{48}$. **Test 4.** $\frac{1}{3}$; $1\frac{5}{8}$; $\frac{8}{9}$; $\frac{31}{40}$; $\frac{37}{40}$; $1\frac{2}{5}$; $1\frac{19}{30}$; $1\frac{13}{36}$; $1\frac{11}{15}$; $\frac{9}{10}$. **Test 5.** $1\frac{5}{8}$; $\frac{23}{24}$; $\frac{14}{15}$; $\frac{2}{3}$; $1\frac{17}{60}$; $1\frac{1}{6}$; $\frac{9}{10}$; $\frac{11}{20}$; $1\frac{9}{16}$; $1\frac{1}{2}$.

Page 23. — **3.** **Test 1.** $\frac{1}{2}$; $\frac{3}{40}$; $\frac{9}{20}$; $\frac{3}{16}$; $\frac{11}{40}$; $13\frac{1}{3}$; $12\frac{1}{3}$; $3\frac{3}{8}$; $1\frac{1}{4}$; $6\frac{3}{5}$. **Test 2.** $\frac{1}{12}$; $\frac{1}{2}$; $\frac{31}{60}$; $\frac{21}{40}$; $\frac{8}{15}$; $11\frac{8}{9}$; $39\frac{7}{8}$; $7\frac{19}{24}$; $66\frac{1}{6}$; $6\frac{1}{2}$. **Test 3.** $\frac{9}{16}$; $\frac{1}{24}$; $\frac{1}{8}$; $\frac{9}{40}$; $\frac{1}{6}$; $31\frac{2}{5}$; $18\frac{2}{3}$ $9\frac{5}{12}$; $7\frac{2}{7}$; $2\frac{11}{12}$.

Page 24. — **Test 4.** $\frac{7}{30}$; $\frac{1}{15}$; $\frac{2}{3}$; $\frac{3}{10}$; $\frac{41}{80}$; $19\frac{23}{24}$; $49\frac{3}{8}$; $11\frac{3}{4}$; $53\frac{1}{4}$; $7\frac{13}{24}$. **Test 5.** $\frac{1}{20}$; $\frac{1}{30}$; $\frac{1}{6}$; $\frac{5}{16}$; $\frac{3}{20}$; $18\frac{1}{6}$; $48\frac{2}{3}$; $6\frac{1}{3}$; $2\frac{17}{24}$; $24\frac{1}{2}$.

Page 25. — **Test 1.** $\frac{2}{3}$; $1\frac{1}{2}$; $4\frac{2}{3}$; $6\frac{2}{3}$; $38\frac{1}{2}$; $\frac{2}{3}$; $2\frac{4}{5}$; 4; $6\frac{1}{4}$; $828\frac{3}{4}$. **Test 2.** 6; 20; $\frac{5}{7}$; 2; $1\frac{7}{8}$; 3; 60; 6; 8; $416\frac{2}{3}$. **Test 3.** $2\frac{2}{5}$; $2\frac{1}{2}$; $1\frac{3}{8}$; 60; $26\frac{2}{3}$; $\frac{2}{3}$; 6; $15\frac{5}{9}$; $\frac{2}{9}$; $937\frac{1}{2}$. **Test 4.** $1\frac{2}{5}$; $3\frac{1}{2}$; 6; 10; $\frac{1}{2}$; $1\frac{7}{8}$; $266\frac{2}{3}$; $78\frac{1}{8}$; 80; $1\frac{1}{2}$. **Test 5.** 2; 2; 1; $\frac{8}{25}$; $2\frac{2}{5}$; $4\frac{4}{5}$; 48; $1\frac{2}{3}$; $33\frac{1}{3}$; $3281\frac{1}{4}$.

Page 26. — **Test 1.** $\frac{1}{8}$; $\frac{2}{7}$; $1\frac{1}{9}$; 12; $1\frac{1}{5}$; $13\frac{1}{3}$; $1\frac{1}{2}$; $\frac{1}{2}$; 2; $15\frac{5}{8}$. **Test 2.** $\frac{2}{5}$; $5\frac{1}{3}$; $\frac{8}{9}$; $2\frac{2}{5}$; $6\frac{1}{4}$; $\frac{5}{8}$; $\frac{8}{16}$; $3\frac{1}{3}$; 3; 9. **Test 3.** $2\frac{1}{2}$; 15; 25; $\frac{2}{9}$; $\frac{5}{12}$; 6; $\frac{2}{3}$; 4; $\frac{8}{16}$; 5. **Test 4.** 4; $1\frac{1}{5}$; $16\frac{2}{3}$; $12\frac{1}{2}$; $\frac{1}{8}$; $\frac{1}{4}$; $1\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{3}$; $21\frac{7}{8}$. **Test 5.** 20; $2\frac{1}{2}$; $\frac{5}{16}$; 15; 4; $\frac{3}{8}$; $1\frac{1}{2}$; $\frac{15}{64}$; $1\frac{2}{5}$; 5.

Page 27. — **3.** $\frac{1}{2}$; $\frac{3}{8}$; $\frac{1}{4}$; $\frac{5}{8}$; $\frac{3}{4}$; $\frac{15}{16}$. **4.** 12¢; 9¢; 6¢; 15¢; 18¢; $22\frac{1}{2}$¢. **5.** 30¢; 20¢; 40¢; 45¢; 50¢. **6.** $4.15; $8.30; $12.45. **7.** 6; 8; 10.

Page 28. — **8.** $3. **1.** $\frac{8}{16}$. **2.** $\frac{5}{6}$. **3.** $\frac{3}{8}$. **4.** $\frac{1}{2}$. **5.** $3\frac{1}{3}$. **6.** $\frac{1}{2}$. **7.** $2\frac{1}{8}$. **8.** $\frac{1}{8}$. **9.** $\frac{1}{8}$. **10.** $\frac{5}{8}$. **11.** $\frac{7}{8}$. **12.** $\frac{1}{3}$. **13.** $\frac{2}{3}$. **14.** $\frac{1}{15}$. **15.** $\frac{1}{12}$. **16.** $\frac{1}{16}$. **17.** $\frac{1}{7}$. **18.** $\frac{1}{80}$. **19.** $\frac{8}{10}$. **20.** $\frac{3}{4}$.

Page 29. — **1**; 20. **2.** 8. **3.** 20.. **4.** $\frac{8}{16}$ in. **5.** $\frac{5}{9}$. **6.** $\frac{1}{5}$; 1056. **7.** 30¢. **8.** 30. **9.** $24; $12; $8. **10.** 64¢.

Page 30. — **11.** 45¢; 30¢; 75¢; 60¢; 6; 90¢. **12.** $74\frac{1}{2}$. **13.** $3\frac{1}{2}$ in.; $\frac{7}{12}$. **14.** $456\frac{1}{4}$ lb.; 45,625 lb. **15.** $\frac{5}{16}$; 80¢. **16.** $20\frac{1}{4}$. **17.** 42¢. **18.** Multiply the quantity of each ingredient given by 6; by 12. **19.** 27 ft.; 9.

Page 31. — **20.** $3\frac{3}{4}$ lb. **21.** 8¢; 40¢. **22.** $9.50. **23.** 7¢; 15. **24.** $\frac{1}{12}$; $\frac{1}{24}$; 48. **25.** $1\frac{7}{15}$. **26.** $\frac{1}{6}$. **28.** $2\frac{1}{8}$ yd.; $1\frac{3}{8}$ yd. **29.** $60\frac{5}{6}$.

Page 32. — **30.** 1. **31.** $2\frac{1}{2}$¢; $\frac{1}{4}$; $\frac{1}{3}$. **34.** $2\frac{6}{7}$. **35.** $4.80. **36.** 412 bu.; 524 bu.; 1156 bu.; 350 bu. **37.** $3\frac{5}{12}$ hr. **38.** $35\frac{3}{4}$.

Page 34. — **10.** .7; .27; .56; .475; .0856; .9875; .0045; 2.5; 3.75; 8.96; 7.035; 9.0256; 8.0406; 7.0014. **12.** .46; .408; 400.008; .0062; $.066\frac{2}{3}$; $.0087\frac{1}{2}$. **13.** $.6\frac{2}{3}$; $.06\frac{2}{3}$; $.14\frac{2}{7}$; $.66\frac{2}{3}$; $.026\frac{2}{3}$; $.018\frac{3}{4}$; $.0008\frac{1}{3}$. **14.** 7.48 ft., 7.5 ft., 7 ft.; $2.76. $2.80, $3; $3.78, $3.80, $4; 2.48 oz., 2.5 oz., 2 oz.; $8.88, $8.90, $9; 3.14, 3.1, 3.

Page 35. — **22.** Reading from left to right: $\frac{1}{5}$; $\frac{1}{4}$; $\frac{1}{20}$; $\frac{1}{8}$; $\frac{5}{8}$; $\frac{18}{40}$; $\frac{17}{40}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{1}{25}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{7}{10}$; $\frac{8}{20}$; $\frac{9}{25}$; $\frac{8}{40}$; $\frac{7}{40}$; $\frac{8}{16}$; $\frac{1}{16}$.

Page 36. — **23.** .5; .4; .25; .12; .02; .28; .55; .125; .875; .175; .2; .8; .08; .35; .06; .45; .85; .625; .025; .225. **24.** $\frac{1}{8}$; $\frac{3}{8}$; $\frac{1}{15}$; $\frac{1}{12}$; $\frac{5}{6}$; $\frac{1}{6}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{2}{3}$; $\frac{1}{7}$; $\frac{1}{16}$; $\frac{8}{16}$. **25.** .625; .1875; .3125; .5625; .6875; .375; .5375; .4375; .78125; .90625. **26.** $.12\frac{1}{2}$; $.62\frac{1}{2}$; $.37\frac{1}{2}$; $.31\frac{1}{4}$; $.16\frac{2}{3}$; $.41\frac{2}{3}$; $.58\frac{1}{3}$; $.56\frac{1}{4}$; $.18\frac{3}{4}$; $.06\frac{1}{4}$.

Page 37. — **27.** .43, .429, .4286; .58, .583, .5833; .83, .833, .8333; .33, .333, .3333; .22, .222, .2222; .67, .667, .6667; .56, .556, .5556; .92, .917, .9167; .13, .133, .1333. **28.** *g.* .32; *h.* .6625; *i.* .015125; *j.* .9125; *k.* .12125; *l.* .00325; *m.* .13; *n.* .1075; *o.* .020625. **1.** 72.692. **2.** 30.3734. **3.** 32.27. **4.** 23.7375.

Page 38. — **5.** 117.4. **6.** 101.9544. **7.** 217,491. **8.** 91.458. **9.** 0.288. **10.** 0.315. **11.** 0.03425. **12.** 0.854. **13.** 1.725. **14.** 1.475. **15.** 4.2975 **16.** 1.691. **17.** 0.7447. **18.** 5.8714. **19.** 1.9991. **20.** 2.012. **21.** 0.01. **22.** 0.1012. **23.** 0.137; 0.16; 0.0105; 0.8385.

Page 39. — **3-6.** The answers given are correct. **7.** *a.* .0288; *b.* 21.75; *c.* 14.4; *d.* 2.688; *e.* .001225; *f.* .576. **8.** *a.* .645; *b.* .003; *c.* 423.2; *d.* 1.845; *e.* .00108; *f.* 14.72. **9.** *a.* $4.3728; *b.* $2.4975; *c.* $0.493; *d.* $66.50; *e.* $2.47625; *f.* $7.476.

Page 40. — **3.** *a.* .15; *b.* .03; *c.* .001; *d.* .0006; *e.* .0007; *f.* .00008. **4.** *a.* .6; *b.* .56; *c.* .56; *d.* .485; *e.* 7.86; *f.* 1.009. **5.** *a.* .175; *f.* .12. **6.** *a.* .125; *b.* .14; *c.*

.55; *d.* .525; *e.* .12125; *f.* .2625. **3.** .0389; .0886; .0911; .6417; .2467; .0048. **4.** 6.3. **5.** 6300.

Page 41. — **7.** *a.* 13.5; *b.* 1.35; *c.* 135.; *d.* .0135. **8.** *a.* 135; *b.* .00135; *c.* 1350; *d.* 1350. **9.** *a.* 7.78; *b.* .0462; *c.* 592; *d.* 8650. **10.** *a.* .684; *b.* 76800; *c.* 8.04; *d.* 7500. **11.** *a.* 6.19; *b.* 8220; *c.* .0607; *d.* 709. **12.** *a.* 246.667, 246.6667; *b.* .229, .2286; *c.* 23.056, 23.0556; *d.* 3.673, 3.6735. **13.** *a.* .229, .2286; *b.* .002, .0022; *c.* .293, .2931; *d.* .047, .0465.

Page 42. — **Test 1.** **1.** 42.17. **2.** 62.74. **3.** 67.21. **Test 2.** **1.** .4; .056; .71; .29. **2.** .15; .499; .391; .601. **3.** .01; .64; .281; .601. **4.** .05; .419; .099; .291. **5.** .0514; .6444; .3192; .1702. **Test 3.** **1.** .12; .003; .018; .00015. **2.** 57.6; .7208; 8; 2.25. **3.** .018; 8.4; 6.96; .00486. **4.** .075; .35; .225; 6.25. **5.** 29.7; .04; .054; 3.2.

Page 43. — **Test 4.** **1.** .02; .02; 20; 200. **2.** .06; .4; .05; .2. **3.** .1; 10; .2; 4.5. **4.** 20; 5; .03; .75. **5.** .2; 5; .2; 30. **Review Test.** (Reading down the columns and from left to right) .075; .027; 1.93; .202; .013; .625; .075; 10; 12.5; 7.25; $33\frac{1}{3}$; 7.5; 4; 9.8; 1.87875; 1; 1; 8; 8; 200.

Page 45. — **8.** $7.76. **9.** $10.36.

Page 46. — **2.** 83¢. **3.** $5.25.

Page 47. — **4.** $25.19.

Page 48. — **1.** $1.78; $3.30; $6.38; $15.95; $26.94; $35.15. **2.** $1.80; $3.32; $6.40; $15.97; $26.97; $35.15. **3.** $7.58. **4.** $4.02. **5.** $19.35 P.O. **6.** $32.65; no difference. **7.** $15.12; 2¢

Page 49. — **1.** 25¢. **2.** $4.65. **3.** The latter by $50. **4.** $1127. **5.** $157.60 **6.** 4¢; 4¢; 4¢; 6¢; 2¢. **7.** $63.75 each.

Page 50. — **8.** $127.50; $127.50. **9.** 10¢; 14¢. **10.** 90¢; $22\frac{1}{2}$¢; 27¢; 36¢. **11.** $1.12. **12.** 45¢. **13.** 4.90 in. **14.** .629; .595.

Page 51. — **15.** .313. **16.** 3¢; 2¢; 3¢; 3¢. **17.** 10¢. **18.** .06. **19.** $750; $600; $450; $450; $450; $300. **20.** $2010. **21.** 4¢; 3¢; 8¢; 10¢. **23.** $5.85.

Page 52. — **24.** $252,683,110.80. **25.** $1124.80. **26.** 1000; 500. **27.** $1.12. **28.** $32. **29.** 112,500. **30.** 200,000; $1,007,750.

Page 55. — **22.** 48. **23.** 24. **24.** 36; 27. **25.** 60; 72. **26.** 3; 5; $3\frac{1}{2}$. **27.** 12, 8, 6, 4, 3, 2, 8, 6; 24, 16, 12, 8, 6, 4, 16, 12. **28.** 4c; $\frac{c}{2}$ $\frac{c}{4}$; $\frac{c}{8}$; $\frac{c}{8}$. **29.** $\frac{c}{2}$; 40¢; 20¢. **30.** 6; 9; 15. **31.** 40; 5.2; 10.5. **32.** $\frac{8r}{4}$ mi.; 90 mi.;

Page 56. — **33.** $2l+2w$; 60; 280. **34.** $a+b+c$; 42; 21; $\frac{a+b+c}{2}$. **35.** 1; 4; 9; 100; $\frac{1}{4}$; .0625. **36.** 1; 8; 27; 1000; $\frac{1}{8}$.

Page 57. — **2.** 18. **3.** 30. **4.** $50. **5.** 10. **6.** 45¢.

Page 58. — **7.** 4. **8.** 10. **9.** 6. **10.** 5. **11.** $4\frac{1}{2}$. **12.** 4. **13.** 5. **14.** 16. **15.** 8. **16.** 5. **17.** 2. **18.** 3. **19.** 4. **20.** 4. **21.** 3. **22.** 4. **23.** 3. **24.** 10. **25.** 4. **26.** 12. **27.** 6. **28.** 7. **29.** 4. **30.** 5. **31.** 5.

Page 59. — **32.** 4 lb. **33.** 3. **34.** 6. **35.** 4. **36.** 4. **37.** 1. **38.** 4. **39.** 2. **40.** 8. **41.** 6. **42.** 8. **43.** 4. **44.** 24. **45.** 40. **46.** 10. **47.** 60. **48.** 10. **49.** 12. **50.** 24. **51.** 200 in.

Page 60. — **52.** 20. **53.** 48; 8; 20. **54.** 5; 20; 48. **55.** 3; 6; 16. **56.** 2; 8. **57.** 4, 12. **58.** 2, 20.

Page 62. — **19.** 3. **20.** 4. **21.** 2. **22.** 1. **23.** $\frac{1}{2}$. **24.** $\frac{3}{4}$. **25.** 2. **26.** 16.

Page 63. — **2.** 200; 150. **3.** $2n$; $n+1$; 8. **4.** $4n$; $5n$; 6, 24. **5.** $\frac{x}{2}$; $x+\frac{x}{2}$; 20. **6.** $4n$; $n+4n$; 10, 40. **7.** $2y$; $y+2y$; 10, 20. **8.** 60. **9.** 12. **10.** 40.

Page 64. — **11.** .252; 3; .8. **12.** 1.8; 12; 5. **13.** 21. **14.** 50. **15.** 1000. **16.** .05. **17.** 25; 64; 144. **18.** 620. **19.** 20. **20.** 314.16. **21.** 256; 400.

Page 66. — **3.** Reading from left to right: .20, $\frac{1}{5}$; .62$\frac{1}{2}$, $\frac{5}{8}$; .25, $\frac{1}{4}$; .12$\frac{1}{2}$, $\frac{1}{8}$; .30, $\frac{8}{10}$; .06$\frac{2}{3}$, $\frac{1}{15}$; .50, $\frac{1}{2}$; .87$\frac{1}{2}$, $\frac{7}{8}$; .75, $\frac{3}{4}$; .08$\frac{1}{3}$, $\frac{1}{12}$; .06$\frac{1}{4}$, $\frac{1}{16}$; .12, $\frac{8}{25}$; .15, $\frac{8}{20}$; .04, $\frac{1}{25}$; .05, $\frac{1}{20}$; .33$\frac{1}{3}$, $\frac{1}{8}$; .80, $\frac{4}{5}$; .66$\frac{2}{3}$, $\frac{2}{3}$; .32, $\frac{8}{25}$; .11$\frac{1}{9}$, $\frac{1}{9}$; .24, $\frac{6}{25}$; .16$\frac{2}{3}$, $\frac{1}{6}$; .22$\frac{2}{9}$, $\frac{2}{9}$; .07$\frac{1}{2}$, $\frac{3}{40}$. **4.** 12$\frac{1}{2}$%; 25%; 20%; 16$\frac{2}{3}$%; 50%; 8$\frac{1}{3}$%; 40%; 60%; 75%; 37$\frac{1}{2}$%; 62$\frac{1}{2}$%; 87$\frac{1}{2}$%; 5%; 4%; 2%; 11$\frac{1}{9}$. **6.** 1; 6; 3; 2; 1.5; 2.5; 5; 7.5; 1.25; 1.06; 1.04; 10. **1.** 9. **2.** 15. **3.** *a.* 3 yd.; *b.* 5 ft.; *c.* 10 in. **4.** *a.* 15 A.; *b.* 28 mi.; *c.* 2$\frac{3}{4}$ oz. **5.** *a.* \$2; *b.* \$4; *c.* 8¢. **6.** *a.* \$21; *b.* \$6; *c.* \$3.

Page 67. — **7.** *a.* 3 lb.; *b.* 6 boys; *c.* 20%. **8.** *a.* \$37$\frac{1}{2}$; *b.* \$0.50; *c.* 10 ft. **9.** *a.* 30 gal.; *b.* 32 bu.; *c.* 2%. **10.** *a.* 3 qt.; *b.* 2¢; *c,* 3%.

Page 68. — **1.** .033; .125; .036; .375; .025; .045; .048; .0675; .0425; .0475; .035; .0525. **2.** \$18.60. **3.** \$11.875. **4.** *a.* 24 bu.; *b.* .1485 ft.; *c.* 2.8125 da. **5.** *a.* 38.75 pt.; *b.* 37.1875¢; *c.* 65 pupils. **6.** *a.* 14.85 in. *b.* .171 lb.; *c.* 31.2 ft. **7.** *a.* \$15.86. *b.* 17.1 yd.; *c.* 11 girls. **8.** *a.* \$2.505; *b.* \$5.271; *c.* 125 soldiers.

Page 69. — **1.** 6. **2.** \$785.70; \$960.30. **3.** \$66. **4.** 512.51778 lb. **5.** \$209. **6.** \$17.57. **7.** 49. **8.** 66 lb.

Page 70. — **9.** (Answers given correct to the nearest pound) 24 lb.; 10 lb.; 9 lb.; 7 lb.; 6 lb.; 5 lb.; 1 lb. **10.** \$81.60. **11.** 1673. **12.** 87.5¢. **13.** \$750; \$600; \$450; \$600; \$600. **14.** \$7.20; \$7.20; \$2.70.

Page 71. — **15.** \$2.952; \$7.20; \$1.80. **16.** 87; 3.3; 4; 5; .7. **17.** 3.54; .95; .12; 5.28; .11. **18.** \$2880. **1.** 12. **2.** 4; 10; 16; 1; .5; 2.4; 5. **3.** 8, 20, 32, 2, 1, 4.8, 10; 10, 25, 40, 2.5, 1.25, 6, 12$\frac{1}{2}$; 20, 50, 80, 5, 2.5, 12, 25. **4.** 16; 24; 4; 2; 2; 12; 20; 6. **5.** 15¢; 20¢; 30¢; 25¢; \$60; \$75; \$6.

Page 72. — **6.** \$15; \$20; \$2.50; \$1.25; 75¢; \$7.50; \$666$\frac{2}{3}$. **7.** \$24; \$32; \$4; \$2; \$1.20; \$12; \$1066$\frac{2}{3}$. **8.** Reading from left to right the numbers to be supplied are: 4; 1; 1; .2; 1; 1.25; 2; 2. **9.** 4¢; 2¢; 1¢. **10.** 10¢; 6¢. **11.** 24¢. **12.** 5¢. **13.** 15¢. **14.** $\frac{9}{10}$; 50. **1.** \$3250.

Page 73. — **2.** *a.* 2000; *b.* 150; *c.* 120. **3.** *a.* 112; *b.* 35; *c.* 30. **4.** *a* 94; *b.* 96; *c.* 200. **5.** *a.* 2000; *c.* 40. **6.** *a.* 1200; *b.* 232; *c.* 248. **7.** *a.* 20,000. *c.* 260. **8.** *a.* 20,000; *b.* 4000; *c.* 326$\frac{2}{23}$, or 326.087 (correct to the nearest .001). **9.** \$2750. **10.** 200. **11.** 2000. **12.** 200. **13.** 2$\frac{1}{2}$. **14.** 2000. **15.** 20 lb.

Page 74. — **12.** *a.* 50%; *b.* 25%; *c.* 20%. **13.** *a.* 33$\frac{1}{3}$%; *b.* 66$\frac{2}{3}$%; *c.* 12$\frac{1}{2}$%. **14.** *a.* 12$\frac{1}{2}$%; *b.* 37$\frac{1}{2}$%; *c.* 62$\frac{1}{2}$%. **15.** *a.* 40%; *b.* 60%; *c.* 80%. **16.** *a.* 16$\frac{2}{3}$%; *b.* 6$\frac{2}{3}$%; *c.* 11$\frac{1}{9}$%. **17.** 25%; 12$\frac{1}{2}$%. **18.** 100. **19.** 200%; 300%; 150%; 125%; 175%.

20) $13\frac{1}{3}$ = $\frac{40}{3} \times \frac{1}{20}$ =

Page 75. — **20.** 500%; 150%; 125%; 275%. **22.** 29%. **23.** .21% (correct to nearest .01%). **24.** 4.55% (correct to nearest .01%). **25.** 50%; 25%. **26.** $16\frac{2}{3}$%. **27.** 19%; 81%. **28.** 5.88% (correct to nearest .01%).

Page 76. — **2.** *a.* 6%; *b.* 4%; *c.* 5%; *d.* 2%. **3.** *a.* 25%; *b.* 48%; *c.* 15%; *d.* 8%. **4.** *a.* 24%; *b.* 8%; *c.* 5%; *d.* 6%. **5.** 14.2%. **6.** 7%. **7.** 112%; 89.29% (correct to nearest .01%). **8.** 40%; $15.

Page 77. — **9.** 20%. **10.** 171%. **11.** 15%. **12.** $\frac{23}{40}$; 57.5%. **13.** 49.75%. **14.** 11.21%; 1.12%. **15.** 63.6%. **16.** 13.9%.

Page 79. — **1.** .045; .0475; .12; .018; .0645; .0175; .042; .0742. **2.** 3%; 25%; $3\frac{3}{4}$%; 6.5%; 3.4%; 3.45%; 6.41%. **3.** $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{1}{8}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{16}$; $\frac{1}{15}$. **1.** $4800.

Page 80. — **2.** 70%. **3.** 90%. **4.** 125%; 25%. **5.** 1199.278224. **6.** 5.13%. **7.** 20¢ **8.** 18. **9.** A peck; a pint. **10.** $37\frac{1}{2}$%; $25; $62\frac{1}{2}$%. **11.** $66\frac{2}{3}$%.

Page 81. — **12.** $46\frac{2}{3}$%; $33\frac{1}{3}$%; 25%; $37\frac{1}{2}$%; 20%. **13.** 95%. **14.** $8316. **15.** 6.9%. **16.** 67.52%. **17.** 16.2%. **18.** 5.4%. **19.** 60¢.

Page 82. — **20.** $525; 52.5. **21.** 15.69%. **22.** 50; 15; 325. **23.** $15.55. **24.** 10%; 90%.

Page 83. — **1.** The chairs and the bedsteads; the rockers and the dining room set; $10; $100; $15; $17.50. **2.** The desk: discount $40, net price $40; the lamp: discount $16, net price $16; the rug: discount $55, net price $55; the calendar: discount 50¢, net price 50¢. **3.** The coat: discount $15, net price $30; the dress: discount $5, net price $10. The blankets discount $3, net price $6; the table: discount $10, net price $20. **4.** $159.29. **5.** $105.42.

Page 84. — **6.** $2.10; $25.20. **7.** $60. **8.** $13,650; 52.30% (correct to nearest .01%). **9.** $44\frac{4}{9}$%. **10.** 20%. **11.** $16\frac{2}{3}$%. **12.** 40%. **13.** 36% **14.** 35%. **15.** $33\frac{1}{3}$%. **16.** $16\frac{2}{3}$%.

Page 85. — **1.** $29.75. **2.** $94.79. **3.** $198.16.

Page 86. — **1.** $190. **2.** $209. **3.** $106.40. **4.** 60%; 44%; 55%; 36%; 19%; 32.5%. **1.** $33\frac{1}{3}$%.

Page 87. — **4.** $6; $7.50.

Page 88. — **5.** $\frac{1}{5}$; $\frac{4}{5}$; $3.75; $3. **6.** Reading from left to right: $12; $7\frac{1}{2}$¢; 66¢; $50; $1400; 4¢; 18¢; $3.15.

Page 88. — **1.** $0.12. **2.** 10¢. **3.** 24¢; 25%. **4.** $1.25; $33\frac{1}{3}$%. **5.** 140%. **6.** $18\frac{3}{4}$%. **7.** 25%; 20%.

Page 89. — **8.** 110%. **9.** 100%; 50%. **10.** 40¢. **11.** 50¢. **12.** 50%; $33\frac{1}{3}$%; $66\frac{2}{3}$%; $33\frac{1}{3}$%. **13.** $25; 25%. **14.** $66\frac{2}{3}$%. **15.** $4000; 20%; $33\frac{1}{3}$%; 25%.

Page 90. — **16.** 20%. **17.** $12; 40%; 20%; 25%. **18.** $1.05; $4.20; $8.75; $5.425. **19.** 50%; $33\frac{1}{3}$%. **20.** $4; 25%; 20%. **21.** $12\frac{1}{2}$%. **22.** $2\frac{1}{2}$%. **23.** $75.60.

Page 91. — **1.** $340. **2.** $30. **5.** $825.

Page 92 — **6.** $130; $142.50. **7.** $252; $1.26. **8.** $61.20. **9.** $1250. **10.** 2%. **11.** $33. **12.** $570; 71.25%. **13.** $471.60; 7.86%.

b $\frac{2}{3}$.

Page 93. — **14.** \$86; \$112. **15.** \$94. **16.** \$6.75. **17.** \$12.50. **18.** Aug., \$80; Sept., \$84.50; Oct., \$93; Nov., \$99; Dec., \$109; Jan., \$93.75. **19.** \$19.50 gain.

Page 94. — **1.** \$6; \$5. **2.** \$12; \$15. **3.** \$36; \$24. **4.** \$45; \$47.50. **5.** \$15; \$22.50. **6.** \$72. **7.** \$72. **8.** \$126. **9.** \$67.50. **10.** \$21. **11.** \$15. **12.** \$27. **13.** \$12.25. **14.** \$44.60. **15.** \$33. **16.** \$13.84. **17.** \$16.88.

Page 95. — **18.** \$35; \$175. **19.** \$2; \$400. **2.** \$0.55. **3.** \$2.08. **4.** \$1.97. **5.** \$15.86. **6.** \$4.80; \$4; \$3.20. **7.** \$1.13; \$0.84; \$0.68.

Page 96. — **8.** \$21.87; \$25.52; \$29.16. **9.** \$3.81. **10.** \$0.95. **11.** Aug. 14; \$1.20. **12.** Dec. 14; \$3.13.

Page 97. — **1.** \$77.50; \$77.33. **2.** \$21.50; \$21.36. **3.** \$31.82; \$31.74. **4.** \$83.39; \$83.32. **5.** \$6.68; \$6.63. **6.** \$29.45; \$29.45. **7.** \$27.06; \$27.03. **8.** \$57.60; \$57.42. **9.** 24; \$24. **10.** 2. **11.** .06. **12.** 200.

Page 99. — **1.** \$101.25, \$120, \$1222.50; \$67.50, \$80, \$815; \$84.38, \$100, \$1018.75. **2.** \$236.25, \$330.75, \$1008. \$157.50, \$220.50, \$672; \$196.88, \$275.63, \$840. **3.** \$18.58, \$18.50; \$12.39, \$12.33; \$15.48, \$15.42. **4.** \$130.07, \$129.68; \$86.71, \$86.46; \$108.39, \$108.07.

Page 100. — **4.** \$2, \$3, \$0.50; \$1.33, \$2, \$0.33; \$1.67, \$2.50, \$0.42. **5.** \$1.50, \$2.25, \$4.50; \$1, \$1.50, \$3; \$1.25, \$1.88, \$3.75. **6.** \$9.99, \$17.76, \$11.88; \$6.66, \$11.84, \$7.92; \$8.33, \$14.80, \$9.90. **7.** \$58.50, \$15.17, \$8.13; \$39, \$10.11, \$5.42; \$48.75, \$12.64, \$6.78.

Page 101. — **1.** \$6575.34. **2.** \$16,438.36. **3.** \$21,832.19. **4.** \$15,102.74. **5.** \$20,027,397.26. **6.** \$30,452,054.79.

Page 102. — **2.** 6; $1\frac{1}{2}$. **3.** \$1.60. **4.** 24. **5.** $57\frac{3}{4}$; $28\frac{7}{8}$; $14\frac{7}{16}$. **6.** 10 qt.; 20 pt., $2\frac{1}{2}$ gal. **7.** $\frac{1}{8}$. **8.** 2.

Page 103. — **9.** $19\frac{1}{4}$¢. **3.** 128. **4.** 7 pk.; 56 qt.; $1\frac{3}{4}$ bu.; 112 pt. **5.** 32; 96. **6.** 220.6. **7.** 14. **8.** \$755.54; \$630.98. **9.** 537.605; 67.200625; 33.6003125. **10.** Compare .8×2150.42 with 1728. **11.** \$10; \$5.

Page 104. — **12.** 18. **13.** 11; \$0.10; \$0.14$\frac{6}{11}$. **14.** \$30.00; \$15.00; 1¢. **15.** \$7.04. **16.** 9.450625. **17.** 3; \$1.20.

Page 105. — **3.** $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{7}{8}$. **4.** $1\frac{1}{4}$ lb.; $2\frac{1}{2}$ lb.; $3\frac{3}{4}$ lb.; $1\frac{5}{8}$ lb.; $1\frac{7}{8}$ lb. **5.** $1\frac{3}{4}$ lb.; 280 lb.; 28 oz.; 1 lb. 12 oz. **6.** 16, 4 lb. over. **8.** 38¢. **9.** 4 lb. 11 oz. **10.** 8 oz.; 2; 50%. **11.** \$2.23, correct to the nearest cent. **12.** 19¢; \$1.14. **13.** \$184.63.

Page 106. — **14.** \$14.40. **15.** \$32.64. **16.** \$36.30. **17.** \$94.86. **18.** \$11.76 **19.** 116.1 gr.; 12.9 gr.; 464.4 gr.; 51.6 gr.

Page 107. — **20.** 371.25 gr.; 185.625 gr.; 92.8125 gr. **21.** 20; 24; 8; 3; 20. **22.** 240. **23.** 72. **24.** \$350. **25.** 9. **26.** 392; \$3010.

Page 108. — **13.** 59 min. **14.** 8 hr. 55 min.

Page 109. — **16.** 55 yr.; 1865. **17.** 12 hr. 31 min. 40 sec. **18.** 38 hr. 33 min. **19.** 120 hr. 46 min. 30 sec. **4.** $18\frac{2}{11}$%; $6\frac{2}{33}$%. **5.** 22; 66; 792. **6.** 7.92.

Page 110. — **7.** 63. **8.** 18 in.; 20 in.; 21 in.; 27 in.; 33 in.; 42 in. **9.** $1\frac{1}{2}$ ft., 1.5 ft.;

$1\frac{1}{4}$ ft., 1.25 ft.; $2\frac{1}{2}$ ft., 2.5 ft.; $7\frac{1}{2}$ ft., 7.5 ft.; $1\frac{1}{2}$ ft., 1.5 ft.; $2\frac{3}{4}$ ft., 2.75 ft. **10.** $\frac{1}{2}$ yd.; $\frac{1}{3}$ yd.; $\frac{3}{4}$ yd.; $1\frac{1}{3}$ yd.; 11 yd.; 13.75 yd. **11.** 5, $5\frac{1}{2}$ ft. remaining. **12.** 10. **13.** 182.4 mi. **14.** 1.1515. **15.** 778.414 mi. **16.** 11.48 mi.

Page 111. — **17.** 3. **18.** 1276.52. **19.** 3.4545. **22.** \$20.62; \$108,873.60.

Page 112. — **3.** $1\frac{1}{4}$ mi.; $\frac{1}{2}$ mi.; $\frac{3}{4}$ mi. **4.** 10′; $12\frac{1}{2}$′; 5′; $7\frac{1}{2}$′. **5.** 1″ = 80′.

Page 115. — **6.** $\frac{3}{4}$ in.; $\frac{1}{4}$ in.; 4 ft. $8\frac{7}{8}$ in.

Page 122. — **1 to 7.** See tables, page 310. **8.** 294; 1296. **9.** 272.25; 2; 30; 544.5. **10.** 481; 96; 2; 67.2. **11.** 2 sq. ft. 112 sq. in. **12.** 2A. 28 sq. rd. **13.** $2\frac{1}{2}$.

Page 123. — **14.** 20. **15.** \$42. **16.** 72. **17.** \$22.40. **18.** 1140. **19.** 216 sq. ft.; 6. **20.** 1314; 146. **21.** 174. **22.** \$156.60.

Page 124. — **23.** 16. **24.** 192. **25.** 72. **26.** 240. **27.** 337.5. **28.** 364.-21875 **29.** \$34.56. **30.** \$46.08. **31.** $12{,}906\frac{2}{3}$.

Page 125. — **32.** 108 sq. ft. **33.** 136.125 sq. ft. **34.** 30.25 sq. yd. **35.** 41.-25 sq. rd. **36.** 67.5 sq. rd. **37.** $1\frac{1}{8}$ sq. ft. **38.** 36 sq. yd. **39.** 57.75 sq. ft. **40.** 4.95 sq. rd. **41.** 8.4375 sq. rd. **42.** 36 sq. in. **43.** 14.5 sq. rd. **44.** 88 sq. ft. **45.** 5073.6 sq. ft. **46.** 24. **47.** 9. **48.** 4. **49.** 72. **50.** 10. **51.** 3.

Page 126. — **52.** 12. **53.** 10. **54.** 4. **55.** The number of square rods in each figure, in turn, is: A, 4; B, 4; C, 8; D, 6; E, 4; F, 5; G, 6; H, $4\frac{1}{2}$; I, 4; J, 10; K, 12; L, 4; M, 12; N, 6.

Page 127. — **3.** 90°; 180°. **5.** 165°. **6.** $69\frac{3}{8}$ mi.; yes.

Page 128. — **7.** 4° 34′. **8.** 10°. **9.** 16° W. **10.** 10° E. **13.** 22′ 14″ N.

Page 129. — **1.** 1 hr.; 2 hr.; 3 hr.; 4 hr.; 5 hr. **2.** 5 hr.; 75°. **3.** 2 hr. 20 min.; 35°.

Page 130. — **5.** 70° 58′ 4″; 16° 15′ 13″; 89° 54′ 53″; 243° 22′ 15″; 87° 36′ 19″; 20° 9′ 26″. **6.** 4 hr. 43 min. $52\frac{4}{15}$ sec.; 1 hr. 5 min. $0\frac{13}{15}$ sec.; 5 hr. 59 min. $39\frac{8}{15}$ sec.; 16 hr. 13 min. 29 sec.; 5 hr. 50 min. $25\frac{4}{15}$ sec.; 1 hr. 20 min. $37\frac{11}{15}$ sec.

Page 132. — **1.** Central, 11 A.M. Mountain, 10 A.M.; Pacific, 9 A.M. **2.** Eastern, 2 P.M.; Central, 1 P.M.; Pacific 11 A.M. **3.** Too fast by 3 hr. **4.** 4 da. 21 hr.

Page 133. — **8.** 4 P.M.; 5 P.M.; 4 P.M.; 5 P.M. **9.** 11.30 A.M.; 9.30 A.M.; 12.30 P.M.; 11.30 AM..; 10.30 A.M.

Page 135. — **16.** 640. **17.** 320.

Page 136. — **7.** 128; 4. **8.** 76.8. **9.** (*a*) 480; (*b*) 5; (*c*) 5; (*d*) 5. **10.** 300.

Page 137. — **11.** 6 ft. **12.** 1690 lb. **13.** 24.75. **14.** 32. **15.** \$260.61. **16.** \$2025. **17.** 7.48. **18.** 1728.

Page 138. — **1.** Plymouth Rocks, $322\frac{11}{12}$, $247\frac{11}{12}$; Wyandottes, 300, $239\frac{7}{12}$; Rhode Island Reds, $314\frac{7}{12}$, $243\frac{3}{4}$; Leghorns, $354\frac{1}{6}$, $287\frac{1}{2}$. **2.** Leghorns, \$184.88; Plymouth Rocks, \$148.54; Rhode Island Reds, \$144.71; Wyandottes, \$138. **3.** Leghorns, \$155.25; Plymouth Rocks, \$120.24; Rhode Island Reds, \$118.22; Wyandottes, \$116.20.

Page 139. — **4.** Plymouth Rocks, \$67.35, \$66.45; Wyandottes, \$60.23,

\$60.30; Rhode Island Reds, \$64.95, \$64.88; Leghorns, \$57.15, \$59.93. **5.** Plymouth Rocks, \$81.19, \$53.79; Wyandottes, \$77.77, \$55.90; Rhode Island Reds, \$79.76, \$53.34; Leghorns, \$127.73, \$95.33. **6.** 1st yr., \$366.45; 2nd yr., \$258.35; \$624.80. **7.** Compare the returns: Plymouth Rocks, \$97.95; Wyandottes, \$95.22; Rhode Island Reds, \$96.81; Leghorns, \$119.68. **9.** \$2165.52.

Page 140. — **1.** 824.1 lb.; 139 lb.; 25.1 lb.; 11.8 lb. **2.** Pride, 396.036; Topsy, 489.0464; Rose, 395.4825; Flora, 452.3613; Lottie, 373.9248.

Page 141. — **3.** 4.24%. **4.** Expenditures, in order, now are: \$3.50, \$2, \$1.50, \$1.30, \$0.60, \$0.50, \$0.60; should be: \$1.20, \$4.40, \$1.30, \$1.70, \$0.60, \$0.30, \$0.50. **5.** *a.* \$8.30; *b.* \$8.70; *c.* \$8.90; *d.* \$9; *e.* \$7.80; *f.* \$7.60; *g.* \$9.10; *h.* \$8.50; *i.* \$9.20; *j.* \$7.70. **7.** \$6.75.

Page 142. — **8.** 72¢; 81¢. **9.** 3.50; 5.13; 4. 54; 5.20; 5.01; 3.92; 4.42; 4.13. **11.** 668.182.

Page 143. — **1.** 36′; 27′; 972. **2.** 36. **3.** $13\frac{1}{2}$. **4.** \$3.40. **5.** 140. **6.** \$8.37. **7.** 7.

Page 144. — **8.** $3\frac{15}{16}$; $2\frac{1}{4}$. **9.** \$25.55. **10.** \$10.20. **11.** \$26.54; yes by \$16.34. **12.** 25; \$33.75; yes.

Page 145. — **1.** Sum, .972625. **2.** $\$1.13\frac{1}{8}$. **3.** 2; 3; 5; 4; 6; 10. **4.** 2.4. **5.** 8000. **6.** 6.9. **7.** 180. **8.** 820. **9.** 1.7. **10.** *a.* \$3.20; *b.* \$2.16; *c.* \$5; *d.* \$11.25; *e.* \$1.80; *f.* \$6.30; *g.* \$3.15; *h.* \$6.00. **11.** \$5.75. **12.** \$9.35. **13.** \$52.

Page 146. — **14.** $\frac{1}{10}$. **15.** \$2.30. **16.** \$6.75. **17.** \$20.25; \$6.75. **18.** \$864,000,000. **19.** 75¢. **20.** 8 da. **21.** \$46.18, correct to nearest cent. **22.** \$121,600; $63\frac{1}{3}$%.

Page 147. — **23.** 25%. **24.** 10¢; 8¢. **25.** 18; \$2565. **26.** 150%. **27.** 80¢. **28.** 283,275; 4.00%. **29.** $2\frac{1}{2}$. **30.** July 24. **31.** $45\frac{5}{8}$.

Page 148. — **33.** 20%. **34.** 255; 270. **35.** \$4168.80. **36.** \$350,000; \$1,400,000.

Page 149. — **38.** \$8; $16\frac{2}{3}$%. **39.** 25%. **40.** \$2.50. **41.** 2.58; 4.02; 1.38; .30; .42; 3.00; 1.74; 5.94. **42.** 8%.

Page 150. — **43.** \$9 gain. **44.** $a+5$; $a-5$; $5a$; $\frac{a}{5}$. **45.** \$186.45.

Page 154. — **6.** 12. **8.** *a.* $\frac{8}{5}$; *b.* $\frac{2}{5}$; *c.* 28; *d.* $\frac{8}{10}$. **9.** *a.* $2\frac{1}{5}$; *b.* $\frac{9}{16}$; *c.* $1\frac{2}{5}$; *d.* $\frac{5}{16}$. **10.** *a.* 11; *b.* $2\frac{7}{20}$; *c.* $\frac{13}{24}$; *d.* $\frac{1}{6}$. **11.** *a.* $\frac{7}{40}$; *b.* $1\frac{7}{80}$; *c.* $\frac{1}{12}$; *d.* $\frac{2}{3}$. **12.** *a.* [illegible]; *b.* 9[illegible] 1; *d.* $1\frac{18}{40}$. **13.** *a.* $\frac{1}{2}$; *b.* $11\frac{1}{8}$; *c.* 10; *d.* $5\frac{7}{40}$. **14.** *a.* 10; *b.* $\frac{7}{8}$; *c.* $\frac{1}{24}$; *d.* $2\frac{1}{6}$. **15.** *a.* $9\frac{5}{8}$; *b.* $1\frac{1}{2}$; *c.* $6\frac{13}{24}$; *d.* $25\frac{1}{2}$. **16.** *a.* 100; *b.* $1\frac{5}{9}$; *c.* $\frac{8}{10}$; *d.* $1\frac{1}{2}$. **17.** *a.* $6\frac{3}{4}$; *b.* $\frac{2}{3}$; *c.* $1\frac{1}{2}$; *d.* $\frac{3}{8}$. **18.** *a.* $1\frac{2}{3}$; *b.* $2\frac{7}{8}$; *c.* $328\frac{1}{2}$; *d.* $1\frac{7}{10}$. **19.** *a.* $\frac{8}{9}$; *b.* $6996\frac{1}{8}$; *c.* $2\frac{2}{9}$; *d.* $\frac{89}{40}$. **20.** *a.* 4; *b.* $1\frac{5}{6}$; *c.* $\frac{5}{6}$; *d.* 15. **21.** *a.* $\frac{2}{3}$; *b.* $187\frac{1}{5}$; *c.* $\frac{3}{16}$; *d.* $3\frac{1}{6}$. **22.** *a.* 7; *b.* 8; *c.* $2058\frac{1}{6}$; *d.* $52\frac{7}{8}$. **23.** *b, c, e, f, h, i.*

Page 155. — **24.** *a.* $\frac{1}{4}$; *b.* $\frac{5}{8}$; *c.* $\frac{8}{5}$; *d.* $\frac{1}{10}$; *e.* $1\frac{1}{5}$; *f.* $\frac{2}{3}$; *g.* $\frac{1}{2}$; *h.* $\frac{5}{9}$; *i.* $\frac{1}{8}$. **26.** $1\frac{2}{9}$. **27.** *a.* .145, .005; *b.* 4.256, 1.256; *c.* 7.054, .946; *d.* 1.008, .392; *e.* .0325, .0175. **28.** *a.* .039, .036; *b.* .079, .076; *c.* .14225, .03225; *d.* 1.5775, 1.4225; *e.* 4, $2.66\frac{2}{3}$. **30.** .005. **31.** 1.25. **32.** 87.5. **33.** .07. **34.** 7.5. **35.** 88.2. **36.** .042. **37.** .05. **38.** 494. **39.** \$1152.

Page 156. — **40.** 11; 2. **41.** 1408; $59,136. **42.** the former by $10.60. **43.** $33,500,000,000. **44.** 40 in.; 12 in.; $9\frac{1}{4}$ in.; 8.3 in.; 3.55 in.; 1.2 in.; 2.35 in. **45.** 6¢. **46.** .0625. **47.** No; no; $\frac{1}{40}$ in.

Page 157. — **48.** $29{,}998\frac{1}{2}$ ft. **49.** .025. **50.** $\frac{5}{18}$; 6,596,389, correct to the nearest unit. **51.** $13.50; $4.12. **52.** 8 min. 19 sec. **53.** 13″; 13″. **54.** $\frac{17}{64}$ in. **55.** $2\frac{1}{8}$ hr. **56.** $1\frac{7}{16}$ in. **57.** The latter by 45¢, correct to nearest cent.

Page 158. — **58.** $9\frac{3}{8}$. **59.** 100.748 lb. **1.** .04, $\frac{1}{25}$; $.87\frac{1}{2}$, $\frac{7}{8}$; .05, $\frac{1}{20}$; $.06\frac{1}{4}$, $\frac{1}{16}$; $.14\frac{2}{7}$, $\frac{1}{7}$; $.08\frac{1}{3}$, $\frac{1}{12}$; $.16\frac{2}{3}$, $\frac{1}{6}$. **2.** Suggestion: Name $\frac{3}{8}$ of each amount. **3.** Suggestion: Name $\frac{7}{8}$ of each amount in problem 2, then $\frac{5}{8}$ of each, then $\frac{1}{16}$ of each. **4.** Suggestion: Name once, then 2×, then $2\frac{1}{2}$× each. **5.** .0075; .025; .075; .0325; .005; .048; .0365; .0645; .065; .0475; .035; .0675; .0075; .025. **6.** *a.* $1.50; *b.* 4 bu.; *c.* $51. **7.** *a.* $3.42; *b.* 15.6 in.; *c.* $18.25. **8.** *a.* $16.25; *b.* 11.4 mi.; *c.* $32.20. **9.** *a.* $37.50; *b.* 8%; *c.* $6. **10.** 50%, $\frac{1}{2}$ and .5; $\frac{1}{2}$% and $\frac{1}{200}$; 5%, $\frac{1}{20}$, and .05; $\frac{1}{5}$ and .20; $\frac{3}{4}$ and 75%.

Page 159. — **11.** $\frac{3}{4}$; 5. **14.** 2000. **15.** *a.* $5500; *b.* $16,600; *c.* $14,400; *d.* $321. **16.** *a.* $23,000; *b.* $200; *c.* $1500; *d.* $1200. **17.** $\frac{5}{8}$; $62\frac{1}{2}$%. **20.** 5.5%.

Page 160. — **21.** *a.* 6%; *b.* 6%; *c.* 4.5%. **22.** *a.* 4%; *b.* 4.8%; *c.* 5.25%. **26.** 5¢; $5; 25¢. **27.** 1%; $\frac{1}{2}$%; $\frac{1}{4}$%. **28.** 50%; 100%; 200%; 25%. **29.** 400%. **30.** 4.5%. **31.** $276. **32.** $1.83; $7.63. **33.** $8\frac{1}{3}$%; $50,000,000. **34.** 96%.

Page 161. — **35.** 50.815%. **36.** 30.9%. **37.** $3667.60. **38.** 75%. **39.** 9.03%. **40.** 200; 60; 1300. **41.** 8.16%; 51.424%. **42.** 8.80%. **43.** 40%.

Page 162. — **1.** $325.00. **2.** The latter by $1.25. **3.** $33\frac{1}{3}$%. **4.** $22\frac{2}{9}$%. **5.** $37\frac{1}{2}$%. **6.** $50. **7.** $226.38. **8.** July 31; $554.40; $546. **9.** The former by $10.80.

Page 163. — **1.** 25%; $33\frac{1}{3}$%. **2.** 24¢; 25%. **3.** $56\frac{1}{4}$%; 36%. **4.** $1.60. **5.** 20%; $16\frac{2}{3}$%. **6.** $22\frac{2}{9}$%. **7.** 100%. **8.** $44\frac{4}{9}$%. **9.** $9.90. **10.** Lost $30.

Page 164. — **1.** $18.32. **2.** $258.95. **3.** $1558.48. **4.** 5.55%. **5.** 8%. **6.** $800; $64; $200. **7.** $102.60; $138.

Page 165. — **1.** $2.50. **2.** $5.19; $4.33. **3.** $6.98; $7.75. **4.** $7.93; $5.41. **5.** $24.19; $27.72. **6.** $1.96; $2.12. **7.** $22.50. **8.** $31.50. **9.** $48.13. **10.** $105. **11.** $10.50. **12.** $16.66. **13.** $8.22. **14.** $23.12. **15.** $71.92. **16.** (1) 72; (2) 400; (3) .06; (4) 3.

Page 170. — **7.** $709.50. **8.** $445.81.

Page 174. — **1.** $6.09. **2.** $12.49. **3.** $16.49. **4.** $10.20. **5.** $125.51 **6.** $12.58. **7.** $338.23. **8.** $343.92. **9.** $346.86. **10.** $377.38. **11.** $133.46. **12.** $578.35. **13.** $5.03. **14.** $0.22.

Page 175. — **2.** $13.26; $27.06; $41.41; $56.34; $71.88.

Page 176. — **3.** $169.03; $281.72. **5.** $68.90. **6.** $137.80.

Page 177. — **1.** $208.52. **2.** $254.90. **3.** $384.08. **4.** $441.22.

Page 179. — **1.** $1, $2. **2.** $1-certificate and savings card with 4 stamps. **3.** $100. **4.** Dec. 1. **5.** $2500. **6.** Aug. 1; 2%; Aug. 1, next year; $0.50. **7.** Mar. 1; Mar. 1, next year; $1. **8.** Dec. 1 this year; $1.60. **9.** $1; see Sav-

ings Cards and Stamps, p. 178. **10.** $50, $20, $5, $2, $1. **11.** One $10, three $5, one $2, one $1; or ,one $10, two $5, three $2, two $1; or, four $5, three $2, two $1.

Page 182. — **2.** June 1; May 31; July 1; June 30.

Page 183. — **3.** Nov. 30. **4.** See p. 169. **5.** See Security, p. 182. **6.** $121.-80. **7.** $3. **8.** See (3), p. 182; $256.25.

Page 187. — **4.** Mar. 20 (Mar. 19 of a leap year.)

Page 189. — **1.** $588.10. **2.** $197.03. **3.** $247.50. **5.** $148.12. **6.** $497.-08. **7.** $196.97. **8.** $202.92. **9.** $355.08. **10.** $502.32. **11.** $594. **12.** $267.18

Page 192. — **1.** $357.48. **2.** $555.90. **3.** $65.73. **4.** $119.82. **5.** $169.60. **6.** $38.40.

Page 196. — **2.** $1795. **3.** $1493. **4.** $2035. **5.** $549.75. **6.** $1608. **7.** $3144. **8.** $2511. **9.** $867.50. **10.** $5067.50. **11.** $1780; $1487; $2027.50; $548.00; $1602. **12.** $798.75. **13.** $1072.50. **14.** $274. **15.** $200.

Page 197. — **16.** $100. **17.** $600. **20.** 4.82%. **21.** 5.69%. **22.** $1597.50.

Page 198. — **23.** 10,000; 5100. **24.** $2; 4%. **25.** 20. **26.** $30,000; $3; $225. **27.** 48; $4612.80. **28.** $60,000; 6%. **29.** $9730.

Page 201. — **1.** $1.75; $1.75.

Page 202. — **2.** June 15, 1932; June 15, 1947. **3.** $52.50; $105. **4.** $69,-630,944.25. **5.** Apr. 1, 1930. **6.** $1,750,000; $35,000,000. **8.** $976.25; $37.50. **9.** $375,000; $26.25.

Page 203. — **1.** $5; $60. **2.** $1440. **3.** $560.

Page 204. — **4.** $0.80. **5.** $14. **6.** $64. **7.** $512. **8.** $2000.

Page 207. — **8.** $72. **9.** $196.56. **10.** $616,000. **11.** $30,800. **12.** $0.95; $9.50; .95%. **13.** $1\frac{1}{4}$¢; $12.50; 1.25%. **14.** $12,500.

Page 208. — **15.** $35.10; 45%. **16.** 8 mills; $0.80; $8. **17.** 16 mills; $1.60; $16. **18.** 11 mills; $1.10; $11. **19.** 17 mills; $1.70; $17. **20.** $75.60. **21.** $7\frac{1}{2}$ mills; $75. **22.** $24.75. **23.** $0.74; Yes, by $0.37. **24.** $50.

Page 210. — **1.** $1,832,993,722. **2.** $537.60. **3.** $306.25. **4.** $6.48 **5.** $100.-80. **6.** $752.50. **7.** $400. **8.** $0.40. **9.** $2925. **10.** $825. **11.** $215.35. **12.** $2790. **13.** $214.20. **14.** $30.

Page 211. — **1.** $60. **2.** $40.

Page 212. — **3.** $20; none. **4.** $20. **5.** $0.18. **6.** $6.75. **7.** $264,104,-919.51. **8.** 2%.

Page 214. — **3.** $48.13. **4.** $4.04; $16.16. **5.** $60; $150; $240. **6.** $8.25. **7.** $0.75.

Page 215. — **8.** $96. **9.** $178.75. **10.** $482.50; $482.50. **11.** $89,448.67; $20,151.33. **12.** The former by $9.75. **13.** $68.85.

Page 216. — **14.** $30; $330. **15.** $1980; $39. **16.** $22,909.20; $343.64.

Page 218. — **1-8.** See Table p. 218.

Page 219. — **9.** $7.67. **10.** $242.30; 45 yr. **11.** $377.60. **12.** $83. **13.** $165. **14.** $2000. **1.** 20. **2.** $4538.75; $87.50.

113 =

Page 220. — **4.** 18 yr. **5.** \$8550. **6.** 2000. **7.** \$75. **8.** \$4,885,563.58. **9.** July 15, 1930; \$2,100,000; \$42,000,000. **10.** \$988.40. **11.** \$59,304,000.

Page 221. — **13.** \$20.19. **14.** $56\frac{1}{4}\%$. **15.** 8 mills; \$7.68. **16.** \$247.50. **17.** Loss \$400.60. **18.** \$34.20. **19.** 5 mills.

Page 222. — **20.** 4.3%. **21.** \$320. **22.** \$11,430.72. **23.** Decreased \$57. **24.** \$50. **25.** \$9330. **26.** $i = \frac{nrd}{360}$.

Page 223. — **27.** \$0.90; 9 mills; \$9. **28.** \$751.51. **29.** \$563.50. **32.** 6.09%, correct to nearest .01%.

Page 225. — **2.** 5. **3.** 2; none.

Page 226. — **8.** 50°. **9.** 50; 80. **11.** 60. **12.** 90; 70.

Page 227. — **13.** Acute. **14.** Obtuse. **15.** Right.

Page 228. — **18.** 12. **19.** 9.875. **20.** 10. **21.** 10.

Page 229. — **27.** $1\frac{1}{2}$ sq. in.; $2\frac{1}{2}$ in. **28.** 30 sq. in.; 13 in. **29.** 125 sq. ft. **30.** 160.

Page 230. — **3.** 30 ft. **4.** 360°. **5.** Equal. **6.** Square.

Page 231. — **10.** 8 in.; 12 in. **12.** 80. **13.** 78.125. **14.** 5. **15.** 8.

Page 232. — **18.** 54. **19.** 118.5. **20.** 4. **21.** 30. **23.** Yes; 608 sq. ft.

Page 235. — **14.** 259.8 sq. ft. **15.** 482.8 sq. in. **16.** 27.5 sq. yd. **17.** 1039.2 sq. ft.

Page 236. — **1.** $C = 2\pi R$. **2.** 3.1416 in.; 37.6992 in.; 7.854 ft.; 12.5664 yd.; 125.664 ft.; 3.1416 mi. **3.** $C = \pi D$. **4.** 1.5708 in.; 18.8496 in.; 3.927 ft. 6.2832 yd.; 62.832 ft.; 1.5708 mi. **5.** $\frac{1}{4}$ as great. **6.** Yes. **7.** $3\frac{1}{2}$ in. (using $\pi = 3\frac{1}{7}$). **8.** 295, correct to nearest unit.

Page 237. — **9.** 210.

Page 238. — **7.** $R = \frac{C}{2\pi}$. **8.** $R = \frac{S}{\frac{1}{2}C}$. **9.** $C = \frac{S}{\frac{1}{2}R}$. **11.** $C = \pi D$. **12.** C and R; R; (4). **13.** See (1), page 238; see (2), page 238; yes, see (4) page 238; yes, $R = \frac{C}{2\pi}$.

Page 239. — **14.** $S = \pi R^2$. **1.** 113.0976 sq. in. **2.** 50.2656 sq. yd. **3.** 452.3904 sq. rd. **4.** 12.5664 sq. mi. **5.** 19.635 sq. ft. **6.** 78.54 sq. yd. **7.** 19.635 sq. mi. **8.** 32.169984 sq. ft. **9.** 5.9395875 sq. rd. **10.** 27.339774 sq. mi. **11.** 28.2744 sq. ft. **12.** 78.54 sq. rd. **13.** 0.7854 sq. in. **14.** 12.5664 sq. in. **15.** 31,416 sq. yd. **16.** 1963.5 sq. ft. **17.** 32 ft.; 64 sq. ft.

Page 240. — **18.** 10.3008 ft. **19.** 21.46 sq. ft. **21.** Use a cord 50 ft. long, also see problem 17, p. 256; 100 ft. **22.** 105.0. **23.** $2\frac{1}{4}$ in. **24.** \$117.81. **25.** 420.

Page 241. — **1.** 6.25; $\frac{1}{4}$; $7\frac{1}{9}$; 0.49; 0.0049; 0.7569; 2025.9001; 0.000025; 0.000484; 9.006001.

Page 242. — **6.** *a.* 18; *b.* 21; *c.* 24; *d.* 28; *e.* 36; *f.* 42; *g.* 43. **7.** *a.* 45; *b.* 54; *c.* 63; *d.* 66; *e.* 34; *f.* 26; *g.* 33. **8.** 100; 121; 10; 11. **9.** 12; 14; 15; 16; 16; 17; 17; 19.

Page 243. — **11.** 32. **12.** 35. **13.** 86. **14.** 45. **15.** 61. **16.** 71. **17.** 91.

18. 57. **19.** 95. **20.** 58. **21.** 88. **22.** 99. **24.** 7.3. **25.** 4.3. **26.** 9.7. **27.** 7.2.

Page **244.** — **28.** 5.66. **29.** 17.92. **30.** 1.85. **31.** 0.15. **32.** $\frac{11}{12}$. **33.** $\frac{16}{27}$. **34.** $2\frac{2}{3}$. **35.** $\frac{88}{84}$. **36.** .8165, correct to nearest .0001.

Page **245.** — **4.** 10. **5.** 15. **6.** 15. **7.** 20. **8.** 13. **9.** 27. **11.** 38.99 ft., correct to nearest .01 ft.

Page **246.** — **12.** Right triangle; 5. **13.** 15 in. **14.** 2.83. **15.** 30 ft. **16.** 28.28 ft., correct to nearest .01 ft. **17.** 40. **18.** 127.28 ft., correct to the nearest .01 ft. **19.** 10.

Page **247.** — **20.** $\overline{25^2} = \overline{15^2} + \overline{20^2}$. **21.** 10; 20; 62.832. **22.** 5; 10; 31.416. **23.** 1243.2. **24.** $\frac{5}{8}$. **25.** 23.63 ft., correct to nearest .01 ft.

Page **248.** — **27.** 20 in. **28.** 120 ft. **29.** 120.42 ft., correct to nearest .01 ft.

Page **249.** — **1.** 4. **2.** 5. **3.** 6. **4.** 7. **5.** 9. **6.** 12. **7.** 16. **8.** 14. **9.** 15. **10.** 18. **11.** 16 ft.; 12 ft.; 11 yd.

Page **252.** — **3.** 96 sq. ft. **4.** 96 sq. ft. **5.** 301.5936 sq. in. **6.** $\frac{3}{8}$.

Page **254.** — **3.** 96 cu. ft. **4.** 2035.7568 cu. in. **5.** 1269 gal. **6.** 5 ft. **7.** 7284.585 lb. **8.** 10 ft. **9.** 1692 gal. **10.** 10 ft.

Page **255.** — **12.** 3392.928. **13.** 43.85 lb., correct to nearest .01 lb. **14.** 9160.9056. **15.** 196.0434, correct to nearest .0001.

Page **256.** — **16.** 218. **17.** 1526.8176.

Page **258.** — **3.** 300 sq. in. **4.** 4.7124 sq. ft. **5.** 5 ft.; 60 sq. ft. **6.** 80 sq. ft.

Page **259.** — **7.** 8 in.; 188.496 sq. in. **8.** 47.124. **9.** \$33.51. **10.** $53\frac{1}{3}$. **11.** 907,136 sq. ft.

Page **260.** — **1.** 187.06 cu. yd. **2.** 56.5488 cu. in. **3.** 108 cu. in. **4.** 84.8232. **5.** 48 cu. ft. **6.** 251.328 sq. in.; 402.1248 cu. in. **7.** 1357.1712 cu. in. **8.** 960 sq. ft.; 3072 cu. ft. **9.** 1570.8 sq. ft.; 6283.2 cu. ft.

Page **262.** — **1.** 50.2656 sq in.; 33 5104 cu. in. **2.** 201.0624 sq. ft. **3.** 128.679936 sq. in. **4.** 452.3904 sq. yd. **5.** 254.4696 sq. ft. **6.** 4.1888 cu. in. **7.** 14.1372 cu. in. **8.** 523.6 cu. ft. **9.** .5236 cu. ft. **10.** 50.2656 sq. ft.; 33.5104 cu. ft. **11.** 3 ft.; 113.0976 cu. ft. **12.** 201,062,400 sq. mi.; 268,083,200,000 cu. mi.

13. Sphere $\begin{cases}\text{Volume 14.1372 cu. in.}\\ \text{Surface 28.2744 sq. in.;}\end{cases}$ Cylinder $\begin{cases}\text{Volume 21.2058 cu. in.}\\ \text{Surface 28.2744 sq. in.;}\end{cases}$

The sphere = $\frac{2}{3}$ the cylinder; Surfaces are equal.

Page **264.** — **2.** 3; no. **3.** 18. **4.** $1\frac{1}{4}$; $\frac{3}{4}$. **6.** $\frac{15}{16}$ in.

Page **265.** — **1.** \$87.13. **2.** \$464. **3.** \$239.40. **4.** \$236.10. **5.** \$699.84. **6.** \$175.41. **7.** \$226.

Page **266.** — **8.** \$947.20. **9.** \$668.61. **10.** \$2042. **11.** \$5785.69. **12.** \$6985.69.

Page **267.** — **1.** \$7.80. **2.** No. 3, between 4 and 5, No. 2, between 1 and

276 / 4260 (42.
1104
1560

2, No. 1, between 8 and 9; 5800; \$7.54. **3.** No. 3, between 6 and 7; No. 2 between 8 and 9; No. 1, between 3 and 4. **4.** No. 3, between 7 and 8; No. 2 between 1 and 2; No. 1 between 2 and 3. **5.** No. 3, between 7 and 8; No. 2, between 6 and 7; No. 1, between 1 and 2. **6.** \$3.77; \$6.37.

Page 268. — **8.** \$5.73.

Page 269. — **2.** \$4.29. **3.** 1.1; 14.3¢. **4.** .5; 6.5¢. **5.** 25%. **6.** 15.6¢. **7.** \$1.056. **8.** 12¢.

Page 270. — **1.** 5.6°; 3.8°; 0.6°; 3°. **2.** 5.2°; 5°.

Page 271. — **4.** 104.4°; 2–13, 8 P.M.; 102.6°, 2–10, 8 A.M.; 5.8°; 4°.

Page 274. — **6.** 2.007^{Km}; 2.00706^{Km}; 7.568^{Km}; 0.3568^{Km}. **7.** 2.116^{m}; 2.75^{m}; 0.857^{m}; 1426^{m}. **8.** 2.7^{cm}; 27^{mm}; 860^{cm}, 8600^{mm}; 127^{cm}; 1270^{mm}; 101.4^{cm}, 1014^{mm}. **9.** 3.937 in. **10.** 0.0063 in. **13.** $\frac{5}{8}$. **14.** 1^{dm} is .063 in. less than 4 in.

Page 275. — **3.** $825^{sq\ mm}$. **4.** 160; 1.6; 4.

Page 276. — **5.** 0.4. **6.** 100. **7.** 6.5. **8.** 12.12.

Page 277. — **3.** 125; 125,000. **4.** 41.16; 41,160; 0.04116. **5.** 125.664.

Page 278. — **4.** A dekaliter; 1.08 qt. **5.** 125; 1000. **6.** 80.

Page 279. — **4.** \$100,000. **5.** 12.5^{g}. **7.** 1.03^{kg}. **8.** 20.94^{g}. **9.** 7920^{mg} **10.** 40.

Page 280. — **1.** 30. **2.** 0.6. **3.** 16 rd.; 72 rd. **4.** \$235.125. **5.** 2310 cu. ft.; 770 sq. ft. **6.** 120 sq. ft.; 60 cu. ft. **7.** 5184 sq. ft. **8.** 512 sq. in. **9.** \$23. **10.** $26\frac{3}{8}$ bu. **11.** 3 in.; 27 cu. in.

Page 281. — **12.** 93.5 sq. ft. **13.** $56\frac{1}{4}$ sq. ft. **14.** 28.2744. **15.** 266. **16.** 46.875 lb. **17.** 32.725. **18.** 2981.25 lb. **19.** 16 ft. **20.** \$27. **21.** 67.0208 **22.** 2827.44 sq. ft. **23.** 34 ft. **24.** 247, correct to the nearest unit.

Page 282. — **25.** 32. **26.** 478.8, correct to nearest .1. **27.** 96. **28.** 34.9 sq. yd., correct to nearest .1 sq. yd. **29.** \$889.125. **30.** 115.5 yd. **31.** 29.92. **32.** \$99.07. **33.** 19.14 ft. **34.** $1\frac{1}{80}$. **35.** 3672; \$587.52.

Page 285. — **13.** 6.2832; 3.1416. **14.** $\frac{8}{40}$. **15.** 1.09. **16.** 5 : 1; 5. **17.** Mrs. W., \$30; Samuel, \$15. **18.** $\frac{1}{4}$. **19.** $\frac{1}{4}$. **20.** $\frac{1}{3}$. **22.** $\frac{1}{2}$; $\frac{1}{2}$. **23.** 50.2656, 201.0624; $\frac{1}{4}$.

Page 286. — **24.** $\frac{R^2}{r^2}$. **25.** 3. **26.** 3. **27.** $\frac{a}{h}$. **28.** $2\frac{1}{4}$. **29.** $\frac{ab}{wl}$. **30.** 2. **31.** $\frac{a}{h}$. **32.** 5. **33.** $\frac{b}{l}$. **34.** $\frac{ab}{wl}$. **35.** $\frac{4}{9}$.

Page 287. — **36.** $\frac{1}{8}$. **37.** Yes; see principle, page 19. **38.** Yes; see principle, page 19. **1.** \$25; \$20. **2.** \$160; \$200. **3.** 800, 800, 400 lb. **4.** \$20, \$10. **5.** 2 lb., 3 lb.

Page 288. — **6.** 20. **7.** \$1.60, \$2.40, \$2. **8.** \$39.60, \$66, \$79.20. **9.** 120, 40, 60, 100, 40. **10.** 320, 400. **11.** 160, 240, 320. **12.** 540, 420.

Page 290. — **1.** 9. **2.** 18. **3.** 4. **4.** 15. **5.** 18. **6.** 22. **7.** 5. **8.** 3. **9.** $2\frac{1}{2}$. **10.** 6. **11.** 2. **12.** 162. **13.** 25. **14.** 6. **15.** $1\frac{1}{5}$. **16.** 25. **17.** 25.

18. $14\frac{1}{16}$. **19.** 8. **20.** 6. **21.** 5. **22.** 24. **23.** $87\frac{1}{2}$. **24.** $\frac{1}{16}$. **25.** 2. **26.** 40. **27.** 4.

Page 293. — **1.** (1) directly; (2) indirectly; (3) directly; (4) indirectly; (5) indirectly; (6) directly; (7) indirectly. **2.** 29.82 lb. **3.** 64. **4.** $17.50. **5.** 12 ft. **6.** $22\frac{9}{28}$.

Page 294. — **7.** 50. **8.** $7\frac{1}{2}$¢. **9.** 225. **10.** 4 hr. **11.** 2.25 in.; 4 in. **12.** 28. **13.** 12 lb.

Page 297. — **1.** 15 ft. **2.** 57.6. **3.** 62.4. **4.** 1 : 2; 1 : 4. **5.** 8. **6.** 4.536. **7.** 64 lb. **8.** 42 ft.

Page 298. — **9.** $49\frac{8}{27}$. **10.** 960 gal. **11.** 1,404,928. **12.** 72 ft.

Page 299. — **13.** 2 ft.; 10 in. **14.** $25\frac{5}{7}$ ft.

Page 302. — **1.** See page 300; see Table page 302. **2.** 18%. **3.** For beef total = 18.4% + 18.5% + .9%, or 37.8%; check, 62.2% + 37.8% = 100%. For the other commodities, find the totals and check in the same way. **4.** 280 C; 363 C; 116.5 C; 160 C; 116 C. **5.** Protein 0.4 oz., 46.4 C; fat, 0.752 oz., 198.528 C; carbohydrates, 11.952 oz., 1386.432 C; 1631.36 C.

Page 304. — **12.** $\frac{1}{30}$¢. **13.** $675,000,000. **15.** $5.07, correct to nearest cent. **17.** 4.7. **18.** 4800.

Page 305. — **20.** 56 ft.; 20 ft. **21.** 132. **22.** 17,010. **23.** $\frac{1}{8}$. **24.** 12 in.; 4 sq. in. **25.** 60¢. **26.** 1.25. **27.** 1000 cu. in.; 600 sq. in.; 17.3 in. ~~**28.** 180.~~ ~~**29.** 256.~~ **31.** $87\frac{1}{2}$%.

Page 306. — **32.** $8.75. **33.** 10. **34.** 150, 45, 30. **35.** 25π; 6. **36.** $12; 25¢. **37.** 5 mills. **38.** 20%; $1.10. **39.** 3. **40.** 50%. **41.** $84.

Page 307. — **42.** $14.97. **43.** 432 sq. ft.; 648 cu. ft. **45.** $485.80. **46.** 5.85%. **47.** 2%. **48.** 62; $9.95. **49.** $8.46. **50.** a, 1; b, 1; c, 63; d, $\frac{24}{R}$. **51.** 231; 53; 7.4.

Page 308. — **52.** 363. **53.** 20. **54.** 360; $39.60; $396. **55.** $2160; $252. **56.** $6. **57.** $297.85, if the rate of discount is 6%; $297.80, if the day of discount is counted. See Table, page 188. **58.** $706.86. **59.** $318.36. **60.** 5%.

Page 309. — **61.** $26.79, correct to nearest $0.01. **62.** 84%; 45.65%, correct to nearest .01%. **63.** $4.40; $1.70; $1.30; $1.20; $0.60; $0.50; $0.30. **64.** In the order named in problem 63, 44%; 17%; 13%; 12%; 6%; 5%; 3%. **65.** 10 rd. **66.** 5050.

.148
200

.8 50000
88
.2 0

43

432

7 8 9 2 1 6 | 8 8 8 . 5 5 6
6 4

1 6 8) 1 4 4 2
1 3 4 4

1 7 6 5) 9 8 1 6
8 8 2 5

1 7 7 0 5) 9 8 5 0 0
8 8 5 2 5

1 7 7 1 0 6) 1 0 9 7 5 0 0
1 0 6 2 6 3 6

326
80
341
10

.1040
50
80

625) 52 83
2500

77
21

77
154

1617
33

4851
4851

R. 155.12
R. 215.12

√5 33 61 | 231
4
43) 133
129
461) 461
461

√2805 | 53
25
103) 305
305

54.75 | 7.4
49
144) 576
576

R. 196 - 80 12

2.0

= . 1260

3

162

X 44.5 X 18 = 81.20

75

532 @ .06 for 135 =

$\frac{50}{1} \times \frac{1}{100} \div \frac{1}{5}$ ~~922~~ × .06 × 135 / 365

305. 24.

274 / 83

150

509 – 207

1.26 gain

60 mo

.50 yr =

243 .07 / 365 × 135 / 1

.243

.06

21.87

1.26 / 1.50

28 lbs × 14 = 348 yr

150) 1348 (8

1200

166

8 = 2

4) 309 (34

27

39

36

24

14

448

32) 448 (14

32

128

128

225

.40

22.00

65

110.0

5.85

..............................Book No.

TEXAS BOOK LABEL

This book is the property of the State of Texas

INSIDE BACK COVER

..............................School. ....Co.

Issued to:		CONDITION OF BOOK Issued	Returned
..............................,	192.......	....	....
..............................,	192.......	....	....
..............................,	192.......	....	....
..............................,	192.......	....	....
..............................,	192.......	....	....

TEXAS EDITION

THE PRICE MARKED HEREON IS FIXED BY THE STATE AND ANY DEVIATION THEREFROM SHOULD BE REPORTED TO THE STATE SUPERINTENDENT OF PUBLIC INSTRUCTION.

CONTRACT PRICE - 68 CENTS
EXCHANGE PRICE - 64 CENTS